F. ALGEBRAIC RELATIONSHIPS

***Content Standard**:* Students in Wisconsin will discover, describe, and generalize simple and complex patterns and relationships. In the context of real-world problem situations, students will use algebraic techniques to define and describe the problem to determine and justify appropriate solutions.

Rationale: Algebra is the language of mathematics. Much of the observable world can be characterized as having patterned regularity where a change in one quantity results in changes in other quantities. Through algebra and the use of variables and functions, mathematical models can be built which are essential to personal, scientific, economic, social, medical, artistic, and civic fields of inquiry.

***Performance Standards*: By the end of grade four, students will:**	***Sample Alternate Performance Indicators:* (1-3 per standard)**	***Sample Performance Activities/Tasks:* (1-2 per indicator)**	***Sources of Data***
F.4.1. Use letters, boxes, or other symbols to stand for any number, measured quantity, or object in simple situations (e.g., N+0=N is true for any number)[1]	1. Use letters, boxes, or other symbols to stand for any number, measured quantity, or object in simple situations (e.g., N+0=N is true for any number)[1]	1.a. Solve problems of the following type: If C + 4 = 7, what is the value of C?(1) 1.b. Solve problems of the following type: What is the length From B to C? (2) :_____5in_____:_____:___4in_____: A B C D The length of AD = 12 in.	
F.4.2. Use the vocabulary, symbols, and notation of algebra accurately (e.g., correct use of the symbol "=", effective use of the associative property of multiplication)[2]	1. Use the vocabulary, symbols, and notation of algebra accurately (e.g., correct use of the symbol "=", effective use of the associative property of multiplication) [1]	1.a. Use correct symbols "=, <, >" to express, "John is at least 10 years old" (e.g., J>9; J<11; 9<J<11) (1)	
F 4.3. Work with simple linear patterns and relationships in a variety of ways, including[2] • recognizing and extending number patterns • describing them verbally	1. Work with simple linear patterns and relationships in a variety of ways, including[3] • recognizing and extending number patterns • describing them verbally	1.a. Use two ways to show a linear pattern (e.g., table, graph, chart, or pictures)(3) • purchase one candy bar for $0.50 each day for two weeks • show the change on a graph when the candy bar is on sale every Wednesday for $0.25	

• representing them with pictures, tables, charts, graphs • recognizing that different models can represent the same pattern or relationship • using them to describe real-world phenomena	• representing them with pictures, tables, charts, and graphs • recognizing that different models can represent the same pattern or relationship • using them to describe real-world phenomena		
F.4.4. Recognize variability in simple functional relationships by describing how a change in one quantity can produce a change in another (e.g., number of bicycles and the total number of wheels) [2]	1. Recognize variability in simple functional relationships by describing how a change in one quantity can produce a change in another (e.g., number of bicycles and the total number of wheels)[2]	1.a. Find the value of X • X is a two-digit number • X > 5, 6, 7, 8 ... 19 • X < 20 • the sum of the two digits is 5 Problem relationships may be presented in L_1 or L_2	
F.4.5. Use simple equations and inequalities in a variety of ways, including[2] • using them to represent problem situations • solving them by different methods (e.g., use of manipulatives, guess and check strategies, recall of number facts) • recording and describing solution strategies	1. Use simple equations and inequalities in a variety of ways, including[2] • using them to represent problem situations • solving them by different methods (e.g., use of manipulatives, guess and check strategies, and recall of number facts) • recording and describing solution strategies	1.a. Find the value of X (2) • X is a two-digit number • X > 5 • X < 20 • the sum of the two digits is 5	
F.4.6. Recognize and use generalized properties and relationships of arithmetic (e.g., commutativity of addition, inverse relationship of multiplication and division)[1]	1. Recognize and use generalized properties and relationships of arithmetic (e.g., comutativity of addition, inverse relationship of multiplication and division)[1]	1.a. Rewrite the equations in different ways by using commutative or associative properties • (x + 5) + 2 = 14; (x + 2) + 5 = 14 • (9 + y) * 7 = 70 Give directions in L_1 or L_2 (1)	

***Performance Standards*: By the end of grade eight, students will:**	***Sample Alternate Performance Indicators:* (1-3 per standard)**	***Sample Performance Activities/Tasks:* (1-2 per indicator)**	***Sources of Data***
F.8.1. Work with algebraic expressions in a variety of ways, including[2] • using appropriate symbolism, including exponents and variables • evaluating expressions through numerical substitution • generating equivalent expressions • adding and subtracting expressions	1. Work with algebraic expressions in a variety of ways, including[2] • using appropriate symbolism, including exponents and variables • evaluating expressions through numerical substitution • generating equivalent expressions • adding and subtracting expressions	1.a. solve (x)(y + 7) when x = 5 and y = 2 (1)	
F.8.2. Work with linear and nonlinear patterns and relationships in a variety of ways, including[2] • representing them with tables, with graphs, and with algebraic expressions, equations, and inequalities • describing and interpreting their graphical representations (e.g., slope, rate of change, intercepts) • using them as models of real-world phenomenon • describing a real-world phenomena that a given graph might represent	1. Work with linear and nonlinear patterns and relationships in a variety of ways, including[3] • representing them with tables, with graphs, and with algebraic expressions, equations, and inequalities • describing and interpreting their graphical representations (e.g., slope, rate of change, intercepts) • using them as models of real-world phenomenon • describing a real-world phenomena that a given graph might represent	1.a. Solve problems of the following type: A phone call costs $0.20 per minute with a service charge of $2.75 per month(3) • present a linear pattern by using tables, graphs, and algebraic expressions and equations • identify the slope and the y-intercept • on the same tables or graphs, present another linear pattern about another phone company charging $.30 per minute with no service charge per month	
F.8.3. Recognize, describe, and analyze functional relationships	1. Recognize, describe, and analyze functional relationships by generalizing	1.a. Develop problems around interest in savings accounts and depreciation of computers, automobiles, and video games(3)	

by generalizing a rule that characterizes the pattern of change among variables. These functional relationships include exponential growth and decay (e.g., cell division, depreciation)[3]	a rule that characterizes the pattern of change among variables. These functional relationships include exponential growth and decay (e.g., cell division, depreciation)[3]		
F.8.4. Use linear equations and inequalities in a variety of ways, including[2] • writing them to represent problem situations and to express generalizations • solving them by different methods (e.g., informally, graphically, with formal properties, with technology) • writing and evaluating formulas (including solving for a specified variable) • using them to record and describe solution strategies	1. Use linear equations and inequalities in a variety of ways, including[2] • writing them to represent problem situations and to express generalizations • solving them by different methods (e.g., informally, graphically, with formal properties, with technology) • writing and evaluating formulas (including solving for a specified variable) • using them to record and describe solution strategies	1.a. Solve problems of the following type: A phone call costs $0.20 per minute with a service charge of $2.75 per month(3) • present a linear pattern by using tables, graphs, and algebraic expressions and equations • identify the slope and the y-intercept • on the same tables or graphs, present another linear pattern about another phone company charging $.30 per minute with no service charge per month (same as activity for F.8.2.-1.a.)	
F.8.5. Recognize and use generalized properties and relations, including[2] • additive and multiplicative property of equations and inequalities • commutativity and associativity of addition and multiplication • distributive property • inverses and identities for addition and multiplication • transitive property	1. Recognize and use generalized properties and relations, including[2] • additive and multiplicative property of equations and inequalities • commutativity and associativity of addition and multiplication • distributive property • inverses and identities for addition and multiplication • transitive property	1.a. Recognize that (2 + 3) - 1 = 2 + (3-1) (1) 1.b. Illustrate by using symbols the additive and multiplicative property of equations and inequalities (e.g., $2x + 5 = 96$ $\underline{x^2 + 3x + 6 = 112}$ $x^2 + 5x + 11 = 208$) commutativity and associativity of addition and multiplication, distributive property, inverses and identities for addition and multiplication, and transitive property (2)	

***Performance Standards*: By the end of grade twelve, students will:**	***Sample Alternate Performance Indicators:* (1-3 per standard)**	***Sample Performance Activities/Tasks:* (1-2 per indicator)**	***Sources of Data***
F.12.1. Analyze and generalize patterns of change (e.g., direct and inverse variation) and numerical sequences, and then represent them with algebraic expressions and equations	1. Analyze and generalize patterns of change (e.g., direct and inverse variation) and numerical sequences, and then represent them with algebraic expressions and equations	1.a. Analyze and generalize patterns of change. Expand the pattern up to three more • 1, 4, 9, 16, ____, ____, ____ • Guy has $250 saved for a used car. Each week he saves an additional $50. Write an equation for the amount he has saved (y) after x number of weeks (e.g., y=250+50x)	
F.12.2. Use mathematical functions (e.g., linear, exponential, quadratic, power) in a variety ways, including • recognizing that a variety of mathematical and real-world phenomena can be modeled by the same type of function • translating different forms of representing them (e.g., tables, graphs, functional notation, formulas) • describing the relationships among variable quantities in a problem • using appropriate technology to interpret properties of their graphical representations (e.g., intercepts, slopes, rates of change, changes in rates of change, maximum, minimum)	1. Use mathematical functions (e.g., linear, exponential, quadratic, and power) in a variety ways, including • recognizing that a variety of mathematical and real-world phenomena can be modeled by the same type of function • translating different forms of representing them (e.g., tables, graphs, functional notation, and formulas) • describing the relationships among variable quantities in a problem • using appropriate technology to interpret properties of their graphical representations (e.g., intercepts, slopes, rates of change, changes in rates of change, maximum, and minimum)	1.a. Solve problems of the following type: Mohamad is renting a bike from his friend for four weeks. He can pay $5.00 per week or one cent on the first day, two cents on the second day, four cents on the third day, eight cents on the fourth day, and continue with that pattern for four weeks (2^{28}). Which one would be least expensive for Mohamad? Which one would be least expensive if he rented a bike for one week (2^{7}), two weeks (2^{14}), three weeks (2^{21})? • use appropriate technology (spreadsheet, charts) to present information • write and solve a system of equations • find the point at which each plan would earn the same amount of money	
F.12.3. Solve linear and quadratic equations, linear inequalities, and systems of linear equations and inequalities	1. Solve linear and quadratic equations, linear inequalities, and systems of linear equations and inequalities	1.a. Solve problems of the type for F.12.2.-1.a.	

• numerically • graphically, including use of appropriate technology • symbolically, including use of the quadratic formula	• numerically • graphically, including use of appropriate technology • symbolically, including use of the quadratic formula		
F.12.4. Model and solve a variety of mathematical and real-world problems by using algebraic expressions, equations, and inequalities	1. Model and solve a variety of mathematical and real-world problems by using algebraic expressions, equations, and inequalities	1.a. Solve problems of the type for F.12.2.-1.a.	