Revised WI Model Academic Standards for Mathematics – Grades 9-12

PK-12 Standard: Concepts and Connections in Number and Algebra
Grades 9-12: Algebraic Reasoning *(Algebra, Number and Operations, Data Analysis)*

Learning Priority 1: Reason, communicate, and compute with numeric and algebraic expressions and equations
Learning Priority 2: Understand functions and their representations, properties, and applications
Learning Priority 3: Understand, recognize, and justify solutions to problems from given algebraic applications
Learning Priority 4: Interpret and create algebraic models to make predictions and evaluate decisions

PK-12 Standard: Concepts and Connections in Geometry and Measurement
Grades 9-12: Geometric Reasoning *(Geometry, Measurement, Number and Operations, Algebra)*

Learning Priority 1: Select, apply and understand measuring tools and procedures
Learning Priority 2: Recognize attributes and properties of common geometric figures, and the relationships between them
Learning Priority 3: Understand the representation of geometric facts, figures, and transformations on a coordinate plane
Learning Priority 4: Understand the representation and application of geometric models to real-world situations

PK-12 Standard: Concepts and Connections in Data Analysis and Probability
Grades 9-12: Data-Driven Reasoning *(Data Analysis, Probability, Number and Operations, Algebra)*

Learning Priority 1: Organize, display, and compare both quantitative and qualitative data to make, justify, and summarize conjectures
Learning Priority 2: Develop, analyze, and justify inferences based on data
Learning Priority 3: Evaluate and derive models based on data to analyze and predict outcomes

Draft…. for discussion and comment

Standards are generally designed to articulate what mathematics students should learn and a relative timeline of when they should learn these expectations. A discipline as expansive and as important as mathematics cannot be easily organized in a format that captures the goals and expectations of all mathematics educators and the needs of all mathematics learners. A challenging balancing act is necessary to identify the key expectations of mathematics within the timeline of a student's education that does not become an unwieldy check-off list of mathematics topics on one hand or a barebones list of minimal expectations on the other hand. Defining what mathematics expectations are important, why these expectations are important, and when these mathematics expectations should be learned represent challenges that are continually examined by the mathematics community.

The format used to define the standards presented in this document is designed to reflect several points that have been learned through previous work with standards. The first important point is that students do not learn a stated standard in isolation. The format used here attempts to articulate key learning expectations as a process of learning, or a *continuum of learning*. At the high school level, the first stage identifies a possible starting point for the process. Students may or may not be at this stage upon entry to high school, however, the continuum is designed to identify how a key learning expectation emerges from the roots of a student's earlier mathematics learning. The subsequent stages move through a student's learning to reflect a progression of understanding of the mathematics articulated in the continuum. In the same way that the first stage is not necessarily the beginning, the last stage identified does not represent an end point, but rather a point at which the next level of learning would begin if the continuum were expanded. In this way, a standard is clarified and expanded within a timeline that attempts to identify the variability of learning that is more reflective of how students learn. Judgments were made both in identifying the key mathematics expectations and the continuum designed. Certainly other points along the continuum could have been identified, however, this should present a range of expectations that provides some needed flexibility in developing the mathematics learning of students.

A second major point is that standards are not intended to be organized in courses, units, or years. The continuum does not represent a delineation of courses, but rather a structure of mathematics learning that is independent of any particular course structure. Courses should be organized around good mathematics and reflect the progression of learning as defined by the continuum within the courses. The points along the continuum can be packaged into courses in multiple ways by different districts all of which will reveal this good mathematics.

Most statements in the continuum represent what all students completing a high school program should learn in order to be prepared for the next steps in starting a college or university program, a technical or vocational program, or general employment. The continuum is organized around *focus statements* that explain the more general levels of reasoning identified as algebraic reasoning, geometric reasoning, and data-driven reasoning. Remember, the continuum is a snapshot of what should define high school learning in mathematics. **Some of the expectations in the continuum are shaded light grey, reflecting that these learning expectations <u>are not part of what all students</u> will be able to accomplish before completing high school.** These more "advanced" expectations help define the continuum of learning for students completing more opportunities in mathematics.

Grades 9-12: Algebraic Reasoning *(Algebra, Number and Operations, Data Analysis)*

As a result of their middle school mathematics experiences, students should have acquired an understanding of rational numbers, and been exposed to the real number system. They should understand the distinction between an arithmetic operation and the algorithms used to perform that operation on particular classes of numbers, and realize that the underlying operation is the same no matter what type of number is involved. They should understand, at least informally, that once the number system has been sufficiently extended there are only two fundamental operations, rather than four: that subtraction and division can now be understood as the inverse operations of addition and multiplication. Although they may not have begun the formal study of algebra, they should enter their first algebra course with an understanding of the basic properties of the arithmetic operations, and with well-developed skills in algebraic reasoning and pattern recognition.

The formal study of algebra builds on these foundations, deepening understanding by adding a layer of abstraction while always returning to concrete examples. Students who recognize that the operations and their properties are the same for all numbers, and that "*x* is just a number" will find the transition from arithmetic to algebra much easier; students who see the relationship between algebraic identities and mental arithmetic will also find it meaningful and useful.

The shift from numbers to using letters or other symbols results in an increased emphasis on the *relationships* between quantities, rather than the quantities themselves. The study of numerical patterns and relationships is largely formalized in the concept of a function. Students need to work with several different representations of functions flexibly, translating between representations efficiently and accurately. The rate of change of one quantity with respect to another is a particularly important topic, which should be connected to the middle school topic of proportional reasoning.

Through their study of algebra, students should come to realize the power of abstraction: working with letters or other symbols means that we do not have to solve a problem from scratch every time the numbers or data in the problem change; it also allows us to see the same mathematical structures underlying applications that may appear very different on the surface. The study of algebra in high school should prepare students to become independent algebraic thinkers, seeing and analyzing the algebraic structures in problem situations they encounter in their future lives.

Grades 9-12 Learning Continuum

Learning Priority	Focus Areas	Stage 1	Stage 2	Stage 3	Stage 4
Learning Priority 1: Reason, communicate, and compute with numeric and algebraic expressions and equations.	**Number and Operations**	Compare real numbers, indicating their relative position on a number line, using order relations (<,>), transitivity, and equivalent forms of numbers (i.e. fractional form, decimals, percents, scientific notation, radicals, exponential, absolute value)	Simplify and generate numerical expressions using *properties* (commutative, distributive, associative) and order of operations and the use of grouping symbols Derive the meaning of negative integer exponents by extending patterns and properties involving whole-number exponents	Derive the properties of integer exponents and apply them to numerical and algebraic expressions, including multiplication and division of numbers written in scientific notation Derive the meaning of rational exponents by extending patterns and properties involving integer exponents Evaluate, simplify, and generate equivalent algebraic expressions including absolute value, polynomial, and rational, expressions	Derive the properties of rational exponents and radicals and apply them to numerical and algebraic expressions Analyze and apply the properties of complex numbers Relate complex numbers to the coordinate plane (or find its polar form)

Learning Priority	Focus Areas	Stage 1	Stage 2	Stage 3	Stage 4
Learning Priority 1: Reason, communicate, and compute with numeric and algebraic expressions and equations.	**Analysis of Algebraic Expressions and Equations**	Analyze patterns, identify relationships, and represent them using algebraic expressions, linear functions, arithmetic sequences and series, and equations with and without real-world applications, including the use of spreadsheets	Analyze patterns, identify relationships, and represent them using exponential functions, finite geometric sequences and series and equations with and without real-world applications	Analyze patterns, identify relationships, and represent them using quadratic and higher power polynomial functions and equations with and without real-world applications	Analyze patterns, identify relationships, and represent them using rational function and equations with and without real-world applications
	Solving Equations and Inequalities	Solve linear equations and inequalities numerically, graphically, including the use of appropriate technology, and symbolically	Solve and understand solutions of systems of linear equations and inequalities numerically symbolically, and graphically, including the use of appropriate technology	Solve quadratic equations and inequalities symbolically (factoring, completing the square, quadratic formula including its derivation) and graphically including the use of appropriate technology	Solve systems of 3 variables symbolically and through the use of technology Solve equations involving rational and polynomial expressions Solve linear absolute value equations and inequalities graphically and using the definition of absolute value
		Solve exponential equations numerically and graphically, including the use of appropriate technology	Solve exponential equations symbolically including the use of logarithms	Solve equations involving logarithmic or trigonometric functions	

Grades 9-12 Learning Continuum

Learning Priority	Focus Areas	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 2: Understand functions and their representations, properties and applications	**Representation of Functions**	Explain whether a relation given in symbolic, graphical or tabular form is a function	Identify and summarize properties of different types of functions from their graphs including linear, quadratic, exponential, absolute value, sine and cosine trigonometric functions	Create and interpret different representations including numerical, graphical, and functional notation of the functions in Stage 2 Analyze and communicate, with and without the use of technology, the effect of transformations on the graphical representations of different types of functions	Identify, summarize, and interpret properties and create different representations of polynomial, step, and other trigonometric functions, including an analysis of even and odd functions
	Basics of Functions	Evaluate a function at a specified point in the domain Understand the concepts of domain and range of functions and determine the domain	Apply arithmetic operations to functions and determine the domain of the resulting function	Determine the range of the function Determine and understand the composition of functions Understand the meaning of the inverse of a function and determine whether a given function has an inverse	Determine the inverse of a function Prove basic properties of logarithms using properties of exponential functions

Grades 9-12 Learning Continuum

Learning Priority	Focus Area	Stage 1	Stage 2:	Stage 3:	Stage 4:
Learning Priority 3: Understand, recognize, and justify solutions to problems from given algebraic applications	**Solving Application Problems**	Create and critically evaluate numerical arguments presented in a variety of classroom and real-world situations	Recognize and solve problems that can be modeled using a linear equation or inequality in one variable, including percentage, ratio, proportion and time/rate/distance problems	Recognize, solve, and justify solutions to problems that can be modeled using a system of two linear equations or inequalities in two variables, including mixture problems	Recognize, solve, and justify solutions to problems that can be modeled using a system of at least three variables
	Analysis of Algebraic Models	Read, interpret, and draw conclusions from the graphical representation of linear equations or inequalities which model real-world applications such as time/rate/distance and break-even point	Read, interpret and draw conclusions from geometric sequences and the graphs of exponential functions, including real-world applications such as population growth or decay and compound interest	Recognize, solve, and justify solutions for problems that can be modeled by a quadratic equation such as the motion of an object under the force of gravity Recognize and solve problems that can be modeled using a finite geometric series, such as home mortgage problems	Recognize, solve, and justify solutions for problems that can be modeled using an exponential function, but whose solution requires facility with logarithms, including growth and decay problems Recognize, solve, and interpret periodic problems that can be modeled from trigonometric functions such as harmonic motion

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 3: Understand, recognize, and justify solutions to problems from given algebraic applications	**Applications of Functions**	Given a real-world situation, determine if a function can model the situation	Given a real-world situation, justify which type of function can model the situation (i.e. linear, quadratic, exponential, periodic, step)	Determine and apply linear, quadratic or exponential functions that model real-world situations, distinguishing relevant from irrelevant information in selecting an appropriate model, and identifying, finding, or estimating missing information Determine the domain of the model	Determine and apply logarithmic, trigonometric, and polynomial functions to solve and interpret real-world situations, distinguishing relevant from irrelevant information in selecting an appropriate model, and identifying, finding, or estimating missing information

Grades 9-12 Learning Continuum

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 4: Interpret and create algebraic models to make predictions and evaluate decisions	**Interpreting and Deriving Algebraic Models**	Extract information from an algebraic model that is used to represent a real-world or applied situation	Write a prediction equation from a regression line or a similar line of best fit which is derived from a scatterplot Make predictions from a linear model Recognize and analyze growth patterns and sequences (both algebraic and geometric) that can be represented as an algebraic model	Identify and interpret features that are used to select and design an algebraic model (i.e. maximum/minimum values, increasing and decreasing trends, and linear, exponential, and quadratic characteristics) Interpret and derive decisions based on developing an algebraic model representing a recursive application	Analyze and explain decisions based on a representation of an application using a logistic model Analyze and explain decisions based on a representation of an application based on logarithmic or periodic algebraic models

Grades 9-12: Geometric Reasoning *(Geometry, Measurement, Number and Operations, Algebra)*

Geometry provides opportunities for students to view mathematics from different perspectives. It initiates their reasoning to solve problems using more visual representations, and opens the door to developing the deductive reasoning that provides a structure to their understanding of proof. The continuum of learning for geometry must allow our students to engage in mathematical thinking as they make, test, and prove their conjectures. It should allow them opportunities to view geometry from several perspectives, including transformations and coordinate systems. Providing these opportunities expands and deepens the background necessary for a correspondingly deep study of topics in advanced mathematics that builds on students' algebraic reasoning and the physical world around them.

The Learning Continuum in Geometric Reasoning is designed to prepare students to continuously engage in a problem-solving setting, routinely making connections to algebraic ideas. Students should be involved in applying the visual connections of proportional thinking to the shapes that are part of the world around them. Geometric reasoning should incorporate the deductive thinking that connects the visual part of their thinking with the more algebraic reasoning they are also learning.

Geometry is able to provide students concrete experiences that stimulate their curiosity. Through these experiences, they will learn more than in many other mathematical situations that their thinking is part of "doing mathematics." The Learning Continuum provides a developmental progression that integrates the tools and perspectives of geometric reasoning so that students will be able to apply and reason their visual representations as a significant component of their mathematical reasoning.

Grades 9-12 Learning Continuum

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 1: Select, apply and understand measuring tools and procedures	**Measures as Estimates**	Understand the meaning of perimeter, area, and volume in terms of the number of units needed to surround, cover, or fill a figure	Understand that measurement involves comparison with a chosen unit Recognize that all measurements are approximations	Determine measurements by estimation and use the estimate to judge the accuracy of a calculation	
	Theory and Practice of Measurement	Select and use tools with appropriate degree of precision to determine measurements directly within specified degrees of accuracy and error (tolerance)	Determine measurements by converting measures within a system, or by relating measures in one system to another (e.g. meters to feet, square feet to square inches, dollars to Euros)	Determine measurements by correctly applying a unit of measure in expressions, equations, or solutions to problems involving linearity, area, and volume	Routinely assess the acceptable limits of error when evaluating strategies, testing the reasonableness of results, and using technology to carry out computations

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 1: Select, apply and understand measuring tools and procedures	**Measurement of Lengths, Areas and Volumes**	Use decomposition to derive lengths and areas of two-dimensional figures, including composite figures Determine arc lengths and areas of sectors of circles	Use decomposition to derive surface area and volume of three-dimensional figures Recognize and use the relationships among the different attributes of a three-dimensional figure, such as explaining how to find the volume of a prism using its height and base area, or why the volume of a cone is 1/3 that of a prism with the same height and base area	Apply concepts of similarity to determine measures of length, area, and volume Recognize that trigonometric ratios remain constant in similar right triangles Use the definition of sine, cosine, and tangent as ratios of sides in a right triangle to solve problems about length	Describe the relationship between the volume of a sphere and the volumes of a cylinder and cone whose height and base radius are equal to the radius of the sphere, and use this relationship to derive the volume of the sphere Use the law of cosines and law of sines to find the lengths of sides of a triangle

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 1: Select, apply and understand measuring tools and procedures	**Measures of Angles**	Solve problems involving angle measures, including angles formed by parallel lines which are intersected by a transversal, the interior and exterior angles of regular and non-regular polygons, and arc and angle measures in a circle	Apply concepts of similarity to determine angle measures	Use the definition of sine, cosine, and tangent as ratios of sides in a right triangle to solve problems involving measures of angles Use the Pythagorean Theorem to derive the exact values for sine, cosine, and tangent in 30°-60°-90° and 45°-45°-90° triangles	Use the law of cosines and law of sines to find the angle measures of a triangle Determine the radian measure of an angle and explain how it is related to the unit circle Derive the exact values of the six trigonometric functions of standard angles in a unit circle (e.g. 30°, 45°, 60°, 120°, 420°)

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 2: Recognize attributes and properties of common geometric figures, and the relationships between them	**Reasoning and Proof**	Determine the role of the hypothesis, logical implications, and conclusion in the proof of a geometric theorem Develop and test geometric conjectures based upon an appropriate set of examples	Understand the difference between examples that support a conjecture and the proof of a conjecture Justify conjectures and conclusions based on previously known definitions, properties, postulates, or theorems	Communicate logical arguments to show the truth of a statement and explain clearly why the reasoning is valid Recognize that proofs can be written in different ways, and that the essence of a proof is not its form but its substance Analyze non-routine problems and the mathematical thinking and strategies of others	Communicate logical arguments, including direct proof and proof by contradiction, to show the truth of theorems and explain clearly why the reasoning is valid Understand the importance of auxiliary lines in many geometric proofs and explain common procedures for choosing these lines

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 2: Recognize attributes and properties of common geometric figures, and the relationships between them	**Rectilinear Figures**	Develop and test conjectures about segments, parallel and perpendicular lines, angles, and regular and non-regular polygons, by means of demonstration, constructions, counter-examples or dynamic software	Communicate logical arguments to show the truth of fundamental theorems related to parallel and perpendicular lines, angles, triangles, and quadrilaterals (e.g. Pythagorean Theorem; Sum of the angles in a triangle equals 180º.)	Apply the properties of quadrilaterals and other polygons to solve problems Apply the Pythagorean Theorem and its converse to solve problems	Understand that $\sin^2 x + \cos^2 x = 1$ is a special representation of the Pythagorean Theorem Derive other trigonometric identities, including the addition formulas for the sine, cosine and tangent functions, and the laws of sine and cosine
	Circles	Develop and test conjectures about the angles and segments in a circle, by means of demonstration, constructions, counter-examples, or dynamic software	Apply the definition and properties of circles (e.g. an angle inscribed in a semicircle is a right angle; a tangent to a circle is perpendicular to the radius through the point of tangency) to solve problems	Communicate logical arguments to show the truth of theorems pertaining to circles and explain clearly why the reasoning is valid	

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 2: Recognize attributes and properties of common geometric figures, and the relationships between them	**Congruence**	Analyze geometric figures and determine if they are related by rigid motion	Understand and apply the definition of congruence in terms of rigid motion Develop and test conjectures about the congruence of angles, line segments, and triangles by demonstration, constructions, counter-examples, or dynamic software	Apply congruence postulates and theorems to solve problems	Communicate logical arguments show the truth of statements pertaining to the congruence of angles, line segments, and triangles
	Similarity	Analyze geometric figures and determine if they are related by similarity transformations	Understand and apply the definition of a similarity transformation Develop and test conjectures about the similarity of triangles and other polygons by demonstration, constructions, counter-examples, or dynamic software	Apply similarity postulates and theorems to solve problems	Communicate logical arguments to show the truth of statements pertaining to the similarity of triangles

Grades 9-12 Learning Continuum

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 3: Understand the representation of geometric facts, figures, and transformations on a coordinate plane	**Rigid Motion**	Identify the different types of symmetries (i.e. point, line, rotational, reflection) of two- and three-dimensional figures	Perform and describe the effect of a single transformation on two -dimensional geometric figures (i.e. translations, rotations, reflections)	Describe translations, rotations, and reflections in mathematical terms (e.g. the equation of the line of reflection and/or vector representation)	Classify a plane rigid motion as one of the four types (reflection, rotation, translation, glide reflection) Perform and describe the effects of successive transformations on a given geometric figure Recognize that all rigid motions are compositions of reflections

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 3: Understand the representation of geometric facts, figures, and transformations on a coordinate plane	**Coordinate Geometry**	Use the two-dimensional rectangular coordinate system and algebraic properties to describe and characterize geometric properties and relationships of lines, such as slope, intercepts, parallelism, and perpendicularity Graph, interpret, and write equations of lines (i.e. point-slope, slope-intercept, standard)	Use coordinate geometry to describe the attributes of geometric figures (i.e. Given the vertices of a triangle, write equations for the sides and medians.) Describe vectors, both algebraic and geometric terms	Justify whether or not two polygons are similar or congruent by using coordinate geometry	Write coordinate proofs to verify properties and show the truth of theorems Use polar coordinates to describe and solve problems Use the three-dimensional rectangular coordinate system and algebraic properties to describe and characterize geometric properties and relationships of lines, planes, and spheres

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 3: Understand the representation of geometric facts, figures, and transformations on a coordinate plane	**Connections Between Algebra and Geometry: Circles, Parabolas, and Other Conic Sections**	Explain the connection between the Pythagorean Theorem, the distance formula, and the equation of a circle	Derive the equation of a circle given its center and radius Derive the center and radius given the equation of a circle	Identify the different types of conic sections Analyze the geometric properties of circles and parabolas	Analyze the geometric properties of general conic sections Graph conic sections whose axes are parallel to the coordinate axes with and without technology Write equations of conic sections in standard and general form

Grades 9-12 Learning Continuum

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 4: Understand the representation and application of geometric models to real-world situations	**Visualization**	Describe two and three-dimensional figures using correct mathematical vocabulary, notation, and symbols	Sketch two and three-dimensional figures based on a written description	Use two-dimensional drawings to represent three-dimensional problems	Create perspective drawings
	Geometric Models	Recognize that a geometric model can be useful in describing real-world or applied situations	Use a given geometric model to solve a real-world situation	Sketch a geometric model that represents a real-world situation	
	Applications of Geometry	Solve problems involving indirect measurement such as finding the height of a building by comparing its shadow with the height and shadow of a known object	Apply similarity and congruence concepts to solve real-world problems	Use the Pythagorean Theorem to solve problems in two and three-dimensional situations Apply trig ratios (sine, cosine, and tangent) to solve real-world situations	Apply trigonometric concepts to solve real-world problems Use vectors to represent physical situations (e.g. velocity, acceleration, force)

Grades 9-12: Data-Driven Reasoning *(Data Analysis, Probability, Number and Operations, Algebra)*

High school students are increasingly involved in important mathematical applications that require them to demonstrate their understanding and reasoning using authentic data. Data presented through the media and global networks require students to utilize more sophisticated tools to understand, interpret, and ultimately value data. In addition, as students use technology tools to sort, organize, display and manipulate data sets, the analysis requires decisions based on data-driven reasoning that incorporate mathematical models and how those models apply to problems involving data.

Students in high school will build upon their previous experiences in connecting data to the mathematical structures they have learned and are learning in their study of algebra and geometry. Data by definition are numbers and other representations in context. As a result, students begin their development of data-driven reasoning by articulating what the data represent, how it was collected, and what summaries can be made that are based on the data. Quantitative data analysis is based upon an understanding of variability as it relates to individuals and samples from given populations. Students can build upon their understanding of the basic notions of shape, center, and spread, and transition to further analyses including associations among variables, the development of probability models, and simulations. These analyses, however, require a transition to more sophisticated mathematical models of the data, and ultimately allow students to develop and evaluate decisions based upon their understanding of probability and the variability inherent in these models. Expanding a student's understanding to inferences based on the analysis of data is important for all students and not only those students who intend to further their study of mathematics and statistics beyond the high school level.

Connecting data to algebraic and geometric models (whether or not those models are linear, exponential, quadratic, or other mathematical functions) requires both an understanding of the data and the mathematics learned through a study of algebra and geometry. These connections further strengthen the learning of algebra, geometry, number and operations, and modeling based upon the use of data. Data-driven reasoning involves an understanding of variability, the context of a data set, an understanding of the likelihood of random events, and an ability to interpret statistical measures that are used to make decisions. As a result, data-driven reasoning requires a careful development of the learning continuum that will move our students from general awareness of data to decision-making skills and inferential thinking.

Grades 9-12 Learning Continuum

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 1: Organize, display, and compare both quantitative and qualitative data to make, justify, and summarize conjectures	**Understanding Quantitative Data Sets**	Understand the concept of a data set and distinguish the difference between qualitative and quantitative data	Estimate the center and spread of a data set using mean, median, and ranges Describe the shape of the data set represented by a histogram or other graphic representations	Calculate and apply measures of center and spread, including the standard deviation, and interquartile range to describe a set of data in context Understand the concept of an outlier and informally determine if any outliers are present in a given data set Examine the impact of potential outliers on further data analysis	Use measures of center and spread to describe, compare, and contrast sets of data in real world contexts Use quantitative methods (e.g., the 1.5 interquartile criterion) to determine if a data set has any potential outliers Determine measures that are affected by outliers and measures that are resistant to the effect of outliers in the data set

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 1: Organize, display, and compare both quantitative and qualitative data to make, justify, and summarize conjectures	**Analysis of Quantitative Data Using Visual Displays**	Create ways to organize and explain data that allow for summaries represented by percentages, ranges, and other numerical representations Create and interpret displays of quantitative data from stem and leaf plots, dot frequency diagrams, and spreadsheets	Summarize and display quantitative data from real-world contexts using histograms, box plots, and spreadsheets	Use displays of quantitative data to describe, compare, and contrast sets of data in real world contexts Create modified box plots that illustrate the location of outliers in a data set	Understand the concept of a normal distribution and use informal methods to determine real world contexts that can be described by normal distributions
	Variability of Quantitative Data Sets	Recognize the variability of a data set by examining displays of data (e.g., box plot)	Compare and contrast the spread of two or more data sets by examining displays	Use measures of spread (range, IQR, standard deviation, variance) to summarize, compare and contrast sets of data	Interpret the relationship between the shape of a distribution and measures of its center (e.g. what might a distribution look like if its median is much larger than its mean?)

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 1: Organize, display, and compare both quantitative and qualitative data to make, justify, and summarize conjectures	**Analysis of Quantitative Data in Two-Variable Data Sets**	Understand how to create and interpret displays of two-variable data sets including tables and measures of center and spread from each variable	Create and interpret scatterplots of two-variable data sets in real world contexts	Summarize and interpret relationships of two-variable data sets represented in tables or spreadsheets using scatterplots by applying visual trend lines and analytical methods (e.g. the median-median line, least-squares regression line)	Read and interpret models that are summarized as exponential functions of the form $y = ab^x + c$ in which the parameters a,b,c are derived from authentic data and appropriate assumptions of either an increasing or decreasing exponential model
	Analysis of Qualitative Data in One and Two-Variable Data Sets	Distinguish between quantitative and qualitative data and understand how to organize and display qualitative data sets using pie graphs and bar graphs	Understand how to create and display qualitative data in order to summarize and describe data in real world contexts	Use displays of qualitative data to describe, compare, and contrast sets of data in real world contexts	Organize two variable qualitative data sets and analyze associations between the two variables Use two-way tables to develop understanding of conditional probability

Grades 9-12 Learning Continuum

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 2: Develop, analyze, and justify inferences based on data	**Sampling Methods**	Compare populations and samples using descriptive statistics (e.g. mean, median, percents)	Understand the concept of bias when selecting data for a sample Compare methods that create and eliminate bias when sampling	Use a random digit chart, dice, and other randomization methods to create random samples from populations	Construct, apply, and understand how various sampling methods are used to remove bias from statistical analysis of real world data (including simple random samples, stratified random samples, probability samples)
	Counting Methods and Development of Probability Concepts	Use tables and counting trees to illustrate the number of outcomes in a random event Compare relative frequencies from trials of random events to probability calculations based on counting tables and counting trees	Use counting trees and the multiplication counting principle to determine the number of outcomes of random events Compare relative frequencies from trials of random events to probability calculations based on counting trees and the multiplication counting principle	Use permutations and combinations to determine the number of outcomes for random events in real world contexts Compare relative frequencies from trials of random events to probability calculations based on permutations and combinations	Apply combinations and Pascal's triangle to make binomial probability calculations in real world contexts Compare relative frequencies from trials of random events to probability calculations based on binomial probabilities

Grades 9-12 Learning Continuum

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 3: Evaluate and derive models based on data to analyze and predict outcomes	**Thinking and Reasoning with Data**	Recognize and interpret that a data driven model can be useful in describing real-world or applied situations	Hypothesize the important aspects of an authentic data set that include how the data was collected, what the data represent, what questions are raised by the data, and what questions remain about the problem that are not addressed by the data	Select an appropriate representation of an authentic data set along with an appropriate model that matches the representation Recognize and explain the accuracy and variability of a selected model as related to the stated problem	Apply appropriate mathematical techniques to a selected model to predict and generalize important insights that address previous questions or hypothesized statements Evaluate conclusions based upon a model by recognizing variability and assumptions used in the development of the model
	Probability Modeling	Evaluate and explain how probability quantifies the likelihood that an event occurs	Evaluate and explain how the relative frequency of a specified outcome of an event can be used to estimate the probability of the outcome (an application of the law of large numbers)	Evaluate probability concepts such as conditional probability and independent or dependent events to determine the likelihood of events involving authentic data (For example, an analysis of a medical test involving the likelihood of a false positive.)	Apply probability concepts to practical situations to make informed decisions involving games, diseases, financial liabilities, and similar risk or gain applications

Learning Priority	Focus Area	Stage 1:	Stage 2:	Stage 3:	Stage 4:
Learning Priority 3: Evaluate and derive models based on data to analyze and predict outcomes	**Critiquing Data**	Identify and explain misleading uses of data	Evaluate reports based on data published in the media by considering the source of the data, the design of the study, and the methods in which the data are analyzed and displayed	Recognize when arguments based on data confuse correlation with causation	
	Experimental Design	Design simple experiments, investigations, or simulations to collect data to answer questions involving the likelihood of events of interest	Explain the differences between randomized experiments and observational studies	Explain the impact of sampling methods, bias, and the phrasing of questions asked during data collection and the conclusions that can rightfully be drawn	