2010 to 2021 Wisconsin Standards for Mathematics Comparison

This document compares the 2021 and 2010 Wisconsin Standards for Mathematics by showing word-level changes in high school standards. It is intended to be used as a scaffold beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of Wisconsin Standards for Mathematics.

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For each high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

- Red font in the 2021 standard indicates that a change was made.
- Strikethrough in the 2010 standard indicates that language was removed.

This document is not meant to build understanding of the revisions made to the 2021 standards. Instead, educators should see the brief professional learning modules designed to develop understanding of the 2021 standards found at Standards Link.

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K-12 Standards for Mathematical Practice, 2021 Standards Revisions compared to 2010 Standards

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Math Practice 1: Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	**Math Practice 1:** Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Math Practice 2: Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **Math Practice 3:** Construct viable arguments, and appreciate and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the	**Math Practice 2:** Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **Math Practice 3:** Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. While communicating their own mathematical ideas is important, students also learn to be open to others' mathematical ideas. They appreciate a different perspective or approach to a problem and learn how to respond to those ideas, respecting the reasoning of others (Gutiérrez 2017, 17-18). Together, students make sense of the mathematics by asking helpful questions that clarify or deepen everyone's understanding. **Math Practice 4:** Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They	Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. ~~Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful~~ questions to clarify or ~~improve the arguments.~~ **Math Practice 4:** Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and

routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Math Practice 5:
Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Math Practice 6:
Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in

reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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Math Practice 6:
Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose,

their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. **Math Practice 7:** Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.	including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. **Math Practice 7:** Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

Math Practice 8:
Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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High School Standards for Mathematical Practice

2021 Standards	2010 Standards
High School Standards for Mathematical Practice	
Math Practice 1: Make sense of problems and persevere in solving them. Mathematically proficient high school students analyze givens, constraints, relationships, and goals. They make assumptions where needed to make the problem more clearly articulated. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. Students identify questions to ask and make observations about the situation through notice and wondering strategies. While following a solution plan, they continually ask themselves, “Does this make sense?” They monitor and evaluate their progress and revise their plan as needed. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. High school students might, depending on the context of the problem, transform algebraic expressions to provide them with different information about the situation. They change the display on their graphing tool to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph and interpret representations of data, and search for regularity or trends. Mathematically proficient students gain deeper insight into problems by using a different approach,	**Math Practice 1:** Make sense of problems and persevere in solving them. Mathematically proficient students ~~start by explaining to themselves the meaning of a problem and looking for entry points to its solution.~~ They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. ~~Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.~~ Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of

understanding the approaches of others to solving complex problems, and identifying correspondences between different approaches. Mathematically proficient students are engaged in the problem-solving process, do not give up when stuck, and accept that it is acceptable to proceed forward when confronted with confusion and struggle.	others to solving complex problems and identify correspondences between different approaches.
Math Practice 2: Reason abstractly and quantitatively. Mathematically proficient high school students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. For example, high school students might work with an exponential function given in symbolic form, but represents a contextual situation. Students are able to manipulate and change the form of the function and reveal different information about the situation based on the numbers stated in the algebraic representation. In addition, students can write explanatory text that conveys their mathematical analyses and thinking, using relevant and sufficient facts, concrete details, quotations, and coherent discussion of ideas. Students can evaluate multiple sources of information presented in diverse formats (and media) to address a question or solve a problem. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how	**Math Practice 2:** Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. ~~They~~ bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

to compute them; and knowing and flexibly using different properties of operations and objects.	
Math Practice 3: Construct viable arguments, and appreciate and critique the reasoning of others (NCTM 2018, 5). Mathematically proficient high school students understand and use stated assumptions, definitions, and previously established results in constructing verbal and written arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples and specific textual evidence to form their arguments. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is and why. They can construct formal arguments relevant to specific contexts and tasks. High school students learn to determine domains to which an argument applies. Students listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students engage in collaborative discussions, respond thoughtfully to diverse perspectives and approaches, and qualify their own views in light of evidence presented. They can present their findings and results to a	**Math Practice 3:** Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. ~~Elementary~~ students can construct arguments ~~using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades~~. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

given audience through a variety of formats such as posters, whiteboards, and interactive materials.	
Math Practice 4: Model with mathematics. Mathematically proficient high school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. Students that engage in modeling have choice when solving problems. By high school, a student might use geometry to solve a design problem or build a function to describe how one quantity depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. They can carry out all phases of the modeling cycle as outlined in the shift for mathematical modeling (reference). Mathematically proficient high school students also retain the widely applicable techniques they first learned in middle school, such as proportional relationships, rates, and percentages, and apply these techniques as needed to real-world tasks of a complexity appropriate to high school. Note: Although physical objects and visuals can be used to model a situation, using these tools absent a contextual situation is not an	**Math Practice 4:** Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. ~~In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.~~ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

example of Math Practice 4. For example, using algebra tiles or an area model to illustrate factoring a quadratic expression would not be an example of Math Practice 4. Practice standard 4 is about applying math to a problem in context.	
Math Practice 5: Use appropriate tools strategically. Mathematically proficient high school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for high school to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students can use slider bars in a dynamic calculator in order to what-if a situation and see how the graph of a function changes when the parameters in the equation are changed. They can use a spreadsheet to model change when cells are dynamically linked together and values are changed. Students can analyze graphs of functions and solutions generated using a graphing calculator; they also know how to sketch graphs of common functions, choosing this approach over a graphing calculator when a sketch will suffice (Math Practice 6). They detect possible errors by strategically using estimation and other mathematical knowledge, for example anticipating the general appearance of a graph of a function by identifying the structure of its defining expression (Math Practice 7). They are able to use software or websites to quickly generate data displays that would otherwise be time-consuming to construct by hand (such as histograms, box plots, or simulation models for random sampling). Students use technological tools to explore and deepen their	**Math Practice 5:** Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

understanding of mathematical concepts and analyze realistic data sets. When making mathematical models, students know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. (Math Practice 4) Mathematically proficient students are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems.	
Math Practice 6: Attend to precision. Mathematically proficient high school students communicate precisely to others both verbally and in writing, adapting their communication to specific contexts, audiences, and purposes. They increasingly use precise language, not only as a mechanism for effective communication, but also as a tool for understanding and solving problems. Describing their ideas precisely helps students understand the ideas in new ways. They use clear definitions in discussions with others and in their own reasoning. They state the meaning of the symbols that they choose. They are careful about specifying units of measure, labeling axes, defining terms and variables, and calculating accurately and efficiently with a degree of precision appropriate for the problem context. They present logical claims and counterclaims fairly and thoroughly in a way that anticipates the audiences' knowledge, concerns, and possible biases. High school students draw specific evidence from informational sources to support analysis, reflection, and research. They critically evaluate the claims, evidence and reasoning of others and attend to important distinctions with their own claims or inconsistencies in competing claims. Students evaluate the conjectures and claims,	**Math Practice 6:** Attend to precision. Mathematically proficient students ~~try to~~ communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

data, analysis, and conclusions in texts that include quantitative elements, comparing those with information found in other sources. Diligence and attention to detail are mathematical virtues: mathematically proficient students care that an answer is right; they minimize errors by keeping a long calculation organized; they check their work; they solve the problem another way; they take responsibility for careless mistakes and correct them.	
Math Practice 7: Look for and make use of structure. Mathematically proficient high school students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, high school students can see the 14 as 2 x 7 and the 9 as 2 + 7. In an equation, high school students recognize that $12 = 3(x-1)^2$ does not require distribution in the expression on the right in order to carry out the process of solving for x. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square, and use that to realize that its value cannot be more than 5 for any real numbers x and y. Students make use of structure for a purpose, for example by applying the conclusion $5 - 3(x - y)^2 \leq 5$ in the context of an applied optimization problem. Students will notice that the structure of a quadratic function written symbolically in a-b-c form(standard), vertex form, or factored form will reveal different information about the graph of the function.	**Math Practice 7:** Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. ~~Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7.~~ They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Math Practice 8: Look for and express regularity in repeated reasoning. Mathematically proficient high school students notice if calculations are repeated, and look both for general and efficient methods. Noticing the regularity in the way terms sum to zero when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead students to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details and continually evaluating the reasonableness of their intermediate results. When students repeatedly compute products of the form $(ax + b)(ax + b)$ and notice the pattern equals $(a^2x^2 + 2abx + b^2)$ they are looking for and expressing regularity in repeated reasoning. Students change their perspective or view and use what they know. They turn or break down structure to something they know. They solve tasks by solving a sub-problem or smaller version. They might add a line or turn geometric structures so they identify something they have worked with before. By doing this it helps students move forward and not be stuck. Students use repetition in reasoning as they work with various expressions involving exponents and develop an understanding of the various structures. Students connect Pascal's triangle to the repetition in reasoning that occurs in the expansion of binomial coefficients	**Math Practice 8:** Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and ~~for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$.~~ Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their intermediate results.

High School Standards

HS Content Standards, 2021 Standards Revisions compared to 2010 Standards

This document is intended to be used beside existing curriculum to determine where revisions may need to be made to ensure curriculum is aligned with the most recent version (2021) of *Wisconsin Standards for Mathematics.*

For each grade level or high school conceptual category of the standards, the first column of each table shows the 2021 standard while the second column shows the corresponding 2010 standard.

- Red font in the 2021 standard indicates that a change was made.
- Strikethrough in the 2010 standard indicates that content was removed entirely.

2021 Standards	2010 Standards
Number and Quantity	
The Real Number System	
A. Extend the properties of exponents to rational exponents. M.N.RN.A.1 (F2Y) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents. M.N.RN.A.2 (F2Y) Rewrite expressions involving radicals and rational exponents using the properties of exponents. **B. Use properties of rational and irrational numbers.** M.N.RN.A.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**A. Extend the properties of exponents to rational exponents.** N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents ~~to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.~~ N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. **B. Use properties of rational and irrational numbers.** N-RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
Quantities	
A. Reason quantitatively and use units to solve problems. (M)	**A. Reason quantitatively and use units to solve problems.**

M.N.Q.A.1 (F2Y) Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. M.N.Q.A.2 (F2Y) Define appropriate quantities for the purpose of descriptive modeling. M.N.Q.A.3 (F2Y) Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N-Q.2 Define appropriate quantities for the purpose of descriptive modeling. N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
The Complex Number System	
A. Perform arithmetic operations with complex numbers. M.N.CN.A.1 Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* with *a* and *b* real. Understand why complex numbers exist. M.N.CN.A.2(+) Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. M.N.CN.A.3 (+) Find the conjugate of a complex number; use conjugates to find moduli (absolute values) and quotients of complex numbers. **B. Represent complex numbers and their operations on the complex plane.**	**A. Perform arithmetic operations with complex numbers.** N-CN.1 Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* with *a* and *b* real. N-CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **B. Represent complex numbers and their operations on the complex plane.** N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers),

M.N.CN.B.4 (+)Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. M.N.CN.B.5(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1+\sqrt{3}i)^3 = 8$ *because* $(-1+\sqrt{3}i)$ *has modulus* 2 *and argument* 120°. M.N.CN.B.6 (+)Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **C. Use complex numbers in polynomial identities and equations.** M.N.CN.C.7 Solve quadratic equations with real coefficients that have complex solutions. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*. M.N.CN.C.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. M.N.CN.C.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	and explain why the rectangular and polar forms of a given complex number represent the same number. N-CN.5(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1+\sqrt{3}i)^3 = 8$ *because* $(-1+\sqrt{3}i)$ *has modulus* 2 *and argument* 120°. N-CN.6 (+)Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **C. Use complex numbers in polynomial identities and equations.** N-CN.7 Solve quadratic equations with real coefficients that have complex solutions. N-CN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. N-CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities	
A. Represent and model with vector quantities. M.N.VM.A.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*). M.N.VM.A.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. M.N.VM.A.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors. **B. Perform operations on vectors.** M.N.VM.B.4 (+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction **v - w** as **v** + (-**w**), where **-w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the	**A. Represent and model with vector quantities.** N-VM.A.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*). N-VM.A.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. N-VM.A.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors. **B. Perform operations on vectors.** N-VM.B.4 (+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction **v - w** as **v** + (-**w**), where **-w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the

appropriate order, and perform vector subtraction component-wise. M.N.VM.B.5 (+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|\|c\mathbf{v}\|\| = \|c\|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $\|c\|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$). **C. Perform operations on matrices and use matrices in applications.** M.N.VM.C.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. M.N.VM.C.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. M.N.VM.C.8 (+) Add, subtract, and multiply matrices of appropriate dimensions. M.N.VM.C.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	appropriate order, and perform vector subtraction component-wise. N-VM.B.5 (+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|\|c\mathbf{v}\|\| = \|c\|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $\|c\|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$). **C. Perform operations on matrices and use matrices in applications.** N-VM.C.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. N-VM.C.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. N-VM.C.8 (+) Add, subtract, and multiply matrices of appropriate dimensions. N-VM.C.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

M.N.VM.C.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. M.N.VM.C.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. M.N.VM.C.12 (+) Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	M.N.VM.C.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. M.N.VM.C.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. M.N.VM.C.12 (+) Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

2021 Standards	2010 Standards
Algebra	
Seeing Structure in Expressions	
A. Interpret the structure of expressions. (M) M.A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. *For example, in the expression representing height of a projective, $-16t^2 + vt + c$ recognizing there are three terms in the expression, factors within some of the terms, and coefficients. Interpret within the context the meaning of the coefficient -16 as related to gravity, the factor of v as the initial velocity, and the c-term as initial height.* b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret the expression representing population growth $P(1+r)^n$ as the product of P and a factor not depending on P. Interpret the meaning of the P-factor as initial population, and the other factor as being related to growth rate and a period of time.*	**A. Interpret the structure of expressions** A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context. (*M*) a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see x^4-y^4 as $(x^2)^2-(y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2-y^2)(x^2+y^2)$.* **B. Write expressions in equivalent forms to solve problems.** A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *

M.A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see x^4-y^4 as $(x^2)^2$-$(y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2-y^2)(x^2+y^2)$.* **B. Write expressions in equivalent forms to solve problems.(M)** M.A.SSE.B.3 Choose and use an efficient process to produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal and explain the properties of the quantity represented by the expression. b. Represent a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions.*For example, if the expression 1.15^t represents growth in an investment account at time t (measured in years), it can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly rate of return is 1.2% based on an annual growth rate of 15%.* M.A.SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *For example, calculating mortgage payments or tracking the amount of an antibiotic in the human body when prescribed for an infection.*	a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as [1.15^(1/12)]^(12t) = 1.012^(12t) to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* A-SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.*

Arithmetic with Polynomials and Rational Expressions	
A. Perform arithmetic operations on polynomials. M.A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **B. Understand the relationship between zeros and factors of polynomials.** M.A.APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. M.A.APR.B.3 Identify zeros of higher degree polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **C. Use polynomial identities to solve problems.** M.A.APR.C.4 Prove polynomial identities and use them to describe numerical relationships. *For example, use $(a+20)^2=a^2+40a+400$ to mentally or efficiently square numbers in the 20's (e.g., $22^2=2^2+2*40+400=484$). Generalize to other double digit numbers. Use $a^2=(a+b)(a-b)+b^2$ and multiples of a *10 to square (e.g. $22^2=(22+12)(22-12)+12^2=340+144=484$. Recognize the visual representation of $(a+2b)^2-a^2-4ab$ as the area of a frame, and find equivalent expressions.* M.A.APR.C.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n,	**A. Perform arithmetic operations on polynomials.** A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **B. Understand the relationship between zeros and factors of polynomials.** A-APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **C. Use polynomial identities to solve problems.** A-APR.C.4 Prove polynomial identities and use them to describe numerical relationships. ~~For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.~~ A-APR.C.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. **D. Rewrite rational expressions.** M.A.APR.D.6 Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x)*, where *a(x)*, *b(x)*, *q(x)*, and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system. M.A.APR.D.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	**D. Rewrite rational expressions.** A-APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. A-APR.D.7 ~~(+)~~ Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Creating Equations	
A. Create equations that describe numbers or relationships. (M) M.A.CED.A.1 (F2Y)Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. M.A.CED.A.2 (F2Y)Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. M.A.CED.A.3 (F2Y)Represent constraints by equations or	**A.** Create equations that describe numbers or relationships. A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* M.A.CED.A.4 (F2Y)Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange C=5/9(F-32) so you solve for F.*	A.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ~~For example, rearrange Ohm's law V = IR to highlight resistance R.~~
Reasoning with Equations and Inequalities	
A. Understand solving equations as a process of reasoning and explain the reasoning. M.A.REI.A.1 (F2Y) Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. M.A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **B. Solve equations and inequalities in one variable.** M.A.REI.B.3 (F2Y) Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. M.A.REI.B.4 (F2Y) Solve quadratic equations by inspection (e.g., for x^2=49), taking square roots, completing the square, the quadratic formula, factoring, and graphing as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	**A. Understand solving equations as a process of reasoning and explain the reasoning** A.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **B. Solve equations and inequalities in one variable.** A.REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. A.REI.B.4 ~~Solve quadratic equations in one variable.~~ ~~a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x - p)² = q that has the same solutions. Derive the quadratic formula from this form.~~ b. Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of

C. Solve systems of equations. M.A.REI.C.5 (F2Y) Justify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. HSA.REI.C.6 (F2Y) Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. M.A.REI.C.7 (F2Y) Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$* M.A.REI.C.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable. M.A.REI.C.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **D. Represent and solve equations and inequalities graphically.** M.A.REI.D.10 (F2Y) Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). M.A.REI.D.11 (F2Y) Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. M.A.REI.D.12 (F2Y) Graph the solutions to a linear inequality in two	the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. **C. Solve systems of equations.** A.REI.C.5 ~~Prove~~ that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. A.REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle $x^2 + y^2 = 3$ A.REI.C.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable. A.REI.C.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **D. Represent and solve equations and inequalities graphically.** A.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. ~~Include cases~~

variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	~~where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*~~ A.REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

2021 Standards	2010 Standards
Functions	
Interpreting Functions	
A. Understand the concept of a function and use function notation. M.F.IF.A.1 (F2Y) Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. M.F.IF.A.2 (F2Y) Use function notation, evaluate functions and interpret statements that use function notation in terms of a context. M.F.IF.A.3 (F2Y) Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, in an arithmetic sequence, f(x)=f(x-1) + C or in a geometric sequence, f(x)=f(x-1)*C, where C is a constant.* **B. Interpret functions that arise in application in terms of the context. (M)** M.F.IF.B.4 (F2Y) For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and*	**A. Understand the concept of a function and use function notation.** F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. ~~If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).~~ F.IF.A.2 Use function notation, evaluate functions ~~for inputs in their domains,~~ and interpret statements that use function notation in terms of a context. F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, ~~the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.~~ **B. Interpret functions that arise in application in terms of the context.** F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of

minimums; symmetries; end behavior; and periodicity. M.F.IF.B.5 Relate the domain of a function to its graph and find an appropriate domain (discrete or continuous) in the context of the given problem. M.F.IF.B.6 Calculate and interpret the average rate of change of a linear or nonlinear function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **C. Analyze functions using different representations.** (M) M.F.IF.C.7 Graph functions expressed symbolically and show key features of the graph using an efficient method. a. (F2Y) Graph linear and quadratic functions and show intercepts, maxima, and minima; and exponential functions, showing intercepts and end behavior. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. M.F.IF.C.8 (F2Y) Write a function defined by an expression in equivalent forms to reveal and explain different properties of the function. Use an efficient process to rewrite $f(x)=ax^2+bx+c$ as $f(x)=a(x-h)^2+k$ or $f(x)=a(x-p)(x-q)$ to determine the characteristics of the function and interpret these in terms of a context. Use the properties of exponents to interpret expressions for exponential	the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* F.IF.B.5 Relate the domain of a function to its graph and~~, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*~~ F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* **C. Analyze functions using different representations.** HF.IF.C.7 Graph functions expressed symbolically and show key features of the graph~~, by hand in simple cases and using technology for more complicated cases.*~~ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. ~~(+)~~ Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph ~~exponential and~~ logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.C.8 Write a function defined by an expression in ~~different but~~ equivalent forms to reveal and explain different properties of the

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<tr>
<td>functions.

For example, identify percent rate of change in functions, where t is in years, such as $y=(1.01)^{12t}$ is approximately $y=(1.127)^{t}$, where t is in years, meaning it is a 1% growth rate each month and a 12.7% growth rate each year. Identify percent rate of change in functions, where t is in years, such as $y=(1.2)^{(t/10)}$ is approximately $y=(1.018)^{t}$, meaning it is a 20% growth rate each decade and a 1.8% growth rate each year.

M.F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</td>
<td>function.

a. Use the process of factoring ~~and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.~~
b. Use the properties of exponents to interpret expressions for exponential functions. For example, ~~identify percent rate of change in functions such as y = (1.02) ŭ, y = (0.97) ŭ, y = (1.01)12 ŭ, y = (1.2) ŭ/ 10, and classify them as representing exponential g rowth or decay.~~

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</td>
</tr>
<tr>
<td colspan="2">Building Functions</td>
</tr>
<tr>
<td>A. Build a function that models a relationship between the quantities. (M)

M.F.BF.A.1 Write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. For example: The temperature of a cup of coffee can be modeled by combining together a function representing the difference in temperature and the actual room temperature,</td>
<td>A. Build a function that models a relationship between the quantities.

F.BF.A.1 Write a function that describes a relationship between two quantities.*
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. For example, ~~build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these~~</td>
</tr>
</table>

which results in an exponential model. An average cost function can be created by dividing the cost of purchasing n items by the number of n items purchased, which results in a rational function.

c. Work with composition of functions using tables, graphs and symbols.
For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

M.F.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

B. Build new functions from existing functions.

M.F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

M.F.BF.B.4 Identify and create inverse functions, using tables, graphs, and symbolic methods to solve for the other variable.
For example: Each car in a state is assigned a unique license plate number and each license plate number is assigned to a unique car; thus there is an inverse relationship. Rearrange the formula C=5/9(F-32) so it is solved for F. You examine a table of values and realize the inputs and outputs are invertible. Two graphs are symmetrical about the line y=x.

HSF.BF.B.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

~~functions to the model.~~

c. ~~(+) Compose functions.~~ For example, if T(*y*) is the temperature in the atmosphere as a function of height, and h(*t*) is the height of a weather balloon as a function of time, then T(*h(t)*) is the temperature at the location of the weather balloon as a function of time.

F.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.~~*~~

B. Build new functions from existing functions.

F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.B.4 ~~Find inverse functions.~~

~~a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x - 1) for x ≠ 1.~~

~~b. (+) Verify by composition that one function is the inverse of another.~~

~~c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.~~

~~d. (+) Produce an invertible function from a non-invertible function by restricting the domain.~~

F.BF.B.5 ~~(+)~~ Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models	
A. Construct and compare linear, quadratic, and exponential models and solve problems. (M) M.F.LE.A.1 (F2Y) Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. M.F.LE.A.2 (F2Y) Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). M.F.LE.A.3 (F2Y) Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. M.F.LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	**A. Construct and compare linear, quadratic, and exponential models and solve problems.** F.LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. F.LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
Trigonometric Functions	

A. Extend the domain of trigonometric functions using the unit circle. M.F.TF.A.1 Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle. M.F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. M.F.TF.A.3(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. MF.TF.A.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **B. Model periodic phenomena with trigonometric functions. (M)** M.F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. M.F.TF.B.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. M.F.TF.B.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. **C. Prove and apply trigonometric identities.** M.F.TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and	**A. Extend the domain of trigonometric functions using the unit circle.** F.TF.A.1 Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. F.TF.A.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **B. Model periodic phenomena with trigonometric functions.** F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* F.TF.B.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F.TF.B.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.* **C. Prove and apply trigonometric identities.** F.TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the

use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) ar the quadrant of the angle. M.TF.C.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	quadrant of the angle. F.TF.C.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

2021 Standards	2010 Standards
Geometry	
Congruence	
A. Experiment with transformations in the plane. M.G.CO.A.1 (F2Y) Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. M.G.CO.A.2 (F2Y) Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). M.G.CO.A.3 (F2Y) Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. M.G.CO.A.4 (F2Y) Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. M.G.CO.A.5 (F2Y) Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto	**A. Experiment with transformations in the plane.** G.CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G.CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

another. **B. Understand congruence in terms of rigid motion.** M.G.CO.B.6 (F2Y)Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. M.G.CO.B.7 (F2Y)Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. M.G.CO.B.8(F2Y) Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **C. Make geometric constructions.** M.CO.D.12 (F2Y) Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. M.G.CO.D.13 (F2Y) Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	**B. Understand congruence in terms of rigid motion.** G.CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **C. Make geometric constructions.** G.CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. G.CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
Similarity, Right Triangles, and Trigonometry	

A. Understand similarity in terms of similarity transformations.	**A. Understand similarity in terms of similarity transformations.**
M.G.SRT.A.1 (F2Y) Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	G.SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor: c. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. d. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
M.G.SRT.A.2 (F2Y) Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	G.SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
M.G.SRT.A.3(F2Y) Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	G.SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
B. Prove theorems involving similarity.	**B. Prove theorems involving similarity.**
M.G.SRT.B.4 (F2Y)Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	G.SRT.B.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
M.G.SRT.B.5 (F2Y)Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	G.SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
C. Define trigonometric ratios and solve problems involving right triangles.	**C. Define trigonometric ratios and solve problems involving right triangles.**
M.G.SRT.C.6(F2Y) Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	G.SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

M.G.SRT.C.7(F2Y) Explain and use the relationship between the sine and cosine of complementary angles. M.G.SRT.C.8(F2Y) Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **D. Apply trigonometry to general triangles.** M.G.SRT.D.9 (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. M.G.SRT.D.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems. M.G.SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	G.SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* **D. Apply trigonometry to general triangles.** G.SRT.D.9 (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.D.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Circles(C)	
A. Understand and apply theorems about circles. M.G.C.A.1 (F2Y) Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. **B. Find the arc length and areas of sectors of circles.** M.G.C.B.2(F2Y)~~-~~Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define	**A. Understand and apply theorems about circles.** G.C.A.1 ~~Prove that all circles are similar.~~ ~~G.C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.~~ ~~G.C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed~~

the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	~~in a circle.~~ ~~G.C.A.4 (+) Construct a tangent line from a point outside a given circle to the circle.~~ **B. Find the arc length and areas of sectors of circles.** G.C.B.~~5~~ Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
Expressing Geometric Properties with Equations	
A. Translate between the geometric description and the equation for a conic section. M.G.GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. M.G.GPE.A.2(+) Derive the equation of a parabola given a focus and directrix. M.G.GPE.A.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **B. Use conditions to prove simple geometric theorems algebraically.** M.G.GPE.B.4 (F2Y) Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined*	**A. Translate between the geometric description and the equation for a conic section.** G.GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G.GPE.A.2 Derive the equation of a parabola given a focus and directrix. G.GPE.A.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant **B. Use conditions to prove simple geometric theorems algebraically.** G.GPE.B.4 Use coordinates t o prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or

by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2). M.G.GPE.B.5 (F2Y) Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). M.G.GPE.B.6 (F2Y) Find the point on a directed line segment between two given points that partitions the segment in a given ratio. M.G.GPE.B.7 (F2Y) Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and contai ning the point (0, 2). G.GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio. G.GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*
Geometric Measurement & Dimension	
A. Explain volume formulas and use them to solve problems. (M) M.G.GMD.A.1 (F2Y) Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. M.G.GMD.A.2 (F2Y) Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. **B. Visualize relationships between two-dimensional and**	**A. Explain volume formulas and use them to solve problems** G.GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. ~~Use dissection arguments, Cavalieri's principle, and informal limit arguments.~~ ~~G.GMD.A.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures~~ ~~G.GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*B. Visualize relationship~~

three-dimensional objects. M.G.GMD.B.3 (F2Y) Identify three-dimensional objects generated by rotations of two-dimensional objects. **C. Apply geometric concepts in modeling situations.(M)** M.G.GMD.A.4 (F2Y) Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). M.G.GMD.A.5 (F2Y) Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). M.G.G.MDA.6 (F2Y) Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	**B. Visualize relationships between two-dimensional and three-dimensional objects.** G.GMD.B.4 ~~Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and~~ identify three-dimensional objects generated by rotations of two-dimensional objects. **~~Modeling with Geometry~~** **A. Apply geometric concepts in modeling situations.** G.MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* G.MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* G.MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

2021 Standards	2010 Standards
Interpreting Categorical & Quantitative Data	
A.Summarize, represent, and interpret data on a single count or measurement variable M.SP.ID.A.1 (F2Y) Represent data with plots on the real number line (dot plots, histograms, and box plots). M.SP.ID.A.2 (F2Y) Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. M.SP.ID.A.3 (F2Y) Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). M.SP.ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **B.Summarize, represent, and interpret data on two categorical and quantitative variables** M.SP.ID.B.5 (F2Y) Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional	**A.Summarize, represent, and interpret data on a single count or measurement variable** S.ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). S.ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **B.Summarize, represent, and interpret data on two categorical and quantitative variables** S.ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the

relative frequencies). Recognize possible associations and trends in the data. M.SP.ID.B.6 (F2Y) Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. **C. Interpret Linear Models** M.SP.ID.C.7 (F2Y) Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. M.SP.ID.C.8 (F2Y) Use technology to create a correlation coefficient for a linear fit and then interpret its meaning for the model. M.SP.ID.C.9 (F2Y) Distinguish between correlation and causation. *For example, cities with a higher number of fast food restaurants tend to have more hospitals. While there is a clear correlation (likely driven by population), we cannot conclude that fast food restaurants are causing more hospitals to open. There is a relationship between height and reading level in elementary school children. This is not because changes in height cause better reading ability. Rather as children get older, they get both taller and improve in their reading skills.*	data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. **C. Interpret Linear Models** S.ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.C.8 ~~Compute (using technology) and~~ interpret the correlation coefficient of a linear fit. S.ID.C.9 Distinguish between correlation and causation.
Making Inferences & Justifying Conclusions	
A.Understand and evaluate random processes underlying statistical experiments M.SP.IC.A.1 Understand statistics as a process for making	**A.Understand and evaluate random processes underlying statistical experiments** S.IC.A.1 Understand statistics as a process for making inferences

inferences about population parameters based on a random sample from that population. M.SP.IC.A.2 Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation). *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **B.Make inferences and justify conclusions from sample surveys, experiments, and observational studies** M.SP.IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. M.SP.IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. M.SP.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. M.SP.IC.B.6 Evaluate reports based on data.	about population parameters based on a random sample from that population. S.IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? **B.Make inferences and justify conclusions from sample surveys, experiments, and observational studies** S.IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S.IC.B.6 Evaluate reports based on data.
Conditional Probability & the Rules of Probability	
A.Understand independence and conditional probability and use them to interpret data. M.SP.CP.A.1 (F2Y) Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other	**A. Understand independence and conditional probability and use them to interpret data.** S.CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or,"

events ("or," "and," "not"). M.SP.CP.A.2 (F2Y) Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. M.SP.CP.A.3 (F2Y) Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. M.SP.CP.A.4 (F2Y) Represent data from two categorical variables using two-way frequency tables and/or venn diagrams. Interpret the representation when two categories are associated with each object being classified. Use the representation as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* M.SP.CP.A.5 (F2Y) Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* **B.Use the rules of probability to compute probabilities of compound events.** M.SP.CP.B.6 (F2Y) Use a representation such as a two-way table or venn diagram to find the conditional probability of A given B as the	"and," "not"). S.CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.A.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. S.CP.A.4 ~~Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.~~ S.CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. **B. Use the rules of probability to compute probabilities of compound events.** S.CP.B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. M.SP.CP.B.7 (F2Y) Use a representation such as a two-way table or venn diagram to apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. M.SP.CP.B.8 (+) Use a representation such as a two-way table or venn diagram to apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. M.SP.CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	S.CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. S.CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. S.CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
Using Probability to Make Decisions	
A.Calculate expected values and use them to solve problems M.SP.MD.A.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. M.SP.MD.A.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. M.SP.MD.A.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* M.SP.MD.A.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are	**A.Calculate expected values and use them to solve problems** S.MD.A.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. S.MD.A.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. S.MD.A.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. S.MD.A.4 (+) Develop a probability distribution for a random

assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* **B.Use probability to evaluate outcomes of decisions** M.SP.MD.B.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* M.SP.MD.B.6 Use probabilities to make fair decisions (e.g., drawing for a party door prize where attendees earn one entry to the drawing for each activity they complete, using an electronic spinner to pick a team spokesperson at random from a group, flip a coin to decide which of two friends gets to choose the movie, using a random number generator to select people to include in a sample for an experiment. M.SP.MD.B.7 Analyze decisions and strategies using probability concepts (e.g., balancing expected gains and risk, medical product testing, choosing an investment option, deciding when to kick an extra point vs. two point conversion after a touchdown in football).	variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? **B.Use probability to evaluate outcomes of decisions** S.MD.B.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident. S.MD.B.6 ~~(+)~~ Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.B.7 ~~(+)~~ Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).