2010 to 2021 Wisconsin Standards for Mathematics Comparison

This document compares the 2010 and 2021 Wisconsin Standards for Mathematics by showing word-level changes in grade-level standards. It is intended to be used as a scaffold beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of Wisconsin Standards for Mathematics.

This document is not meant to build understanding of the revisions made to the 2021 standards. Instead, educators should see the brief professional learning modules designed to develop understanding of the 2021 standards found at Standards Link.

For each grade level or high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

- Red font in the 2021 standard indicates that a change was made.
- Strikethrough in the 2010 standard indicates that language was removed.

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K-12 Standards for Mathematical Practice, 2010 Standards compared to 2021 Revisions

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K-12 Standards for Mathematical Practice	
Math Practice 1: Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves,	**Math Practice 1:** Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves,

"Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **Math Practice 2:** Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **Math Practice 3:** Construct viable arguments, and appreciate and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making	"Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **Math Practice 2:** Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **Math Practice 3:** Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making

plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. While communicating their own mathematical ideas is important, students also learn to be open to others' mathematical ideas. They appreciate a different perspective or approach to a problem and learn how to respond to those ideas, respecting the reasoning of others (Gutiérrez 2017, 17-18). Together, students make sense of the mathematics by asking helpful questions that clarify or deepen everyone's understanding. **Math Practice 4:** Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map	plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. ~~Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful~~ questions to clarify or ~~improve the arguments.~~ **Math Practice 4:** Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map

their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **Math Practice 5:** Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **Math Practice 5:** Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Math Practice 6:
Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Math Practice 7:
Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and

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use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

Math Practice 8:
Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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K-5 Standards for Mathematical Practice

2021 Standards	2010 Standards
A comparison of 2021, K-5 Standards for Mathematical Practice (SMP) and 2010, K-12 SMP	
Math Practice 1: Make sense of problems and persevere in solving them. Mathematically proficient elementary students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. For example, instead of hunting for "key words" in a word problem, students might use concrete objects or pictures to show the actions of a problem, such as counting out and joining two sets. If students are not at first making sense of a problem or seeing a way to begin, they ask questions about what is happening in the problem that will help them get started. As they work, they continually ask themselves, "Does this make sense?" When they find that their solution pathway does not make sense, they look for another pathway that does. They may consider simpler forms of the original problem; for example, they might replace multi-digit numbers in a word problem with single-digit numbers to better appreciate the quantities in the problem and how they relate. Mathematically proficient students consider different solution pathways, both their own and those of other students, in order to identify and analyze connections among approaches. They can explain connections among physical models, pictures, diagrams, equations, verbal descriptions, tables, and graphs. Once students have a solution, they often check their answers to problems using a	**Math Practice 1:** Make sense of problems and persevere in solving them. Mathematically proficient students ~~start by~~ explain~~ing~~ to themselves the meaning of a problem ~~and~~ look~~ing~~ for entry points to ~~its~~ solution. ~~They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and~~ plan a solution pathway ~~rather than simply jumping into a solution attempt.~~ They consider ~~analogous problems, and try special cases and simpler forms~~ of the original problem ~~in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students~~ can explain ~~correspondences between~~ equations, verbal descriptions, tables, and graphs or draw diagrams ~~of important features and relationships, graph data, and search for regularity or trends. Younger students~~ might ~~rely on~~ us~~ing~~ concrete objects or pictures to ~~help conceptualize and solve a problem~~. Mathematically proficient students check their answers to problems using a different ~~method~~, and they continually ask themselves, "Does this make sense?" ~~They can understand~~ the approaches of

different approach. **Math Practice 2:** Reason abstractly and quantitatively. Mathematically proficient elementary students make sense of quantities and their relationships in problem situations. They can contextualize quantities and operations by using visual representations or stories. They interpret symbols as having meaning, not just as directions to carry out a procedure. Even as they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numerical relationships, numbers, and geometric objects. They can contextualize an abstract problem by placing it in a context that they can then use to make sense of the mathematical ideas. For example, if a student chooses to evaluate the expression 13 x 25 mentally, the student might think of a context to help produce a strategy—for example, by thinking “Thirteen groups of 25 is like having 13 quarters.” This prompts a strategy of thinking "I know that 10 quarters is $2.50 and 3 quarters is $0.75. $2.50 and $0.75 is $3.25." In this example the student uses a context to think through a strategy for solving the problem, using their knowledge of money and of decomposing one factor based on place value (13 = 10+3). The student then uses the context to identify the solution to the original problem. Mathematically proficient students can also make sense of a contextual problem and express the actions or events that are described in the problem using numbers and symbols. If they work	others ~~to solving complex problems and~~ identify ~~correspondences between different~~ approaches. **Math Practice 2:** Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. ~~They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—~~to abstract a given situation and represent it symbolically and manipulate the ~~representing~~ symbols ~~as if they have a life of their own, without necessarily attending to their referents—and the ability to~~ *contextualize*, to pause as needed ~~during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to~~ the meaning of ~~quantities, not just how to compute them; and~~ know~~ing~~ and flexibly us~~ing~~ different properties of operations and objects.

with the symbols to solve the problem, they can then interpret their solution in terms of the context. Consider the problem: A teacher wants to bring 10 pumpkins to school to decorate the classroom. Some are big pumpkins and some are small pumpkins. How many of each size pumpkin might the teacher bring to school? When students create the number sentence 4+6=10, they have decontextualized the problem and expressed it with numbers and symbols. When they can explain that the number sentence means, "4 big pumpkins plus 6 small pumpkins equals 10 pumpkins," they demonstrate their ability to recontextualize the numbers and equation back to the word problem.

Math Practice 3:
Construct viable arguments, and appreciate and critique the reasoning of others.

Mathematically proficient elementary students construct verbal and written mathematical arguments that explain the reasoning underlying a strategy, solution, or conjecture. Arguments might use concrete referents such as objects, drawings, diagrams, and actions. Arguments might also rely on definitions, previously established results, properties, or structures. For example, a student might argue that $^1/_5 > ^1/_9$ on the basis that one of 5 equal parts of a whole is larger than one of 9 equal parts of that whole, because with more equal parts, the size of each part must be smaller. Another example is reasoning that two different shapes have equal area because it has already been demonstrated that they are each half of the same rectangle. Students might also use counterexamples to argue that a conjecture is not true—for example, a rhombus is an example that shows that not all quadrilaterals with 4 equal sides are squares; or, multiplying by 1 shows that a product of two whole numbers is not always greater than each factor.

Math Practice 3:
Construct viable arguments and critique the reasoning of others.

Mathematically proficient students ~~understand and use stated assumptions,~~ definitions, and previously established results in constructing arguments. ~~They make conjectures and build a logical progression of statements to explore the truth of their~~ conjectures~~.~~ ~~They are able to analyze situations by breaking them into cases, and~~ can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. ~~They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construc~~t arguments usi~~ng~~ concrete referents such as objects, drawings, diagrams, and actions. ~~Such arguments can make sense and be correct, even though they are not~~ generaliz~~ed or made~~

Mathematically proficient students present their arguments in the form of representations, actions on those representations, explanations in words (oral or written), or a combination of these three. Some of their arguments apply to individual problems, but others are about conjectures based on regularities they have noticed across multiple problems (Math Practice 8). As they articulate and justify generalizations, students consider to which mathematical objects (numbers or shapes, for example) their generalizations apply. For example, primary grade students may believe a generalization about the behavior of addition applies to positive whole numbers less than 100 because those are the numbers with which they are currently familiar. As they expand their understanding of the number system, they may reexamine their conjecture for numbers in the hundreds and thousands. Intermediate grade students return to their conjectures and arguments about whole numbers to determine whether they apply to fractions and decimals. While communicating their own mathematical ideas is important, elementary students also learn to be open to others' mathematical ideas. They appreciate a different perspective or approach to a problem and learn how to respond to those ideas, respecting the reasoning of others (Gutiérrez 2017, 17-18). Together, students make sense of the mathematics, asking helpful questions that clarify or deepen everyone's understanding, and reconsider their own arguments in response to the collaboration. **Math Practice 4:** Model with mathematics. "In the course of a student's mathematics education, the word 'model' is used in many ways. Several of these, such as manipulatives, demonstration, role modeling, and conceptual	~~formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or~~ read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **Math Practice 4:** Model with mathematics. ~~Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an~~

models of mathematics, are valuable tools for teaching and learning. However, they are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer big, messy, reality-based questions (Bliss and Libertini 2016, 7). “ Mathematically proficient elementary students formulate their own problems that emerge from natural circumstances as they mathematize the world around them. They can identify the mathematical elements of a situation and generate questions that can be addressed using mathematics (e.g., noticing and wondering). Students dig into the context and make assumptions as they decide “what matters”. Mathematically proficient elementary students understand that there are multiple solutions to a modeling problem so they are working to find a solution rather than the solution. Students make judgements about what matters and assess the quality of their solution (Bliss and Libertini 2016, 10). They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving their mathematical modeling approach if it has not served its purpose. As they decontextualize the situation and represent it mathematically, they are also reasoning abstractly (Math Practice 2). In the elementary grades, students encounter mathematical modeling opportunities each and every day at school and at home. Students might consider how the classroom’s set of blocks should be shared throughout recess time. Students might then need to make assumptions about how many blocks each student should have as well as the length of time each student should have the blocks. Once a solution is determined, students could be asked to refine their model by posing the question, “What if one of our friends will not be at recess?” Children might also be presented with a bag of apples and simply asked “Is this enough for our class/family?” or consider	~~addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable~~ making assumptions ~~and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions.~~ They ~~routinely~~ interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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the question, “Is the carpet in our classroom big enough for our bodies?”
Note: Although physical objects and visuals can be used to model a situation, using these tools absent a contextual situation is not an example of Math Practice 4. For example, solving a word problem using counters or a tape diagram would not be modeling with mathematics, instead this is modeling the mathematics. Math Practice 4 is about engaging in solving authentic real-world problems.
Math Practice 5:
Use appropriate tools strategically.
Mathematically proficient elementary students strategically consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. These tools might include physical objects (e.g., manipulatives, pencil and paper, rulers); conceptual tools (e.g., properties of operations, algorithms); drawings or diagrams (e.g., number lines, tally marks, tape diagrams, arrays, tables, graphs) and available technologies (e.g., calculators, online apps).
Mathematically proficient students are sufficiently familiar with tools appropriate for their grade and areas of content to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained from their use as well as their limitations. Students choose tools that are relevant and useful to the problem at hand. For example, when determining how to measure length, students may compare the benefits of using non-standard units of measure (e.g., their own hands, paperclips)
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Math Practice 5:
Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, <s>concrete models</s>, a ruler, <s>a protractor</s>, a calculator, <s>a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.</s> Proficient students are sufficiently familiar with tools appropriate for their grade <s>or course</s> to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained <s>and</s> their limitations. <s>For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify</s>
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versus standard units and tools (e.g., an inch or centimeter ruler). As another example, when presented with 1002-3 or 101-98, students subtract strategically, which may involve reasoning, counting, or decomposing rather than using a written algorithm. **Math Practice 6:** Attend to precision. Mathematically proficient elementary students use precise language to communicate orally and in written form. They come to appreciate, understand, and use mathematical vocabulary not in isolation, but in the context of doing mathematical thinking and problem solving. They may start by using everyday language to express their mathematical ideas, and gradually select words with greater clarity and specificity. For example, they may initially use the word “triangle” to refer only to equilateral triangles resting on their bases, but come to understand and use a more precise definition of a triangle as a closed figure with three straight sides. As another example, they may initially explain a solution by saying, “it works" without explaining what “it" means but later clarify their explanation with specific details. In using mathematical representations, students provide appropriate labels to precisely communicate the meaning of their representations (e.g., charts, graphs, and drawings). When making mathematical arguments about a solution, strategy, or conjecture (Math Practice 3), mathematically proficient students learn to craft careful explanations that communicate their reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to their representations. Students use mathematical symbols correctly and can describe the	~~relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.~~ **Math Practice 6:** Attend to precision. Mathematically proficient students ~~try~~ to communicate precise~~ly to others. They try to use clear definitions in discussion with others and in their own reasoning.~~ They ~~state~~ the meaning of the symbols they choose, including using the equal sign consistently and appropriately. ~~They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree~~ of precision appropriate ~~for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.~~

meaning of the symbols they use. For example, they use the equal sign consistently and appropriately. They state the meaning of the symbols they choose in relation to the problem at hand. Students use tools and strategies (e.g., measuring tools, estimation) effectively, to maintain a level of precision that is appropriate to the situation. They specify units of measure where needed. Perseverance and attention to detail are mathematical habits of mind; mathematically proficient students check for reasonableness and accuracy by solving a problem a second way, analyzing errors and learning from them. **Math Practice 7:** Look for and make use of structure. Mathematically proficient elementary students use structures such as place value, the properties of operations, and attributes of shapes to solve problems. In many cases, they have identified and described these structures through repeated reasoning (Math Practice 8). When students use an algorithm to solve 53-17 in order to fully understand how to decompose the tens and ones, they must understand that 53 can be seen as 4 tens and 13 ones, not just 5 tens and 3 ones. When younger students recognize that adding 1 results in the next counting number, they are identifying the basic structure of whole numbers. When older students calculate 16 x 9, they might apply the structure of place value and the distributive property to find the product: 16 x 9 = (10 + 6) x 9 = (10 x 9) + (6 x 9). To determine the volume of a 3 x 4 x 5 rectangular prism, students might see the structure of the prism as five layers of 3 x 4 arrays of cubes.	**Math Practice 7:** Look for and make use of structure. Mathematically proficient students ~~look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of~~ shapes ~~according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.~~

Students in elementary grades look for and make use of structure when they view expressions as objects to observe and interpret. For example, students might observe that 120 – 41 must be one less than 120 – 40 because "if you subtract one more, the result will be one less" (Math Practice 8). Students can interpret the expression 5 x 3 + 6 x 3 as "five groups of three and six more groups of three" or notice there are a total of 11 groups of 3. A word problem that involves distributing 29 marbles among 4 vases could lead (Math Practice 4) to an equation model (29 – 1) ÷ 4 = 7, where the expression on the left-hand side not only has the value 7 but also suggests, based on its structure, a process of discarding 1 marble and dividing the rest of the marbles equally into 4 groups of 7. **Math Practice 8:** Look for and express regularity in repeated reasoning. Mathematically proficient elementary students look for and identify regularities as they solve multiple related problems. Students make and test conjectures, reason about and express these regularities as generalizations about structures and relationships, and then use those generalizations to solve problems (Math Practice 7). For example, younger students might notice that when tossing two-color counters to find combinations of a given number, over time students will notice a pattern (commutative property of addition). For example, when tossing six 2-sided counters, they may get 2 red, 4 yellow and 4 red, 2 yellow and when tossing 4 counters, they get 1 red, 3 yellow and 3 red, 1 yellow. In the elementary grades students can recognize and use patterns to help them become flexible with addition. For example, given the	**Math Practice 8:** Look for and express regularity in repeated reasoning. Mathematically proficient students ~~notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation~~ $\cancel{(y - 2)/(x - 1) = 3}$~~. Noticing the regularity in the way terms cancel when expanding~~ $\cancel{(x - 1)(x + 1)}$~~,~~ $\cancel{(x - 1)(x^2 + x + 1)}$~~, and~~ $\cancel{(x - 1)(x^3 + x^2 + x + 1)}$ ~~might lead them to the general formula for the sum of a geometric series. As they work to~~ solve ~~a~~ problem, ~~mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.~~

number string below, students may recognize they can take one away from the 5 and add it to the first number to make a multiple of ten. They also may notice a pattern related to the first digit increasing by 10, therefore the answer increases by 10. 9+5 19+5 29+5 39+5 When drawing and representing fractions, students might notice a consistent relationship between the numerator and denominator of fractions that equal one half (e.g., that the numerator is half the denominator and the denominator is two times the numerator). They can generalize from these repeated examples that all fractions equal to one half show this relationship. As students practice articulating their observations both verbally and in writing, they learn to communicate with greater precision (Math Practice 6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (Math Practice 3).	

Kindergarten Standards, 2010 Standards compared to 2021 Revisions

This document is intended to be used beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of *Wisconsin Standards for Mathematics.*

For each grade level or high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

- Red font in the 2021 standard indicates that a change was made.
- Strikethrough in the 2010 standard indicates that language was removed.

For Kindergarten Standards, the following revisions were made (in addition to revisions in the language of the grade-level standards):

- Specific narratives were written as Standards for Mathematical Practice (K-5) to illustrate what the Standards for Mathematical Practice (K-12) look like in the elementary grades.
- The kindergarten introduction now reads as a narrative rather than a numbered list. In addition, language was added to more fully describe the idea of focusing on critical areas while still reminding readers that it is critical to teach all of the standards.
- The kindergarten introduction includes language to describe what mathematical modeling might look like at the K-2 grade band in support of all students as flexible users of mathematics (shift #2).
- The *Addition and Subtraction Situations by Grade Level* (Appendix 1, Table 1) has been updated to include generalizable equations for each problem situation. Shading is also used to indicate which problem situations students should use or master at each grade level. These adjustments now match the *Progressions for the Common Core State Standards for Mathematics* (2019) situation tables.

2021 Standards	2010 Standards
Counting and Cardinality - Kindergarten	
A. Know number names and the count sequence. M.K.CC.A.1 Count to 100 by ones and by tens. M.K.CC.A.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1). M.K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **B. Tell the number of objects.** M.K.CC.A.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object (one to one correspondence). b. Understand that the last number name said tells the number of objects counted (cardinality). The number of objects is the same regardless of their arrangement or the order in which they were counted (number conservation). c. Understand that each successive number name refers to a quantity that is one larger and the previous number is one smaller (hierarchical inclusion).	**Know number names and the count sequence.** K.CC.1 Count to 100 by ones and by tens. K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1). K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **~~Count to~~ tell the number of objects.** K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.

M.K.CC.B.5 Quickly recognize and name the quantity of up to 5 objects briefly shown in structured or unstructured arrangements without counting (perceptual subitizing). M.K.CC.B.6 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. **C. Compare numbers.** M.K.CC.C.7 Identify whether the number of objects (up to 10) in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. M.K.CC.C.8 Compare two numbers between 1 and 10 presented as written numerals using student generated ways to record the comparison.	K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. **Compare numbers.** K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] K.CC.7 Compare two numbers between 1 and 10 presented as written numerals.
Operations and Algebraic Thinking - Kindergarten	
A. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. M.K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, or numbers. Drawings need not show details, but should show the mathematics in the problem.	**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, or numbers.

M.K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. *See Appendix, Table 1 for specific problem situations and category information.* M.K.OA.A.3 Compose and decompose quantities within 10 a. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition with drawings or numbers. b. Quickly name the quantity of objects briefly shown in structured arrangements anchored to 5 (e.g., fingers, ten frames, math rack/rekenrek) with totals up to 10 without counting by recognizing the arrangement or seeing the quantity in subgroups that are combined (conceptual subitizing). M.K.OA.A.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or numbers. M.K.OA.A.5 Flexibly and efficiently add and subtract within 5 using mental images and composing/decomposing numbers up to 5.	K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or ~~equation (e.g., 5 = 2 + 3 and 5 = 4 + 1)~~. K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or ~~equation~~. K.OA.5 ~~Fluently~~ add and subtract within 5.

Number and Operations in Base Ten - Kindergarten	
A. Work with numbers 11-19 to gain foundations for place value. M.K.NBT.A.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or numbers; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	**Work with numbers 11-19 to gain foundations for place value.** K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or ~~equation (e.g., 18 = 10 + 8)~~; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
Measurement and Data - Kindergarten	
A. Describe and compare measurable attributes. M.K.MD.A.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. M.K.MD.A.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of" / "less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **B. Classify objects and count the number of objects in each category.**	**Describe and compare measurable attributes.** K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of" / "less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **Classify objects and count the number of objects in each category.**

M.K.MD.B.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. Limit category counts to be less than or equal to 10.	K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.-~~[3]~~
Geometry - Kindergarten	
A. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). M.K.G.A.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. M.K.G.A.2 Correctly name shapes regardless of their orientations or overall size. M.K.G.A.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). **B. Analyze, compare, create, and compose shapes.** M.K.G.B.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). M.K.G.B.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. K.G.2 Correctly name shapes regardless of their orientations or overall size. K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). **Analyze, compare, create, and compose shapes.** K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

M.K.G.B.6 Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	K.G.6 Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

Grade 1 Standards, 2010 Standards compared to 2021 Revisions

This document is intended to be used beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of *Wisconsin Standards for Mathematics*.

For each grade level or high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

- Red font in the 2021 standard indicates that a change was made.
- Strikethrough in the 2010 standard indicates that language was removed.

For Grade 1 Standards, the following revisions were made (in addition to revisions in the language of the grade-level standards):

- Specific narratives were written as Standards for Mathematical Practice (K-5) to illustrate what the Standards for Mathematical Practice (K-12) look like in the elementary grades.
- The first grade introduction now reads as a narrative rather than a numbered list. In addition, language was added to more fully describe the idea of focusing on critical areas while still reminding readers that it is critical to teach all of the standards.
- The first grade introduction includes language to describe what mathematical modeling might look like at the K-2 grade band in support of all students as flexible users of mathematics (shift #2).
- The *Addition and Subtraction Situations by Grade Level* (Appendix 1, Table 1) has been updated to include generalizable equations for each problem situation. Shading is also used to indicate which problem situations students should use or master at each grade level. These adjustments now match the *Progressions for the Common Core State Standards for Mathematics* (2019) situation tables.

2021 Standards	2010 Standards
Operations and Algebraic Thinking - Grade 1	
A. Represent and solve problems involving addition and subtraction. M.1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. *See Appendix, Table 1 for specific problem situations and category information.* M.1.OA.A.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **B. Understand and apply properties of operations and the relationship between addition and subtraction.** M.1.OA.B.3 Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Informal use of the commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Informal use of the associative property of addition.)*	**Represent and solve problems involving addition and subtraction.** 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2] 1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **Understand and apply properties of operations and the relationship between addition and subtraction.** 1.OA.3 Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

M.1.OA.B.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.* **C. Add and subtract within 20.** M.1.OA.C.5 Use counting and subitizing strategies to explain addition and subtraction. a. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). b. Use conceptual subitizing in unstructured arrangements with totals up to 10 and structured arrangements anchored to 5 or 10 (e.g., 10 frames, double ten frames, math rack/rekenrek) with totals up to 20 to relate the compositions and decompositions to addition and subtraction. M.1.OA.C.6 Use multiple strategies to add and subtract within 20. a. Flexibly and efficiently add and subtract within 10 using strategies that may include mental images and composing/decomposing up to 10. b. Add and subtract within 20 using objects, drawings or equations. Use multiple strategies that may include counting on; making a ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14) ; decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or	1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.* **Add and subtract within 20.** 1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). 1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies ~~such as~~ counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14) ; decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). **D. Work with addition and subtraction equations.** M.1.OA.D.7 Understand the meaning of the equal sign as "has the same value/amount as" and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*	**Work with addition and subtraction equations.** 1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.* ~~1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.~~ ~~*For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? - 3, 6 + 6 = ?.*~~
Number and Operations in Base Ten - Grade 1	
A. Extend the counting sequence. M.1.NBT.A.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **B. Understand place value.** M.1.NBT.B.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:	**Extend the counting sequence.** 1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **Understand place value.** 1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones -- called a "ten". b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	a. 10 can be thought of as a bundle of ten ones -- called a "ten". b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
M.1.NBT.B.3 Compare two two-digit numbers based on meanings of the tens and ones digits and describe the result of the comparison using words and symbols (>, =, and <).	1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, ~~recording~~ the result~~s~~ of the comparison~~s with the~~ symbols (>, =, and <).
C. Use place value understanding and properties of operations to add and subtract.	**Use place value understanding and properties of operations to add and subtract.**
M.1.NBT.C.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
M.1.NBT.C.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
M.1.NBT.C.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between	1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition

addition and subtraction; relate the strategy to a written method and explain the reasoning used.	and subtraction; relate the strategy to a written method and explain the reasoning used.
Measurement and Data - Grade 1	
A. Measure lengths indirectly and by iterating length units. M.1.MD.A.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object. M.1.MD.A.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **B. Tell and write time.** M.1.MD.B.3 Tell and write time in hours and half-hours using analog and digital clocks. **C. Represent and interpret data.** M.1.MD.C.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**Measure lengths indirectly and by iterating length units.** 1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object. 1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **Tell and write time.** 1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks. **Represent and interpret data.** 1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry - Grade 1	
A. Reason with shapes and their attributes.	**Reason with shapes and their attributes.**
M.1.G.A.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
M.1.G.A.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Student use of formal names such as "right rectangular prism" is not expected.	1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Student use of formal names such as "right rectangular prism" is not expected.
M.1.G.A.3 Partition circles and rectangles into two and four equal shares, describe and count the shares using the words *halves and fourths*, and use the phrases *half of and fourth of the whole.* Describe the whole as being two of the shares, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, ~~and *quarters*~~, and use the phrases *half of*, *fourth of*, ~~and *quarter of*~~. Describe the whole as being two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Grade 2 Standards, 2010 Standards compared to 2021 Revisions

This document is intended to be used beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of *Wisconsin Standards for Mathematics*.

For each grade level or high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

- Red font in the 2021 standard indicates that a change was made.
- Strikethrough in the 2010 standard indicates that language was removed.

For Grade 2 Standards, the following revisions were made (in addition to revisions in the language of the grade-level standards):

- Specific narratives were written as Standards for Mathematical Practice (K-5) to illustrate what the Standards for Mathematical Practice (K-12) look like in the elementary grades.
- The second grade introduction now reads as a narrative rather than a numbered list. In addition, language was added to more fully describe the idea of focusing on critical areas while still reminding readers that it is critical to teach all of the standards.
- The second grade introduction includes language to describe what mathematical modeling might look like at the K-2 grade band in support of all students as flexible users of mathematics (shift #2).
- The *Addition and Subtraction Situations by Grade Level* (Appendix 1, Table 1) has been updated to include generalizable equations for each problem situation. Shading is also used to indicate which problem situations students should use or master at each grade level. These adjustments now match the *Progressions for the Common Core State Standards for Mathematics* (2019) situation tables.

2021 Standards	2010 Standards
Operations and Algebraic Thinking - Grade 2	
A. Represent and solve problems involving addition and subtraction. M.2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. *See Appendix, Table 1 for specific problem situations and category information.*	**Represent and solve problems involving addition and subtraction.** 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.~~[1]~~
B. Add and subtract within 20. M.2.OA.B.2 Flexibly and efficiently add and subtract within 20 using multiple mental strategies which may include counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	**Add and subtract within 20.** 2.OA.2 ~~Fluently~~ add and subtract within 20 using mental strategies.~~[2]~~ ~~By end of Grade 2, know from memory all sums of two one-digit numbers.~~
C. Work with equal groups of objects to gain foundations for multiplication. M.2.OA.C.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting	**Work with equal groups of objects to gain foundations for multiplication.** 2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting

them by 2s; write an equation to express an even number as a sum of two equal addends. M.2.OA.C.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	them by 2s; write an equation to express an even number as a sum of two equal addends. 2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
Number and Operations in Base Ten - Grade 2	
A. Understand place value. M.2.NBT.A.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens -- called a "hundred". b. The numbers 100,200,300,400,500,600,700,800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). M.2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, and 100s. M.2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	**Understand place value.** 2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens -- called a "hundred". b. The numbers 100,200,300,400,500,600,700,800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). 2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s. 2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

M.2.NBT.A.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, and describe the result of the comparison using words and symbols (>, =, and <).	2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols ~~to record~~ the result~~s~~ of comparison~~s~~.
B. Use place value understanding and properties of operations to add and subtract.	**Use place value understanding and properties of operations to add and subtract.**
M.2.NBT.B.5 Flexibly and efficiently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In Grade 2, subtraction with decomposition is an exception and may include drawings/representations.	2.NBT.5 ~~Fluently~~ add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
M.2.NBT.B.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.	2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.
M.2.NBT.B.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
M.2.NBT.B.8 Mentally add 10 or 100 to a given number 100 - 900, and mentally subtract 10 or 100 from a given number 100 - 900.	2.NBT.8 Mentally add 10 or 100 to a given number 100 - 900, and mentally subtract 10 or 100 from a given number 100 - 900.
M.2.NBT.B.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. These explanations may be supported by drawings or objects.	2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. [a]

Measurement and Data - Grade 2	
A. Measure and estimate lengths in standard units.	**Measure and estimate lengths in standard units.**
M.2.MD.A.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
M.2.MD.A.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
M.2.MD.A.3 Estimate lengths using units of inches, feet, centimeters, and meters.	2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.
M.2.MD.A.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
B. Relate addition and subtraction to length.	**Relate addition and subtraction to length.**
M.2.MD.B.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as number lines) and equations with a symbol for the unknown number to represent the problem.	2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as ~~drawings of rulers~~) and equations with a symbol for the unknown number to represent the problem.
M.2.MD.B.6 Represent whole numbers as lengths from 0 on a number line with equally spaced points corresponding to the numbers 0, 1, 2, ... and represent whole-number sums and differences within 100 on a number line.	2.MD.6 Represent whole numbers as lengths from 0 on a number line ~~diagram~~ with equally spaced points corresponding to the numbers 0, 1, 2, ...~~,~~ and represent whole-number sums and differences within 100 on a number line ~~diagram~~.
C. Work with time and money.	**Work with time and money.**

M.2.MD.C.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. M.2.MD.C.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?* **D. Represent and interpret data.** M.2.MD.D.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. M.2.MD.D.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems using information presented in a bar graph. *See Appendix, Table 1 for specific problem situations.*	2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. 2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?* **Represent and interpret data.** 2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. 2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems[4] using information presented in a bar graph.
Geometry - Grade 2	
A. Reason with shapes and their attributes. M.2.G.A.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.	**Reason with shapes and their attributes.** 2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. Sizes are compared directly or visually, not compared by measuring. M.2.G.A.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. M.2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe and count the shares using the words *halves*, *thirds*, and *fourths*, and use phrases *half of*, *a third of*, and *a fourth of the whole*. Describe the whole as composed of two halves, three thirds, and four fourths. Recognize that equal shares of identical wholes need not have the same shape.	2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. 2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of*, ~~etc., and~~ describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Grade 3 Standards, 2010 Standards compared to 2021 Revisions

This document is intended to be used beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of *Wisconsin Standards for Mathematics*.

For each grade level or high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

- Red font in the 2021 standard indicates that a change was made.
- Strikethrough in the 2010 standard indicates that language was removed.

For Grade 3 Standards, the following revisions were made (in addition to revisions in the language of the grade-level standards):

- Specific narratives were written as Standards for Mathematical Practice (K-5) to illustrate what the Standards for Mathematical Practice (K-12) look like in the elementary grades.
- The third grade introduction now reads as a narrative rather than a numbered list. In addition, language was added to more fully describe the idea of focusing on critical areas while still reminding readers that it is critical to teach all of the standards.
- The third grade introduction includes language to describe what mathematical modeling might look like at the 3-5 grade band in support of all students as flexible users of mathematics (shift #2).
- The *Multiplication and Division Problem Situations* (Appendix 1, Table 2A) and *Multiplication and Division - Measurement examples (Appendix 1, Table 2B)* replace *Common Multiplication and Division Situations (*Table 2 from *Wisconsin Standards for Mathematics [2010]).* Table 2A has been updated to include generalizable equations for each type of unknown. The number of groups and group size are no longer distinguished when considering arrays of objects since both a row or a column can be a group. Table 2B now shows measurement examples in a separate table and also includes generalizable equations. The notes underneath both Tables 2A and 2B now indicate which problem situations appear in which grade levels and connect partitive or sharing language and quotative or measurement language to the different types of division. These adjustments now match the *Progressions for the Common Core State Standards for Mathematics* (2019) situation tables.

2021 Standards	2010 Standards
Operations and Algebraic Thinking - Grade 3	
A. Represent and solve problems involving multiplication and division. M.3.OA.A.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7.* M.3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* M.3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. *See Appendix, Tables 2A and 2B for specific problem situations.*	**Represent and solve problems involving multiplication and division.** 3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7.* 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. ~~[1]~~ ~~3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.~~

	~~*For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = ? ÷ 3, 6 x 6 = ?.*~~
B. Understand properties of multiplication and the relationship between multiplication and division.	**Understand properties of multiplication and the relationship between multiplication and division.**
M.3.OA.B.4 Apply properties of operations as strategies to multiply and divide. Student use of the formal terms for these properties is not necessary. *Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56. (Distributive property.)*	3.OA.5 Apply properties of operations as strategies to multiply and divide. ~~2~~ *Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56. (Distributive property.)*
M.3.OA.B.5 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*	3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
C. Multiply and divide within 100.	**Multiply and divide within 100.**
M.3.OA.C.6 Use multiplicative thinking to multiply and divide within 100. a. Use the meanings of multiplication and division, the relationship between the operations (e.g., knowing that 8 x 5 = 40, one could reason that 40 ÷ 5 = 8), and properties of	3.OA.7 ~~Fluently~~ multiply and divide within 100, using strategies ~~such as~~ the relationship between ~~multiplication and division~~ (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ *5 = 8)* ~~or~~ properties of operations. ~~By the end of Grade 3, know from memory all products of two one-digit numbers.~~

operations (e.g., the distributive property) to develop and understand strategies to multiply and divide within 100. b. Flexibly and efficiently use strategies, the relationship between the operations, and properties of operations to find products and quotients with multiples of 0, 1, 2, 5, & 10 within 100.	
D. Solve problems involving the four operations, and identify and explain patterns in arithmetic. M.3.OA.D.7 Solve two-step word problems, posed with whole numbers and having whole number answers, using the four operations. Represent these problems using one or two equations with a letter standing for the unknown quantity. If one equation is used, grouping symbols (i.e. parentheses) may be needed. Assess the reasonableness of answers using mental computation and estimation strategies. M.3.OA.D.8 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	**Solve problems involving the four operations, and identify and explain patterns in arithmetic.** 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies ~~including rounding. [3]~~. 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*
Number and Operations in Base Ten - Grade 3	
A. Use place value understanding and properties of operations to perform multi-digit arithmetic, using a variety of strategies.	**Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]**

M.3.NBT.A.1 Use place value understanding to generate estimates for problems in real-world situations, with whole numbers within 1,000, using strategies such as mental math, benchmark numbers, compatible numbers, and rounding. Assess the reasonableness of their estimates (e.g., Is my estimate too low or too high? What degree of precision do I need for this situation?). M.3.NBT.A.2 Flexibly and efficiently add and subtract within 1,000 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. M.3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.	3.NBT.1 Use place value understanding ~~to round whole numbers to the nearest 10 or 100.~~ 3.NBT.2 ~~Fluently~~ add and subtract within 1,000 using strategies ~~and algorithms~~ based on place value, properties of operations, and/or the relationship between addition and subtraction. 3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.
Number and Operations - Fractions - Grade 3	
Number and Operations - Fractions Grade 3 assessment expectations in this domain are limited to fractions with denominators 2, 3 , 4, 6, and 8, but students should have instructional experiences with other sized fractions. **A. Develop understanding of fractions as numbers.** M.3.NF.A.1 Understand a unit fraction as the quantity formed when a whole is partitioned into equal parts and explain that a unit fraction is one of those parts (e.g., 1/4).	**Number and Operations - Fractions**[5] **Develop understanding of fractions as numbers.** 3.NF.A.1 Understand a fraction ~~1/*b*~~ as the quantity formed ~~by 1 part~~ when a whole is partitioned into ~~*b*~~ equal parts~~;~~ understand ~~a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.~~

Understand fractions are composed of unit fractions, for example, 7/4 is the quantity formed by 7 parts of the size 1/4.

M.3.NF.A.2 Understand and represent a fraction as a number on the number line.

a. Understand the whole on a number line is defined as the interval from 0 to 1 and the unit fraction is defined by partitioning the interval into equal parts (i.e., equal-sized lengths).

b. Represent fractions on a number line by iterating lengths of the unit fraction from 0. Recognize that the resulting interval represents the size of the fraction and that its endpoint locates the fraction as a number on the number line.

For example, 5/3 indicates the length of a line segment from 0 by iterating the unit fraction 1/3 five times and its end point locates the fraction 5/3 on the number line.

M.3.NF.A.3 Explain equivalence of fractions and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size or name the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., *1/2 = 2/4*, 4/6 = 2/3) and explain why the fractions are equivalent by using a visual fraction model (e.g., tape diagram or number line).

c. Express whole numbers as fractions (3 = 3/1), and recognize fractions that are equivalent to whole numbers (4/4 = 1).

3.NF.2 Understand a fraction as a number on the number line; represent fraction~~s~~ on a number line diagram.

a. Represent a fraction ~~*1/b*~~ on a number line ~~diagram~~ by defining the interval from 0 to 1 as the whole and partitioning it into ~~*b*~~ equal parts. Recognize that each part has size ~~*1/b*~~ and that the endpoint of the part based at 0 locates the number ~~*1/b*~~ on the number line.

b. Represent ~~a~~ fraction ~~*a/b*~~ on a number line ~~diagram~~ by ~~marking off a~~ lengths ~~*1/b*~~ from 0. Recognize that the resulting interval ~~has size *a/b*~~ and that its endpoint locates the number ~~*a/b*~~ on the number line.

3.NF.3 Explain equivalence of fractions and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size or name the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., *1/2 = 2/4*, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. ~~*Examples: Express 3 in the form*~~ *3=3/1*~~*; recognize that 6/1 =*~~

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Justify the conclusions by using a visual fraction model (e.g., tape diagram or number line), and describe the result of the comparison using words and symbols (>, =, and <).	*~~6; locate~~ 4/4 ~~and 1 at the same point of a number line diagram.~~* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. ~~Record~~ the result~~s~~ of comparison~~s with~~ the symbols >, =, ~~or~~ < and justify the conclusions, ~~e.g.,~~ by using a visual fraction model.
Measurement and Data - Grade 3	
A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. M.3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line. M.3.MD.A.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l), excluding compound units such as cm^3 and finding the geometric volume of a container. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. *See Appendix, Table 2B for problem situations. Do not include multiplicative comparison problems.* **B. Represent and interpret data.**	**Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.** 3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line. 3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).~~[6]~~ Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.~~[7]~~ **Represent and interpret data.**

<table>
<tr>
<td>M.3.MD.B.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.

For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</td>
<td>3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</td>
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<tr>
<td>M.3.MD.B.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units -- whole numbers, halves, fourths.</td>
<td>3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units -- whole numbers, halves, <s>quarters</s>.</td>
</tr>
<tr>
<td>C. Geometric measurement: understand concepts of area and relate area to multiplication and to addition.</td>
<td>Geometric measurement: understand concepts of area and relate area to multiplication and to addition.</td>
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<tr>
<td>M.3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square" is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</td>
<td>3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square" is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</td>
</tr>
<tr>
<td>M.3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units).</td>
<td>3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).</td>
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<tr>
<td>M.3.MD.C.7 Relate area to the operations of multiplication and addition.</td>
<td>3.MD.7 Relate area to the operations of multiplication and addition.</td>
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</table>

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b + c* is the sum of *a x b and a x c.* Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **D. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** M.3.MD.D.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b + c* is the sum of *a x b and a x c.* Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** 3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry - Grade 3	
A. Reason with shapes and their attributes. M.3.G.A.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. M.3.G.A.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	**Reason with shapes and their attributes.** 3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. 3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Grade 4 Standards, 2010 Standards compared to 2021 Revisions

This document is intended to be used beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of *Wisconsin Standards for Mathematics*.

For each grade level or high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

- Red font in the 2021 standard indicates that a change was made.
- Strikethrough in the 2010 standard indicates that language was removed.

For Grade 4 Standards, the following revisions were made (in addition to revisions in the language of the grade-level standards):

- Specific narratives were written as Standards for Mathematical Practice (K-5) to illustrate what the Standards for Mathematical Practice (K-12) look like in the elementary grades.
- The fourth grade introduction now reads as a narrative rather than a numbered list. In addition, language was added to more fully describe the idea of focusing on critical areas while still reminding readers that it is critical to teach all of the standards.
- The fourth grade introduction includes language to describe what mathematical modeling might look like at the 3-5 grade band in support of all students as flexible users of mathematics (shift #2).
- The *Multiplication and Division Problem Situations* (Appendix 1, Table 2A) and *Multiplication and Division - Measurement examples (Appendix 1, Table 2B)* replace *Common Multiplication and Division Situations (*Table 2 from *Wisconsin Standards for Mathematics [2010]).* Table 2A has been updated to include generalizable equations for each type of unknown. The number of groups and group size are no longer distinguished when considering arrays of objects since both a row or a column can be a group. Table 2B now shows measurement examples in a separate table and also includes generalizable equations. The notes underneath both Tables 2A and 2B now indicate which problem situations appear in which grade levels and connect partitive or sharing language and quotative or measurement language to the different types of division. These adjustments now match the *Progressions for the Common Core State Standards for Mathematics* (2019) situation tables.

2021 Standards	2010 Standards
Operations and Algebraic Thinking - Grade 4	
A. Use the four operations with whole numbers to solve problems. M.4.OA.A.1 Interpret a multiplication equation as a multiplicative comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. M.4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. *See Appendix, Tables 2A & 2B.* M.4.OA.A.3 Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies. **B. Gain familiarity with factors and multiples.** M.4.OA.B.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range	**Use the four operations with whole numbers to solve problems.** 4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] 4.OA.3 Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies ~~including rounding~~. **Gain familiarity with factors and multiples.** 4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a

1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. **C. Generate and analyze patterns.** M.4.OA.C.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* **D. Multiply and divide within 100.** M.4.OA.D.6 Flexibly and efficiently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations [e.g., knowing that 7 x 6 can be thought of as 7 groups of 6 so one could think 5 groups of 6 is 30 and 2 more groups of 6 is 12 and 30 + 12 = 42 (informal use of the distributive property)].	multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. **Generate and analyze patterns.** 4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*
Number and Operations in Base Ten - Grade 4	
Number and Operations in Base Ten (4.NBT) Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.	**Number and Operations in Base Ten (4.NBT)**[2]

A. Generalize place value understanding for multi-digit whole numbers. M.4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* M.4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place and describe the result of the comparison using words and symbols (>, =, and <). M.4.NBT.A.3 Use place value understanding to generate estimates for real-world problem situations, with multi-digit whole numbers, using strategies such as mental math, benchmark numbers, compatible numbers, and rounding. Assess the reasonableness of their estimates. (e.g. Is my estimate too low or too high? What degree of precision do I need for this situation?) **B. Use place value understanding and properties of operations to perform multi-digit arithmetic.** M.4.NBT.B.4 Flexibly and efficiently add and subtract multi-digit whole numbers using strategies or algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. M.4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using	**Generalize place value understanding for multi-digit whole numbers.** 4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* 4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols ~~to record~~ the result~~s~~ of comparison~~s~~. 4.NBT.3 Use place value understanding to ~~round~~ multi-digit whole numbers ~~to any place.~~ **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.4 ~~Fluently~~ add and subtract multi-digit whole numbers using ~~the standard~~ algorithm. 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies

strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. M.4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Number and Operations - Fractions - Grade 4	
Number and Operations - Fractions (4.NF) Grade 4 assessment expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100 but students should have instructional experiences with other sized fractions. **A. Extend understanding of fraction equivalence.** M.4.NF.A.1 Understand fraction equivalence. a. Explain why a fraction is equivalent to another fraction by using visual fraction models (e.g., tape diagrams and number lines), with attention to how the number and the size of the parts differ even though the two fractions themselves are the same size. b. Understand and use a general principle to recognize and generate equivalent fractions that name the same amount.	**Number and Operations - Fractions (4.NF)**[3] **Extend understanding of fraction equivalence ~~and ordering.~~** 4.NF.1 Explain why a fraction ~~a/b~~ is equivalent to a fraction ~~$(n \times a) / (n \times b)$~~ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

M.4.NF.A.2 Compare fractions with different numerators and different denominators while recognizing that comparisons are valid only when the fractions refer to the same whole. Justify the conclusions by using visual fraction models (e.g., tape diagrams and number lines) and by reasoning about the size of the fractions, using benchmark fractions (including whole numbers), or creating common denominators or numerators. Describe the result of the comparison using words and symbols (>, =, and <).	4.NF.2 Compare ~~two~~ fractions with different numerators and different denominators, ~~e.g.,~~ by creating common denominators or numerators, or by ~~comparing to~~ a benchmark fraction ~~such as ½.~~ Recognize that comparisons are valid only when the ~~two~~ fractions refer to the same whole. ~~Record~~ the results of comparisons ~~with~~ symbols >, =, ~~or~~ <, and justify the conclusions~~, e.g.,~~ by using a visual fraction model.
B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	**Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**
M.4.NF.B.3 Understand composing and decomposing fractions. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of unit fractions and/or multiples of that unit fraction in more than one way, recording each decomposition by an equation. Justify decompositions with explanations, visual fraction models, or equations. *For example: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract fractions, including mixed numbers, with like denominators (eg., 3/8 + 2/8) and related denominators (e.g., 1/2 +1/4 , 1/3 + 1/6) by using visual fraction models (e.g., tape diagrams and number lines), properties of operations, and the relationship between addition and subtraction.	4.NF.3 Understand ~~a fraction a/b with a>1 as a sum of fractions 1/b.~~ a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions ~~with the same denominator~~ in more than one way, recording each decomposition by an equation. Justify decompositions~~, e.g., by using a~~ visual fraction model. Examples: *3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, ~~e.g., by replacing each mixed number with an equivalent fraction, and/or~~ by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions ~~referring to the same whole and~~ having like

d. Solve word problems involving addition and subtraction of fractions with like and related denominators, including mixed numbers, by using visual fraction models and equations to represent the problem. Students are not required to rename fractions in lowest terms nor use least common denominators. M.4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a whole number times a fraction. a. Understand a fraction as a group of unit fractions or as a multiple of a unit fraction. *For example, 5/4 can be represented visually as 5 groups of 1/4 , as a sum of unit fractions 1/4 + 1/4 + 1/4 + 1/4 + 1/4, or as a multiple of a unit fraction 5 x 1/4.* b. Represent a whole number times a non-unit fraction (e.g., 3 x 2/5) using visual fraction models and understand this as combining equal groups of the non-unit fraction (3 groups of 2/5) and as a collection of unit fractions (6 groups of 1/5), recognizing this product as 6/5. c. Solve word problems involving multiplication of a whole number times a fraction by using visual fraction models and equations to represent the problem. Understand a reasonable answer range when multiplying with fractions. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	denominators~~, e.g.,~~ by using visual fraction models and equations to represent the problem. 4.NF.4 Apply and extend previous understandings of multiplication to multiply a ~~fraction by a whole number.~~ a. Understand a fraction ~~*a/b*~~ as a multiple of ~~*1/b*~~. *For example, ~~use a visual fraction model to represent~~ 5/4 as ~~the product~~ 5 x (1/4)~~, recording the conclusion by the equation 5/4 = 5 x (1/4).~~* b. ~~Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number.~~ *~~For example, use a visual fraction model to express~~ 3 x (2/5) ~~as 6 x (1/5)~~, recognizing this product as 6/5. ~~(In general, n x (a/b) = (n x a)/b.)~~* c. Solve word problems involving multiplication of ~~a fraction by a whole number, e.g.,~~ by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

C. Understand decimal notation for fractions, and compare decimal fractions. M.4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* M.4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100, connect decimals to real-world contexts, and represent with visual models (e.g., number line or area model). *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line.* M.4.NF.C.7 Compare decimals to hundredths by reasoning about their size and using benchmarks. Recognize that comparisons are valid only when the decimals refer to the same whole. Justify the conclusions, by using explanations or visual models (e.g., number line or area model) and describe the result of the comparison using words and symbols (>, =, and <).	**Understand decimal notation for fractions, and compare decimal fractions.** 4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* 4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line ~~diagram~~.* 4.NF.7 Compare ~~two~~ decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the ~~two~~ decimals refer to the same whole. ~~Record~~ the result~~s~~ of comparison~~s~~ with the symbol~~s~~ >, =, or <, and justify the conclusions, ~~e.g.,~~ by using a visual model.
Measurement and Data - Grade 4	
A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. M.4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min., sec. Within a single system of measurement, express measurements in a	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** 4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a

larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. *For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* M.4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as a number line that feature a measurement scale. M.4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* **B. Represent and interpret data.** M.4.MD.B.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* 4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as a number line that feature a measurement scale. 4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* **Represent and interpret data.** 4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

C. Geometric measurement: understand concepts of angle and measure angles. M.4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle" and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. M.4.MD.C.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. M.4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	**Geometric measurement: understand concepts of angle and measure angles.** 4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle" and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. 4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. 4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
Geometry - Grade 4	
A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	**Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**

M.4.G.A.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. M.4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. M.4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. 4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Grade 5 Standards, 2010 Standards compared to 2021 Revisions

This document is intended to be used beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of *Wisconsin Standards for Mathematics*.

For each grade level or high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

- Red font in the 2021 standard indicates that a change was made.
- Strikethrough in the 2010 standard indicates that language was removed.

For Grade 5 Standards, the following revisions were made (in addition to revisions in the language of the grade-level standards):

- Specific narratives were written as Standards for Mathematical Practice (K-5) to illustrate what the Standards for Mathematical Practice (K-12) look like in the elementary grades.
- The fifth grade introduction now reads as a narrative rather than a numbered list. In addition, language was added to more fully describe the idea of focusing on critical areas while still reminding readers that it is critical to teach all of the standards.
- The fifth grade introduction includes language to describe what mathematical modeling might look like at the 3-5 grade band in support of all students as flexible users of mathematics (shift #2).
- The *Multiplication and Division Problem Situations* (Appendix 1, Table 2A) and *Multiplication and Division - Measurement examples (Appendix 1, Table 2B)* replace *Common Multiplication and Division Situations (*Table 2 from *Wisconsin Standards for Mathematics [2010]).* Table 2A has been updated to include generalizable equations for each type of unknown. The number of groups and group size are no longer distinguished when considering arrays of objects since both a row or a column can be a group. Table 2B now shows measurement examples in a separate table and also includes generalizable equations. The notes underneath both Tables 2A and 2B now indicate which problem situations appear in which grade levels and connect partitive or sharing language and quotative or measurement language to the different types of division. These adjustments now match the *Progressions for the Common Core State Standards for Mathematics* (2019) situation tables.

2021 Standards	2010 Standards
Operations and Algebraic Thinking - Grade 5	
A. Write and interpret numerical expressions. M.5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. M.5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x(18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	**Write and interpret numerical expressions.** 5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. 5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x(18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*
B. Analyze patterns and relationships. M.5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	**Analyze patterns and relationships.** 5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Number and Operations in Base Ten - Grade 5	
A. Understand the place value system. M.5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. M.5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. M.5.NBT.A.3 Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000). b. Compare decimals to thousandths based on meanings of the digits in each place and describe the result of the comparison using words and symbols (>, =, and <). M.5.NBT.A.4 Use place value understanding to generate estimates for problems in real-world situations, with decimals, using strategies such as mental math, benchmark numbers, compatible numbers, and rounding. Assess the reasonableness of their estimates (e.g. Is my estimate too low or too high? What degree of precision do I need for this situation?	**Understand the place value system.** 5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. 5.NBT.3 Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000). b. Compare ~~two~~ decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to ~~record~~ the result~~s~~ of comparison~~s.~~ 5.NBT.4 Use place value understanding to ~~round~~ decimals ~~to any place.~~

B. Perform operations with multi-digit whole numbers and with decimals to hundredths. M.5.NBT.B.5 Flexibly and efficiently multiply multi-digit whole numbers using strategies or algorithms based on place value, area models, and the properties of operations. M.5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. M.5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**Perform operations with multi-digit whole numbers and with decimals to hundredths.** 5.NBT.5 ~~Fluently~~ multiply multi-digit whole numbers using ~~the standard~~ algorithm. 5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
Number and Operations - Fractions - Grade 5	
Number and Operations - Fractions Students are not required to rename fractions in lowest terms nor use least common denominators. **A. Use equivalent fractions as a strategy to add and subtract fractions.** M.5.NF.A.1 Add and subtract fractions and mixed numbers using flexible and efficient strategies, including renaming fractions with	**Number and Operations - Fractions** **Use equivalent fractions as a strategy to add and subtract fractions.** 5.NF.1 Add and subtract fractions ~~with unlike denominators (including~~ mixed numbers) ~~by replacing given fractions with~~

equivalent fractions. Justify using visual models (e.g., tape diagrams or number lines) and equations. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12.* M.5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 <1/2 .* **B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** M.5.NF.B.3 Interpret a fraction as an equal sharing division situation, where a quantity (the numerator) is divided into equal parts (the denominator). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, by using visual fraction models (e.g., tape diagrams or area models) or equations to represent the problem. *For example, when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* M.5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction times a whole number (e.g., 2/3	equivalent fractions ~~in such a way as to produce an equivalent sum or difference of fractions with like denominators.~~ *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12.* ~~*(In general, a/b + c/d = (ad + bc/bd.)*~~ 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, ~~including cases of unlike denominators, e.g., by~~ using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 <1/2 .* **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** 5.NF.3 Interpret a fraction as division ~~of the~~ numerator ~~by the~~ denominator ~~*(a/b = a ÷ b)*~~. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, ~~e.g.,~~ by using visual fraction models or equations to represent the problem. *For example,* ~~*interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that*~~ *when 3 wholes are shared equally among 4 people each person has a share of size 3/4 . If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

x 4) or a fraction times a fraction (e.g, 2/3 x 4/5), including mixed numbers. a. Represent word problems involving multiplication of fractions using visual models to develop flexible and efficient strategies. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15.* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. M.5.NF.B.5 Interpret multiplication as scaling (resizing) by estimating whether a product will be larger or smaller than a given factor on the basis of the size of the other factor, without performing the indicated multiplication. a. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number and explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number. b. Relate the principle of fraction equivalence to the effect of multiplying or dividing a fraction by 1 or an equivalent form of 1 (e.g., 3/3, 5/5).	a. ~~Interpret the product (*a*/*b*) x *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* x *q* ÷ *b*.~~ *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. ~~(In general, (a/b) x (c/d) = ac/bd.)~~* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. 5.NF.5 Interpret multiplication as scaling (resizing) by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaini~~ng~~ why multiplying a given number by a fraction greater than 1 results in a product greater than the given number ~~(recognizing multiplication by whole numbers greater than 1 as a familiar case)~~; explaini~~ng~~ why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; ~~and~~ relating the

	principle of fraction equivalence ~~a/b = (n x a)/(n x b)~~ to the effect of multiplying ~~a/b~~ by 1.
M.5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers by using visual fraction models (e.g., tape diagrams, area models, or number lines) and equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers~~, e.g.,~~ by using visual fraction models ~~or~~ equations to represent the problem.
M.5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers (e.g.,*1/3* ÷ 4) and whole numbers by unit fractions (e.g., 4 ÷ 1/5). Students able to multiply fractions can develop strategies to divide fractions by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.	5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers (e.g.,*1/3* ÷ 4) and whole numbers by unit fractions (e.g., 4 ÷ 1/5). ~~[1]~~
a. Interpret and represent division of a unit fraction by a non-zero whole number as an equal sharing division situation. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*	a. Interpret division of a unit fraction by a non-zero whole number~~, and compute such quotients~~. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*
b. Interpret and represent division of a whole number by a unit fraction as a measurement division situation. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship*	b. Interpret division of a whole number by a unit fraction, ~~and compute such quotients~~. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 4 people share 1/3 lb of chocolate equally? Each person gets 1/12 lb of chocolate. How many 1/5-cup servings are in 4 cups of raisins? There are 20 servings of size 1/5-cup of raisins.*	c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions~~, e.g.,~~ by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if ~~3~~ people share ~~1/2~~ lb of chocolate equally? How many ~~1/3~~-cup servings are in ~~2~~ cups of raisins?*
Measurement and Data - Grade 5	
A. Convert like measurement units within a given measurement system. M.5.MD.A.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **B. Represent and interpret data.** M.5.MD.B.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2,1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.	**Convert like measurement units within a given measurement system.** 5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **Represent and interpret data.** 5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2,1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.	
C. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**
M.5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube", is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube", is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
M.5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.	5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
M.5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with	5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = \cancel{b} \times h$ for rectangular prisms to find volumes of right rectangular prisms with

whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.
Geometry - Grade 5	
A. Graph points on the coordinate plane to solve real-world and mathematical problems. M.5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). M.5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **B. Classify two-dimensional figures into categories based on their properties.**	**Graph points on the coordinate plane to solve real-world and mathematical problems.** 5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). 5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **Classify two-dimensional figures into categories based on their properties.**

M.5.G.B.3 Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* M.5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.	5.G.3 Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* 5.G.4 Classify two-dimensional figures in a hierarchy based on properties.

6-8 Standards for Mathematical Practice

2021 Standards	2010 Standards
A comparison of 2021, 6-8 Standards for Mathematical Practice (SMP) and 2010, K-12 SMP	
Math Practice 1: Make sense of problems and persevere in solving them. Mathematically proficient middle school students set out to understand a problem and then look for entry points to its solution. Students identify questions to ask and make observations about the situation by using noticing and attending to aspects of the problem that look familiar. Students make assumptions where needed to make the problem more clearly defined. They analyze problem conditions and goals, translating, for example, verbal descriptions into equations, diagrams, or graphs as part of the process. They consider analogous problems, and try special cases and simpler forms of the original in order to gain insight into its solution. For example, to understand why a 20% discount followed by a 20% markup does not return an item to its original price, they might translate the situation into a tape diagram or a general equation; or they might first consider the situation for an item priced at $100. Mathematically proficient students can explain how alternate representations of problem conditions relate to each other. For example, they can identify connections between the solution to a word problem that uses only arithmetic and a solution that uses variables and algebra; and they can navigate among verbal descriptions, tables, graphs, and equations representing linear	**Math Practice 1:** Make sense of problems and persevere in solving them. Mathematically proficient students ~~start by explaining to themselves the meaning of~~ a problem and look~~ing~~ for entry points to its solution. They analyze ~~givens, constraints, relationships,~~ and goals. ~~They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.~~ They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. ~~Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need.~~ Mathematically proficient students can explain ~~correspondences between~~ equations, verbal descriptions, tables, and graphs or draw diagrams ~~of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.~~ Mathematically proficient students check their ~~answers to problems using a different method, and they~~ continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and ~~identify correspondences between different~~ approaches.

relationships to gain insights into the role played by constant rate of change. Mathematically proficient students check their approach, continually asking themselves "Does this approach make sense?" and "Can I solve the problem in a different way?" Students ask themselves these types of questions as a way to persevere through problem solving. While working on a problem, they monitor and evaluate their progress and change course if necessary. Students will reflect and revise their solution as needed. They can understand the approaches of others to solving complex problems and compare approaches. **Math Practice 2:** Reason abstractly and quantitatively. Mathematically proficient middle school students make sense of quantities and relationships in problem situations. For example, they can apply ratio reasoning to convert measurement units and proportional relationships to solve percent problems. They represent problem situations using symbols and then manipulate those symbols in search of a solution (decontextualize). They can, for example, solve problems involving unit rates by representing the situations in equation form. Mathematically proficient students also pause as needed during problem solving to validate the meaning of the symbols involved. In the process, they can look back at the applicable units of measure to clarify or inform solution pathways (contextualize). Students can integrate quantitative information and concepts expressed in text and visual formats. Students can examine the constant and coefficient used in a linear function and express the meaning of those numbers related to a contextual situation. They can work with the function in different representations, such as a graph, keeping in mind the slope and vertical intercept have	**Math Practice 2:** Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. ~~They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to~~ *decontextualize*~~—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to~~ *contextualize*, ~~to~~ pause as needed during ~~the manipulation process in order~~ to ~~probe into the referents for~~ the symbols involved~~:~~ Quantitative reasoning entails ~~habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and~~ knowing and flexibly using different properties of operations and objects.

meaning related to the context. Quantitative reasoning also entails knowing and flexibly using different properties of operations and objects. For example, in middle school, students use properties of operations to generate equivalent expressions and use the number line to understand multiplication and division of rational numbers. **Math Practice 3:** Construct viable arguments, and appreciate and critique the reasoning of others. Mathematically proficient middle school students understand and use assumptions, definitions, and previously established results in constructing verbal and written arguments. They understand the importance of making and exploring the validity of conjectures. They can recognize and appreciate the use of counterexamples. For example, using numerical counterexamples to identify common errors in algebraic manipulation, such as thinking that 5 – 2x is equivalent to 3x. Conversely, given a pair of equivalent algebraic expressions, they can show that the two expressions name the same number regardless of which value is substituted into them by showing which properties of operations can be applied to transform one expression into the other. They can explain and justify their conclusions to others using numerals, symbols, and visuals. They also reason inductively about data, making plausible arguments that take into account the context from which the data arose. For example, they might argue that the great variability of heights in their class is explained by growth spurts, and that the small variability of ages is explained by school admission policies. While communicating their own mathematical ideas is important, middle school students also learn to be open to others' mathematical ideas. They appreciate a different perspective or approach to a problem and learn how to respond to those ideas,	**Math Practice 3:** Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use ~~stated~~ assumptions, definitions, and previously established results in constructing arguments. They make conjectures ~~and build a logical progression of statements to explore the truth of their conjectures.~~ They ~~are able to analyze situations by breaking them into cases, and~~ can recognize and use counterexamples. They justify their conclusions, ~~communicate them to others, and respond to the arguments of others.~~ They reason inductively about data, making plausible arguments that take into account the context from which the data arose. ~~Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or~~ read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

respecting the reasoning of others (Gutiérrez 2017, 17-18). Together, students make sense of the mathematics, asking helpful questions such as "How did you get that?" "Why is that true?" and "Does that always work?" that clarify or deepen everyone's understanding. Mathematically proficient students are able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and if there is a flaw in an argument, explain what it is. Students engage in collaborative discussions, drawing on evidence from problem texts and arguments of others, follow conventions for collegial discussions, and qualify their own views in light of evidence presented. They can present their findings and results to a given audience through a variety of formats such as posters, whiteboards, and interactive materials. **Math Practice 4:** Model with mathematics. "In the course of a student's mathematics education, the word 'model' is used in many ways. Several of these, such as manipulatives, demonstration, role modeling, and conceptual models of mathematics, are valuable tools for teaching and learning. However, they are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer big, messy, reality-based questions" (Bliss and Libertini 2016, 7). Mathematically proficient middle school students formulate their own problems that emerge from natural circumstances as they mathematize the world around them. They can identify the mathematical elements of a situation and generate questions that can be addressed using mathematics (e.g., noticing and wondering). Middle school students can see a complicated problem and	**Math Practice 4:** Model with mathematics. Mathematically proficient students ~~can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know~~ are comfortable making assumptions ~~and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables,~~

understand how that problem contains smaller problems to be solved. They are comfortable making assumptions as they decide "what matters". Mathematically proficient middle students understand that there are multiple solutions to a modeling problem so they are working to find a solution rather than the solution. Students make judgements about what matters and assess the quality of their solution (Bliss and Libertini 2016, 10). They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving their mathematical modeling approach if it has not served its purpose. As they decontextualize the situation and represent it mathematically, they are also reasoning abstractly (Math Practice 2). In the middle school grades, students encounter mathematical opportunities each and every day at school and at home. Mathematically proficient middle school students might consider how to plan a route to get to school with their friends. Students might then need to make assumptions about travel time and when to leave the house. In the morning they implement their plan and revise it by changing the departure time or including additional friends to the route. As a classroom, students might plan a fundraising event involving selling popcorn after school. In this example, sometimes students will be engaged in only a part of the modeling cycle such as making assumptions about how much to charge or how much popcorn to make (Godbold, Malkevitch, Teague, and van der Kooij 2016, 50). Note: Although physical objects and drawings can be used to model a situation, using these tools absent a contextual situation is not an example of Math Practice 4. For example, drawing an area model to illustrate the distributive property in 4(t + s) = 4t + 4s would not be	~~graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely~~ interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **Math Practice 5:** Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include

an example of Math Practice 4. Math Practice 4 is about applying math to a problem in context. **Math Practice 5:** Use appropriate tools strategically. Mathematically proficient middle school students strategically consider the available tools when solving a mathematical problem or exploring a mathematical relationship. These tools might include pencil and paper, concrete models, a ruler, a protractor, a dynamic graphing tool , a spreadsheet, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insights to be gained and their limitations. For example, they use estimation to check reasonableness; graph functions defined by expressions to picture the way one quantity depends on another; use algebra tiles to see how the properties of operations familiar from the elementary grades continue to apply to algebraic expressions; use an area model to visualize multiplication of rational numbers; use dynamic graphing tools to approximate solutions to systems of equations; use spreadsheets to analyze data sets of realistic size; or use dynamic geometry software to discover properties of parallelograms. Students are also strategic about when not to use tools, such as by simplifying an expression before substituting values into it (Math Practice 7), or rounding the inputs to a calculation and calculating on paper when an approximate answer is enough (Math Practice 6). When making mathematical models, students know that technology can enable them to visualize the results of their assumptions, to explore consequences, and to compare predictions with data (Math Practice 4). Mathematically proficient students are able to identify relevant external	pencil and paper, concrete models, a ruler, a protractor, a ~~calculator~~, a spreadsheet, ~~a computer algebra system~~, a statistical package, or dynamic geometry software. Proficient students ~~are sufficiently familiar with tools appropriate for their grade or course to~~ make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, ~~mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge.~~ When making mathematical models, ~~they~~ know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students ~~at various grade levels~~ are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. ~~They are able to use technological tools to explore and deepen their understanding of concepts.~~

mathematical resources, such as digital content located on a website, and use them to pose or solve problems. **Math Practice 6:** Attend to precision. Mathematically proficient middle school students communicate precisely to others both verbally and in writing. They present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound and valid reasoning, well-chosen details, and precise language. They use clear definitions in discussion with others and in their own reasoning and determine the meaning of symbols, terms, and phrases as used in specific mathematical contexts. For example, they can use the definition of rational numbers to explain why the sqrt(2) is irrational. Middle school students can describe congruence and similarity in terms of transformations in the plane. They decide which parts of a problem need to be defined by a variable, state the meaning of the symbols, consistently and appropriately, such as independent and dependent variables of a linear equation. They are careful about specifying units of measure, and label axes to display the correct correspondence between quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate to the context. For example, they accurately apply scientific notation to large numbers and use measures of center to describe data sets. In statistics and probability, students must attend to precision in the manner they write their statistical questions, the manner in which they collect their data, and the process they use to develop a simulation. Mathematically proficient middle school students care that an answer is right or reasonable; they attend to precision when they	**Math Practice 6:** Attend to precision. Mathematically proficient students ~~try to~~ communicate precisely to others. They ~~try to~~ use clear definitions in discussion with others and in their own reasoning. ~~They state~~ the meaning of the symbols ~~they choose, including using the equal sign consistently and appropriately.~~ They are careful about specifying units of measure, and label~~ing~~ axes to ~~clarify~~ the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate ~~for the problem~~ context. ~~In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.~~

check their work; they solve the problem another way; they make revisions where appropriate. **Math Practice 7:** Look for and make use of structure. Mathematically proficient middle school students look closely to discern a pattern or structure. They might use the structure of the number line to demonstrate that the distance between two rational numbers is the absolute value of their difference, ascertain the relationship between slopes and solution sets of systems of linear equations, and see that the equation 3x = 2y represents a proportional relationship with a unit rate of 3/2 = 1.5. They might recognize how the Pythagorean Theorem is used to find distances between points in the coordinate plane and identify right triangles that can be used to find the length of a diagonal in a rectangular prism. They also can step back for an overview and shift perspective, as in finding a representation of consecutive numbers that shows all sums of three consecutive whole numbers are divisible by six. They can see complicated things as single objects, such as seeing two successive reflections across parallel lines as a translation along a line perpendicular to the parallel lines or understanding 1.05*a* as an original value, *a*, plus 5% of that value, 0.05*a*. They can evaluate numeric expressions without combining each term in the order they are given. Example $2\frac{7}{8}$ - (-31/7) - $2\frac{7}{8}$. **Math Practice 8:** Look for and express regularity in repeated reasoning. Mathematically proficient middle school students notice if calculations are repeated, and look for both general methods and general and efficient methods. Working with tables of equivalent	**Math Practice 7:** Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. ~~Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.~~ They also can step back for an overview and shift perspective. They can see complicated things, ~~such as some algebraic expressions,~~ as single objects ~~or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.~~ **Math Practice 8:** Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and ~~for shortcuts~~. ~~Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again,~~

ratios, they might deduce the corresponding multiplicative relationships and make generalizations about the relationship to rates. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $^{(y - 2)}/_{(x - 1)} = 3$. Noticing the regularity with which interior angle sums increase with the number of sides in a polygon might lead them to the general formula for the interior angle sum of an *n*-gon. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their results throughout all stages of the process.	~~and conclude they have a repeating decimal.~~ By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity ~~in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$~~ might lead them to the general formula for the ~~sum of a geometric series~~. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their ~~intermediate~~ results.

Grade 6 Standards, 2010 Standards compared to 2021 Revisions

This document is intended to be used beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of *Wisconsin Standards for Mathematics*.

For each grade level or high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

- Red font in the 2021 standard indicates that a change was made.
- Strikethrough in the 2010 standard indicates that language was removed.

For Grade 6 Standards, the following revisions were made (in addition to revisions in the language of the grade-level standards):

- Specific narratives were written as Standards for Mathematical Practice (6-8) to illustrate what the Standards for Mathematical Practice (K-12) look like in the middle grades.
- The sixth grade introduction now reads as a narrative rather than a numbered list. In addition, language was added to more fully describe the idea of focusing on critical areas while still reminding readers that it is critical to teach all of the standards.
- The sixth grade introduction includes language to describe what mathematical modeling might look like at the 6-8 grade band in support of all students as flexible users of mathematics (shift #2).
- Mathematical modeling is best interpreted, not as a collection of isolated topics, but rather in relation to other standards. To support these relationships, content standards that may be particularly valuable in middle school have been indicated with an (M) symbol. The (M) symbol appears following those cluster statements throughout grades 6-8.

2021 Standards	2010 Standards
Ratios and Proportional Relationships - Grade 6	
A. Understand ratio concept and use ratio reasoning to solve problems. (M) M.6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* M.6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* Expectations for unit rates in this grade are limited to non-complex fractions. M.6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the	**Understand ratio concept and use ratio reasoning to solve problems.** 6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* 6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*~~[1]~~ 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line ~~diagram~~s, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the

tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
The Number System - Grade 6	
A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions. M.6.NS.A.1 Interpret, represent and compute division of fractions by fractions; and solve word problems by using visual fraction models (e.g., tape diagrams, area models, or number lines), equations, and the relationship between multiplication and division. *For example, create a story context for (2/3) ÷ (3/4) such as "How many ¾-cup servings are in ⅔ of a cup of yogurt" or "How wide is a rectangular strip of land with length 3/4 mile and area 2/3 square mile?" Explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3.*	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** 6.NS.1 Interpret and compute ~~quotients~~ of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction ~~model to show the quotient; use~~ the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. ~~(In general, (a/b) ÷ (c/d) = ad / bc.) How much chocolate will each person get if 3 people share ½ lb of chocolate equally?~~ How many 3/4-cup*

B. Flexibly and efficiently compute with multi-digit numbers and find common factors and multiples. M.6.NS.B.2 Flexibly and efficiently divide multi-digit whole numbers using strategies or algorithms based on place value, area models, and the properties of operations. M.6.NS.B.3 Flexibly and efficiently add, subtract, multiply, and divide multi-digit decimals using strategies or algorithms based on place value, visual models, the relationship between operations and the properties of operations. M.6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* **C. Apply and extend previous understandings of numbers to the system of rational numbers. (M)** M.6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	*servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mile and area 2/3 square mi?* **Compute fluently with multi-digit numbers and find common factors and multiples.** 6.NS.2 ~~Fluently~~ divide multi-digit whole numbers using ~~the standard~~ algorithm. 6.NS.3 ~~Fluently~~ add, subtract, multiply, and divide multi-digit decimals using ~~the standard~~ algorithm for each operation. 6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* **Apply and extend previous understandings of numbers to the system of rational numbers.** 6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

M.6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line; find and position pairs of integers and other rational numbers on a coordinate plane.	6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line ~~diagram~~; find and position pairs of integers and other rational numbers on a coordinate plane.
M.6.NS.C.7 Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.	6.NS.7 Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.*

For example, write -3 °C > -7 °C to express the fact that -3°C is warmer than -7 °C. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.* M.6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write -3 °C > -7 °C to express the fact that -3°C is warmer than -7 °C.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.* 6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
Expressions and Equations - Grade 6	
A. Apply and extend previous understandings of arithmetic to algebraic expressions. M.6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.	**Apply and extend previous understandings of arithmetic to algebraic expressions.** 6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

M.6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\ s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$. M.6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x +*	6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\ s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$. 6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. M.6.EE.A.4 Identify when two expressions are equivalent (e.g., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*	M.6.EE.A.4 Identify when two expressions are equivalent (~~i.e.~~, when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
B. Reason about and solve one-variable equations and inequalities.	**Reason about and solve one-variable equations and inequalities.**
M.6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
M.6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
M.6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
M.6.EE.B.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely	6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely

many solutions; represent solutions of such inequalities on number line diagrams. **C. Represent and analyze quantitative relationships between dependent and independent variables. (M)** M.6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d=65t to represent the relationship between distance and time.*	many solutions; represent solutions of such inequalities on number line diagrams. **Represent and analyze quantitative relationships between dependent and independent variables.** 6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*
Geometry - Grade 6	
A. Solve real-world and mathematical problems involving area, surface area, and volume. (M) M.6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. M.6.G.A.2 Find volumes of right rectangular prisms with fractional edge lengths by using physical or virtual unit cubes. Develop	**Solve real-world and mathematical problems involving area, surface area, and volume.** 6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. 6.G.2 Find volumes of ~~a~~ right rectangular prisms~~s~~ with fractional edge lengths by ~~packing it with~~ unit cubes ~~of the appropriate unit~~

(construct) and apply the formulas $V = l\ w\ h$ and $V = B\ h$ to find volumes of right rectangular prisms in the context of solving real-world and mathematical problems. M.6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. M.6.G.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	~~fraction edge length, and show that the volume is the same as would be found by multiplying the edge lengths of the prism~~. Apply the formulas $V = l\ w\ h$ and $V = \cancel{b}\ h$ to find volumes of right rectangular prisms ~~with fractional edge lengths~~ in the context of solving real-world and mathematical problems. 6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. 6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
Statistics and Probability - Grade 6	
A. Develop understanding of statistical variability. (M) M.6.SP.A.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* M.6.SP.A.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	**Develop understanding of statistical variability.** 6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* 6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

M.6.SP.A.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **B. Summarize and describe distributions. (M)** M.6.SP.B.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots. M.6.SP.B.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered and the quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation) were given. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **Summarize and describe distributions.** 6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots. 6.SP.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. ~~Giving~~ quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), ~~as well as~~ describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Grade 7 Standards, 2010 Standards compared to 2021 Revisions

This document is intended to be used beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of *Wisconsin Standards for Mathematics*.

For each grade level or high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

- Red font in the 2021 standard indicates that a change was made.
- Strikethrough in the 2010 standard indicates that language was removed.

For Grade 7 Standards, the following revisions were made (in addition to revisions in the language of the grade-level standards):

- Specific narratives were written as Standards for Mathematical Practice (6-8) to illustrate what the Standards for Mathematical Practice (K-12) look like in the middle grades.
- The seventh grade introduction now reads as a narrative rather than a numbered list. In addition, language was added to more fully describe the idea of focusing on critical areas while still reminding readers that it is critical to teach all of the standards.
- The seventh grade introduction includes language to describe what mathematical modeling might look like at the 6-8 grade band in support of all students as flexible users of mathematics (shift #2).
- Mathematical modeling is best interpreted, not as a collection of isolated topics, but rather in relation to other standards. To support these relationships, content standards that may be particularly valuable in middle school have been indicated with an (M) symbol. The (M) symbol appears following those cluster statements throughout grades 6-8.

2021 Standards	2010 Standards
Ratios and Proportional Relationships - Grade 7	
A. Analyze proportional relationships and use them to solve real-world and mathematical problems. (M) M.7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2 / 1/4 miles per hour, equivalently 2 miles per hour.* M.7.RP.A.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between*	**Analyze proportional relationships and use them to solve real-world and mathematical problems.** 7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2 / 1/4 miles per hour, equivalently 2 miles per hour.* 7.RP.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

the total cost and the number of items can be expressed as t = pn. d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. M.7.RP.A.3 Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
The Number System - Grade 7	
A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. M.7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line. a. Describe situations in which opposite quantities combine to make 0. *For example, if you earn $10 and then spend $10, you are left with $0.*	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** 7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line ~~diagram~~. a. Describe situations in which opposite quantities combine to make 0. *For example, ~~a hydrogen atom has 0 charge because its two constituents are oppositely charged~~.*

b. Understand *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, *p* - *q* = *p* + (-*q*). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. M.7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then -(*p*/*q*) = (-*p*)/*q* = *p*/(-*q*). Interpret quotients of rational numbers by describing real world contexts.	b. Understand *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, *p* - *q* = *p* + (-*q*). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then -(*p*/*q*) = (-*p*)/*q* = *p*/(-*q*). Interpret quotients of rational numbers by describing real world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. M.7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.	c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.[1]
Expressions and Equations - Grade 7	
A. Use properties of operations to generate equivalent expressions. M.7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. M.7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations. (M)**	**Use properties of operations to generate equivalent expressions.** 7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

M.7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50.* M.7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Flexibly and efficiently apply the properties of operations and equality to solve equations of these forms. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational	7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50.* ~~*If you want to place a towel bar 9 ¾ inches long in the center of a door that is 27 ½ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*~~ 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms ~~fluently~~. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and

numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
Geometry - Grade 7	
A. Draw, construct, and describe geometrical figures and describe the relationships between them. M.7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. M.7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. M.7.G.A.3 Describe the two-dimensional figures that result from slicing three dimensional figures parallel to the base, as in plane sections of right rectangular prisms and right rectangular pyramids. **B. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (M)**	**Draw, construct, and describe geometrical figures and describe the relationships between them.** 7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. 7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. 7.G.3 Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**

M.7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. M.7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. M.7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three- dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. 7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three- dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Statistics and Probability - Grade 7	
A. Use random sampling to draw inferences about a population. (M) M.7.SP.A.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. M.7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election*	**Use random sampling to draw inferences about a population.** 7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. 7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

based on randomly sampled survey data. Gauge how far off the estimate or prediction might be. **B. Draw informal comparative inferences about two populations. (M)** M.7.SP.B.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* M.7.SP.B.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* **C. Investigate chance processes and develop, use, and evaluate probability models. (M)** M.7.SP.C.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates	**Draw informal comparative inferences about two populations.** 7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* 7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* **Investigate chance processes and develop, use, and evaluate probability models.** 7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. M.7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
M.7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

M.7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Grade 8 Standards, 2010 Standards compared to 2021 Revisions

This document is intended to be used beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of *Wisconsin Standards for Mathematics*.

For each grade level or high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

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For Grade 8 Standards, the following revisions were made (in addition to revisions in the language of the grade-level standards):

- Specific narratives were written as Standards for Mathematical Practice (6-8) to illustrate what the Standards for Mathematical Practice (K-12) look like in the middle grades.
- The eighth grade introduction now reads as a narrative rather than a numbered list. In addition, language was added to more fully describe the idea of focusing on critical areas while still reminding readers that it is critical to teach all of the standards.
- The eighth grade introduction includes language to describe what mathematical modeling might look like at the 6-8 grade band in support of all students as flexible users of mathematics (shift #2).
- Mathematical modeling is best interpreted, not as a collection of isolated topics, but rather in relation to other standards. To support these relationships, content standards that may be particularly valuable in middle school have been indicated with an (M) symbol. The (M) symbol appears following those cluster statements throughout grades 6-8.

2021 Standards	2010 Standards
The Number System - Grade 8	
A. Know that there are numbers that are not rational, and approximate them by rational numbers. M.8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and use patterns to rewrite a decimal expansion that repeats into a rational number. M.8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	**Know that there are numbers that are not rational, and approximate them by rational numbers.** 8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and ~~convert~~ a decimal expansion ~~which~~ repeats ~~eventually~~ into a rational number. 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line ~~diagram~~, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
Expressions and Equations - Grade 8	
A. Work with radicals and integer exponents. M.8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*	**Work with radicals and integer exponents.** 8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*

M.8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational. M.8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.* M.8.EE.A.4 Use technology to interpret and perform operations with numbers expressed in scientific notation. Choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). **B. Understand the connections between proportional relationships, lines, and linear equations.** M.8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* M.8.EE.B.6 Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the	8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational. 8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.* 8.EE.4 Perform operations with numbers expressed in scientific notation ~~including problems where both decimal and scientific notation are used. Use scientific notation and~~ choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). ~~Interpret scientific notation that has been generated by technology.~~ **Understand the connections between proportional relationships, lines, and linear equations.** 8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. **C. Analyze and solve linear equations and pairs of simultaneous linear equations. (M)** M.8.EE.C.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into equivalent forms. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. M.8.EE.C.8 Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.	8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. **Analyze and solve linear equations and pairs of simultaneous linear equations.** 8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into ~~simpler~~ forms~~, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).~~. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. 8.EE.8 Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables by graphing and analyzing tables. Solve simple cases represented in algebraic symbols by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	b. Solve systems of two linear equations in two variables ~~algebraically, and estimate solutions~~ by graphing ~~the equations.~~ Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
Functions - Grade 8	
A. Define, evaluate, and compare functions. M.8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a numerically valued function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8. M.8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* M.8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.	**Define, evaluate, and compare functions.** 8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. 8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* 8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as*

For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line. **B. Use functions to model relationships between quantities. (M)** M.8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. M.8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear, continuous or discrete). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	*a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* **Use functions to model relationships between quantities.** 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Geometry - Grade 8	
A. Understand congruence and similarity using physical models, transparencies, or geometry software. M.8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length.	**Understand congruence and similarity using physical models, transparencies, or geometry software.** 8.G.1 Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. M.8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. M.8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. M.8.G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them. M.8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* **B. Understand and apply the Pythagorean Theorem. (M)** M.8.G.B.6 Justify the relationship between the lengths of the legs and the length of the hypotenuse of a right triangle, and the converse of the Pythagorean theorem.	b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. 8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. 8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. 8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them. 8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* **Understand and apply the Pythagorean Theorem.** 8.G.6 ~~Explain a proof of~~ the Pythagorean theorem and ~~its~~ converse.

M.8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. M.8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **C. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. (M)** M.8.G.C.9 Know the relationship among the formulas for the volumes of cones, cylinders, and spheres (given the same height and diameter) and use them to solve real-world and mathematical problems.	8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** 8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Statistics and Probability - Grade 8	
A. Investigate patterns of association in bivariate data. (M) M.8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. M.8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	**Investigate patterns of association in bivariate data.** 8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. 8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

M.8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* M.8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* 8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*