Instructional Practice Guide for Equitable Teaching and Learning in Mathematics

Grades 9 through 12

Wisconsin Department of Public Instruction

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Instructional Practice Guide for Equitable Teaching and Learning in Mathematics
Grades 9 through 12

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Introduction:

Instructional Practice Guide for

Equitable Teaching and Learning in Mathematics

Educational equity means that every student has access to the resources and educational rigor they need at the right moment in their education, across race, gender, ethnicity, language, ability, sexual orientation, family background, and/or family income.[1]

To support educators and systems in ensuring educational equity in mathematics, the Wisconsin Department of Public Instruction's Literacy and Mathematics Team - in collaboration with mathematics educators - offers this collection of instructional practices.

In Wisconsin, mathematics teaching and learning strives to ensure that[2]:

- Mathematics is experienced as coherent, connected, intrinsically interesting, and relevant.
- Every student must have access to and engage in meaningful, challenging, and rigorous mathematics on a regular basis.
- Problem solving, understanding, reasoning, and sense-making are at the heart of mathematics teaching and learning and are central to mathematical proficiency.
- Effective mathematics classroom practices include the use of collaboration, discourse, and reflection to engage students in the study of important mathematics.

For more information, see Wisconsin's Vision for Mathematics.

Historically, this is not a reality that Wisconsin's educational system has made available to every student. In nearly every measurable area (e.g., academic achievement, discipline practices, gifted and talented placement, and graduation rates) across all ages and grades, Wisconsin's education system has yielded persistent inequitable outcomes for learners along demographic lines. In particular, learners of color and students identified as English learners have experienced significantly lower rates of success than their peers. In addition, students with Individualized Education Programs (IEPs), particularly students of color with IEPs, and learners eligible for free and reduced lunch also experience significantly lower rates of success than their peers.

We believe this collection of instructional practices - when implemented along with curriculum (or scope and sequence) and with factors beyond instruction including motivational, managerial, and environmental factors - will increase every Wisconsin child's opportunity to develop as a mathematically confident and competent individual.

To support educators and systems in ensuring educational equity in mathematics, the Wisconsin Department of Public Instruction's Literacy and Mathematics Team - in collaboration with Wisconsin mathematics educators - offers this collection of instructional practices.

How the Guide Supports Educators

The instructional practices described focus on what the teacher is doing in the classroom at the universal level of instruction to engage every student in accessing and meeting grade-level academic standards. The practices are not a curriculum or program; rather, they are practices that can be used alongside an array of approaches to teaching mathematics, school schedules, or frameworks (including Universal Design for Learning (UDL)). While it is possible that many of the same practices apply in intervention or enrichment settings, universal instruction in mathematics is the primary focus of the guide. As a local control state, districts and schools in Wisconsin make important decisions regarding instructional materials. While reviewing the practices in this guide, schools and districts could also review their curricular resources to determine if their current resources are sufficient to support these recommended research-aligned practices. Instructional Materials and Professional Learning (IMPL) is supported by mounting evidence that suggests providing teachers with access to high-quality, standards-aligned instructional materials and curriculum-based professional learning can result in improvement in student outcomes.

The guide emphasizes the integrated nature of mathematics, representing the complexity of the development of mathematical proficiency.[3]

Mathematical proficiency includes the development of:

- *conceptual understanding*
    - comprehension of mathematical concepts, operations, and relations
- *procedural fluency*
    - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *adaptive reasoning*
    - the capacity for logical thought, reflection, explanation, and justification
- *strategic competence*
    - the ability to formulate, represent, and solve mathematical problems
- *productive disposition*
    - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Defining and communicating to students a broader perspective of mathematical proficiency has a powerful impact on their mathematics identities and how they exercise their agency as learners and doers of mathematics.[4]

Development of the Guide

This document was developed by Wisconsin mathematics educators and the Wisconsin Department of Public Instruction Literacy and Mathematics team. Educators who contributed to this work came from all across Wisconsin, and currently work with students, pre-service teachers, or teachers in schools spanning from kindergarten through higher education. This includes educators who specialize in working with English learners and students with differing abilities.

Participants reviewed research and materials from mathematics education and mathematics-related professional organizations that highlighted impactful practices. In addition, participants read and reflected on scholarly work related to equity in mathematics teaching and learning. All sources reviewed during development, along with additional citations, are found at the conclusion of this document.

Organization of the Guide

The instructional practice guides are organized by K-2, 3-5, 6-8, and 9-12 grade bands and are based on how Wisconsin's Vision for Mathematics [5]and responsive teaching intersect with the eight Mathematical Teaching Practices as articulated in *Principles to Actions: Ensuring Mathematical Success for All.* [6] In line with NCTM's renewed focus on access, equity, and empowerment[7], this instructional guide intentionally leads with a practice that highlights responsive mathematics teaching that positions each and every student as a valued mathematics contributor and supports the development of their identity, agency, and competence as a learner of mathematics.

The sections include the following vision statements, coordinated with NCTM's Mathematical Teaching Practices:

- Schools and classrooms are responsive to and value the students they serve conveying the message that all students are capable doers of mathematics;
- Mathematics is a coherent and connected discipline;
    - Establish mathematics goals to focus learning.[8]
    - Use and connect mathematical representations.[8]
- Every student must have access to and engage in meaningful, challenging, and rigorous mathematics;
    - Elicit and use evidence of student thinking.[8]
    - Support productive struggle in learning mathematics.[8]
- Problem solving, understanding, reasoning, and sense-making are at the heart of mathematics teaching and learning and are central to mathematical proficiency; and
    - Implement tasks that promote reasoning and problem solving.[8]
    - Build procedural fluency from conceptual understanding.[8]
- Effective mathematics learning leverages collaboration, discourse, and reflection to engage students.
    - Facilitate meaningful mathematical discourse.[8]
    - Pose purposeful questions.[8]

[8] From *Principles to Actions: Ensuring Mathematical Success for All* (p. 10) by the National Council of Teachers of Mathematics, 2014. Reston, VA: NCTM. Copyright 2014 by National Council of Teachers of Mathematics. Used with permission.

While the practices in this guide are listed individually, it is important to note that the eight research-informed NCTM teaching practices (NCTM, 2014) are a coherent and connected set of instructional pedagogies that when implemented together create a powerful framework for effective and responsive mathematics teaching[9]. The Mathematics Teaching Framework (See Figure 1) illustrates the relationship between and among the eight teaching practices and how they work together to support equitable and responsive mathematics instruction. Articulating the relationships among the eight teaching practices offers insight into the highly interactive and dependent nature of high-quality and impactful mathematics instruction. All students should experience learning mathematics in classrooms where teachers intentionally plan and facilitate mathematics lessons that enact this framework.

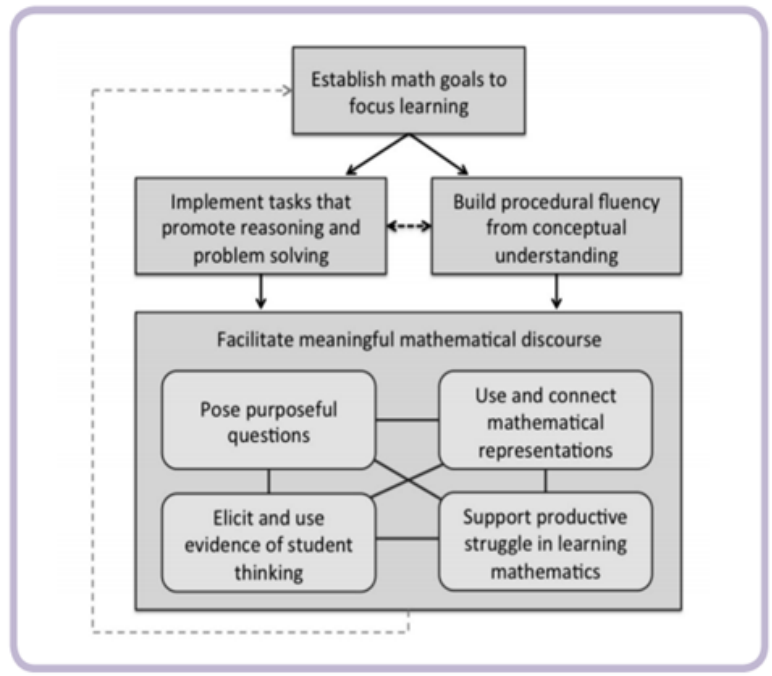


Fig. 1. Teaching Framework for Mathematics. Reprinted from Taking Action: Implementing Effective Mathematics Teaching Practices Series by the National Council of Teachers of Mathematics (2017). Reston, VA: NCTM. (Boston, et al., 2017, 215; Huinker & Bill, 2017; Smith, Steele, & Raith, 2017, 194). Used with permission.

Components of an Equitable Mathematics Classroom – Beyond Instruction

While this guide is solely focused on the mathematics instructional practices teachers employ with students, equity-minded educators understand that instruction is only one component of an effective classroom or instructional setting. Instruction works together with the following to create equitable teaching and learning:

- teacher knowledge factors
- motivational factors
- managerial factors
- curricular factors
- environmental factors

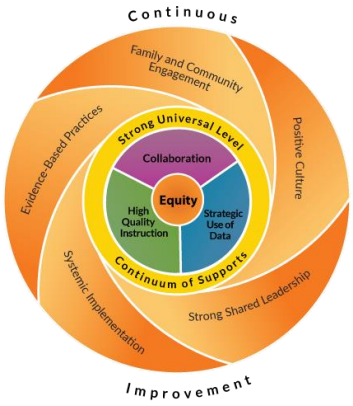
These factors, along with Wisconsin's Guiding Principles for Teaching and Learning[10], are interdependent and must be addressed simultaneously to ensure that all learners benefit equitably from the research-aligned instructional practices.

While these additional factors are critical to successful implementation of the research-aligned instructional practices, they are outside of the scope of this document and, therefore, not addressed here.

The Guide and Wisconsin's Framework for an Equitable Multi-Level System of Support

It must also be noted that an effective classroom or instructional setting is situated within a larger equitable multi-level system of support[11].

For Wisconsin schools and districts, implementing an equitable multi-level system of support means providing equitable services, practices, and resources to every learner based upon responsiveness to effective instruction and intervention. In this system, high quality instruction, strategic use of data, and collaboration interact within a continuum of supports to facilitate learner success. Schools provide varying types of supports at differing levels of intensity to proactively and responsively adjust to the needs of the whole child. These include the knowledge, skills, and habits learners need for success beyond high school, including developmental, academic, behavioral, social, and emotional skills.

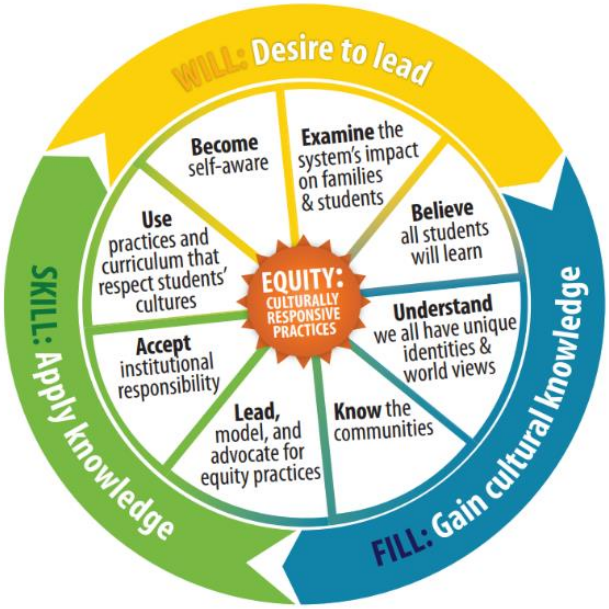


An equitable multi-level system of support exists to meet the needs of all learners. This includes learners with IEPs, learners who are advanced, and learners who are bilingual or learning English as a second or other language. This guide has been created and reviewed with all of these populations in mind. While individual learners may need additional supports, challenges, or services beyond what is included in this guide (especially practices that are included in a student's IEP), the practices outlined in the guide were carefully considered as ways to move all learners toward meeting grade-level academic standards.

The Guide and Wisconsin's Model to Inform Culturally Responsive Practice

Wisconsin's framework for an equitable multi-level system of support has - at its core - educational equity. Wisconsin's Model to Inform Culturally Responsive Practices "describes the beliefs, knowledge, and practices Wisconsin educators, schools, and districts need to reach and teach diverse students within their culturally responsive multi-level systems of support" (p. 2).[12]

The ongoing process of becoming culturally responsive is represented in the outer circle as will, fill, and skill. The inner circle describes eight actions associated with will, fill, and skill. The actions are not discrete or sequential; instead, the actions are recursive throughout one's journey toward culturally responsive practice.



The instructional practices described in this guide are tangible actions toward educational equity. Wisconsin's Model to Inform Culturally Responsive Practice provides a critical reminder that actions toward equity alone will not result in education equity. Instructional practices must be supported by a continual exploration of beliefs (of individuals and systems), recognition and elimination of barriers, and ongoing learning about our communities.

The practices within this guide can be supported by seeking opportunities within our classrooms and systems to validate, affirm, build, and bridge (VABB) our students, families, colleagues, and communities.

Validate	To make legitimate that which the institution (academia) and mainstream has made illegitimate.
Affirm	To make positive that which the institution (academia) and mainstream media has made negative.
Build	To make the connections between home culture and language and the school culture and language through instructional strategy and activity.
Bridge	To give opportunities for situational appropriateness or the utilization of the appropriate cultural or linguistic behavior.

(Adapted by the Wisconsin RtI Center from the work of Dr. Sharroky Hollie)

Suggestions for Using this Guide

We recommend collaborating with colleagues around the guide. First, develop an overall understanding of the guide and then select areas of focus to apply to your practice.

Many caring adults support our students. Ensure that your study and implementation of these practices is as inclusive as possible. Collaborate across and within general education, special education, specialists (such as educators focused on mathematics teaching and learning, English language learning, gifts and talents, and technology/library media), and support staff (such as educational assistants).

Continually engage with students and families about the selection and implementation of practices, including ongoing feedback from these critical partners.

Ideas for Developing an Overall Understanding of the Guide:

- Examine the document at each grade band (such as K - 2). Begin with the elements of the vision and their explanations. Move on to the NCTM Math Practices below each element of the vision. Finally, move on to the sample instructional practices beneath each NCTM Math Practice.
- As you read this guide:
    - Indicate practices that are already part of your teaching.
    - Indicate practices that you would like to know more about.
    - Indicate practices where you see clear connections to particular standards.

Ideas for Selecting an Area of Focus for Reading This Guide:

- Review varied data sources about student performance, including the performance of subgroups. Select several practices related to an area of need indicated by varied data.
- Engage in collegial observations to better understand which practices are being used or how a particular practice is already being used.
- Engage in a study of the practices that interest you in order to better understand what implementing particular practices might look like.
- Seek input from students and families about the use of particular practices and refine those practices as suggested.

Ideas for Implementation and Refinement:

- Specifically define what a practice looks like when in use. Collect videos and examples of the practice.
- Record yourself implementing the selected practices. Share the recordings with colleagues for feedback.
- Seek feedback from students and families about the impact of practices.
- Work with a coach around your implementation of the selected practices.
- Collect data about how a practice is being used to determine professional learning priorities.

How to Read the Instructional Practice Guide

Begin with the introduction. The introduction situates the Guide within other work, including multi-level systems of support. The introduction provides suggestions for how to use the Guide.

The Guide is organized around Wisconsin's Vision for Mathematics. Each section of the Guide begins with a statement from this vision along with the addition of Responsive Teaching. We refer to these as **vision statements**. These are highlighted in blue. There are five vision statements. Each vision statement is followed by an explanatory paragraph. See Figure 2.

Each vision statement is supported by two or more **anchor statements** that further define each vision statement. Many of the anchor statements are the NCTM Mathematical Teaching Practices (*indicated as NCTM Mathematics Teaching Practice)*. Anchor statements appear in **bold**. Each anchor statement is followed by an explanatory paragraph.

Anchor statements are further supported by specific **instructional practices.** The instructional practices illustrate and operationalize the anchor statements.

This guide begins with a vision statement centered on responsive teaching and squarely focuses our efforts as teachers on developing students' identity and agency as learners of mathematics. The remainder of the vision statements, coordinated with the eight mathematics teaching practices as articulated in *Principles to Actions: Ensuring Mathematical Success for All*, (NCTM 2014) complete the remainder of the list.

The guide includes a glossary. Throughout the document, words are hyperlinked to glossary definitions.

Note. The Guide is available for K - 2, 3 - 5, 6 - 8, and 9 - 12. Visit www.dpi.wi.gov/math to find other grade-bands as well as a document that shows the differences between grade-bands.

Figure 2.

Vision Statement 1: Responsive Teaching

Responsive Teaching: Effective mathematics educators engage in responsive teaching grounded in research-aligned practices.

NCTM's Principles to Actions: Ensuring Mathematical Success (2014) states that effective mathematics instruction must be "responsive to students' backgrounds, experiences, and knowledge" (p. 60). A major part of the daily work that takes place in mathematics classrooms cannot be singled out as one or more isolated strategies. Much of this work is directly related to educators' understanding of their students and responsiveness while teaching. Responsive teachers of mathematics consider and respect the experiences and knowledge that their students bring to the classroom. They work to validate students for whom they are and engage them in the study of challenging and relevant mathematics.

This first instructional practice represents some of the work that can be done to place an explicit focus on equitable access to engaging, relevant, and challenging mathematics content for all learners. The learning based on this content will build on the strengths students bring to the classroom and expand students' mathematical identities. Learners will see themselves and each other as capable and confident doers of mathematics.

Anchor Statement 1: Responsive Teaching
Math proficiency must be defined for everyone involved in a mathematical learning community.

Anchor Statement 2: Responsive Teaching
The student and teacher relationship is viewed as a partnership (Hammond, 2015) and as part of a classroom community in which all students feel safe and valued.

VISION STATEMENT

ANCHOR STATEMENTS

Figure 2 is a sample page that shows a vision statement. Each vision statement is a sentence followed by explanatory paragraphs. The anchor statements that relate to the vision statement are listed below the vision statement.

Figure 3.

Anchor Statement 1: Responsive Teaching

Math proficiency must be defined for everyone involved in a mathematical learning community.

Students and teachers as well as family and community members must understand that mathematical proficiency is more than developing computation skills and memorizing algorithms. Mathematical proficiency is a balance of developing conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Adding It Up 2001). By expanding the idea of what being a mathematician means and what counts as mathematics competency beyond speed and accuracy, mathematics ability is viewed as a function of opportunity and experience - not of innate intelligence. Therefore, mathematics lessons and learning opportunities are intentionally structured and facilitated around meaningful, challenging, and rigorous mathematics.[7] Adjustments in how mathematical proficiency is defined can also support the development of positive identification with mathematics among students.

Sample Equity-Based Instructional Practices

- Publically draw attention to competencies aligned with the Wisconsin State Standards for Mathematics, including both the Content Standards and the Standards for Mathematical Practice (Yeh & Chao, 2019).
- Ensure that mathematical discussions are driven by ideas and concepts rather than by those who arrive at a solution first.
- Explicitly tell students that fast work is not what is valued. Mathematical thinking is about depth, not speed.

ANCHOR STATEMENT

Figure 3 shows an anchor statement. Each anchor statement is supported by explanatory paragraphs and followed by sample instructional practices.

INSTRUCTIONAL PRACTICES

Equitable mathematics teaching practices aim to ensure all students view themselves as capable of participating in and achieving mathematics at the highest levels. The recommendations below are not intended to be exhaustive; rather, the intent is to provoke ideas and serve as a first step for teachers who are intentional about implementing equitable teaching practices in the mathematics classroom. The practices are content-focused instructional pedagogies used to advance the mathematics learning of each and every student. Because of these complex intersections, no practice can stand on its own. The practices intersect with each other throughout a lesson, unit, or course; each practice depends upon others to be effective.

Vision Statement 1: Responsive Teaching

Responsive Teaching: Schools and classrooms are responsive to and value the students they serve, conveying the message that all students are capable doers of mathematics.

NCTM's Principles to Actions: Ensuring Mathematical Success (2014) states that effective mathematics instruction must be "responsive to students' backgrounds, experiences, and knowledge" (p. 60). A major part of the daily work that takes place in mathematics classrooms cannot be singled out as one or more isolated strategies. Much of this work is directly related to educators' understanding of their students and responsiveness while teaching. Responsive teachers of mathematics consider and respect the experiences and knowledge that their students bring to the classroom. They work to validate students for whom they are and engage them in the study of challenging and relevant mathematics.

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Anchor Statement 1: Responsive Teaching
Math proficiency is defined for everyone involved in a mathematical learning community.

Anchor Statement 2: Responsive Teaching
The student and teacher relationship is viewed as a partnership[13] and as part of a classroom community in which all students feel safe and valued.

Anchor Statement 3: Responsive Teaching
Meaningful relationships between teachers, students, and their families and caregivers lay the foundation for responsive environments that support equitable teaching and learning of mathematics.

Anchor Statement 4: Responsive Teaching
Students view themselves as competent learners and doers of mathematics (Aguirre, et al. 2013).

Anchor Statement 5: Responsive Teaching
The mathematics students learn must be relevant to their life experiences and learning environment.

Anchor Statement 1: Responsive Teaching

Math proficiency is defined for everyone involved in a mathematical learning community.

Students and teachers as well as family and community members must understand that mathematical proficiency is more than developing computation skills and memorizing algorithms. Mathematical proficiency is a balance of developing conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Adding It Up 2001). By expanding the idea of what counts as mathematics competency beyond speed and accuracy, mathematics ability is viewed as a function of opportunity and experience - not of innate intelligence. Therefore, mathematics lessons and learning opportunities are intentionally structured and facilitated around meaningful, challenging, and rigorous mathematics. Adjustments in how mathematical proficiency is defined can also support the development of positive identification with mathematics among students.

Sample Equity-Based Instructional Practices:

- Publically draw attention to competencies aligned with the Wisconsin State Standards for Mathematics, including both the Content Standards and the Standards for Mathematical Practice.[14]

- Ensure that mathematical discussions are driven by ideas and concepts rather than by those who arrive at a solution first.

- Explicitly tell students that fast work is not what is valued. Mathematical thinking is about depth, not speed.

- Encourage creativity, different ways of thinking, and different explanations when working toward solutions and reflecting on math tasks.

- Include examples of student thinking that illustrate this broader definition of math proficiency in family communications.

- Provide families with questions that can promote true mathematical proficiency when engaging in math at home. For example, ask: "How did you start solving the problem? Why was that a good strategy for this problem? Will your strategy always work? What else did or could you try?"

- Help families understand the value of think time when engaging in math at home. For example, share a video clip of a class discussion that highlights how important it is to give children space to think deeply.

- Support families as they partner with school to shape a positive mathematics identity and foster a strong sense of mathematical agency in their children. For example, parents can be encouraged to regularly try new things with their children, share instances when they themselves have needed perseverance to accomplish a difficult task, and recognize their child's efforts during math problem solving as well as their achievements.

Anchor Statement 2: Responsive Teaching

The student and teacher relationship is viewed as a partnership (Hammond 2015) and as part of a classroom community in which all students feel safe and valued.

Responsive teachers act as guides, mediators, consultants, instructors, and advocates for students, helping to effectively connect their culturally- and community-based knowledge to classroom learning experiences.

Responsive teachers care for and demonstrate a commitment to the mathematical learning and growth of every student, and exhibit through visible action an awareness and belief in the capacity of every student to learn. They take responsibility for obstacles to student success and work intentionally to ensure equitable access to high-quality learning opportunities. This includes a teacher's willingness to identify and acknowledge unproductive beliefs regarding students' abilities to learn significant mathematics at high levels.

Sample Equity-based Instructional Practices:

- Place student thinking at the center of classroom activity.
- Establish norms for community first or group first work and discussions. For example, students are told that one student from their group will be responsible for reporting about the group's work. Students do not know ahead of time who will be chosen. The group, therefore, has a community mindset and ensures that all in the group are engaged and understand the mathematics involved.
- Name students as authors and owners of mathematical ideas. For example, the teacher facilitates a class discussion comparing and contrasting Beth's use of a table of values with Keisha's algebraic solution to solve a system of equations.
- Provide opportunities for students to share their own mathematical thinking, receive feedback and critique from others, and evolve their thinking in a visible, public space[15].
- Invite students to add on or paraphrase a peer's idea in order to experience diverse views and strategies as being mathematically valuable[16].
- Build an alliance with students. This alliance balances rigor and high expectations with support (Hammond 2015).

Anchor Statement 3: Responsive Teaching

Meaningful relationships between teachers, students, and their families and caregivers lay the foundation for responsive environments that support equitable teaching and learning of mathematics.

Developing meaningful relationships with students is how teachers come to learn about and respect students' experiences and knowledge (Gay 2018; Hammond 2015; Kitchens 2018; Nieto 2010). Meaningful teacher-student relationships play a critical role in nurturing students' mathematical identity and agency (Aguirre, et al. 2013; Boaler 2002; NCTM 2014).

Understanding mathematical learners includes knowing about their backgrounds, interests, strengths, and personalities as well as knowing how students think about and learn mathematics. Responsive teachers learn about and demonstrate an awareness of and appreciation for cultural identities and social diversity, particularly as they are present in one's classroom, and draw on diversity as a resource in instruction.

Families and caregivers are engaged as critical partners in their child's mathematics learning. Responsive teaching honors the mathematics of the home and helps families gain competence and comfort in the mathematics their children are learning. Productive communications between school and families are attentive to considerations of language and culture and designed to support parents and guardians in fostering their child's success with mathematics in and out of school.

Responsive teaching honors and builds upon students' existing mathematical ideas and ways of knowing and learning to support and enhance mathematics learning and engagement, including attending to each student's culture, race/ethnicity, language, gender, socioeconomic status, cognitive and physical abilities, and personal interests.

Sample Equity-based Instructional Practices:

- Use family communications to highlight the mathematics being learned, explaining the significance of strategies, visual models, or student work so that families can engage in the mathematics with their children.

- Use home visits, one-to-one interviews, phone calls, and surveys to get to know students and their families.

- Invite families into the mathematics classroom in a variety of ways. Families may come in-person, engage through reading information purposefully shared by the teacher, or complete take-home activities with their children and share feedback about the activity with the teacher. For example, families can be provided with information to foster math-based conversations or explorations at home that would share what is being learned.

- Connect with students and families to learn about their language and communication style preferences. Use these preferences to promote two-way communication about the math classroom and student learning.

- Ask students to share their math story. This may include drawings, narratives, or verbal recordings that allow the student to share their thoughts and feelings about learning mathematics. Keep the stories and refer back to them throughout the year.

- Be aware of different mathematical strategies or gestures based on cultural practices. Honor and build new learning from these forms of mathematical thinking. For example, families may use non-standard measurements when cooking (e.g., fistful) or use different grouping practices, or different algorithms for math computation.

Anchor Statement 4: Responsive Teaching

Students view themselves as competent learners and doers of mathematics (Aguirre et al., 2013).

In order to ensure that each and every student not only understands and can make use of foundational mathematics concepts and relationships but also comes to experience the joy, wonder, and beauty of mathematics, each and every student must be positioned as mathematically competent. "This requires creating classrooms--structures and norms--that support students to take risks to engage in discourse and see themselves as capable and worthy of being heard."[17]

Mathematical identity plays a critical role in how students see themselves in relation to mathematics and their ability to engage in mathematics; therefore, responsive teaching aims to increase each and every student's confidence as learners of mathematics and nurture robust and powerful mathematical identities.

Sample Equity-based Instructional Practices:

- Co-create classroom norms to orchestrate participation and engagement in daily mathematics that give all students a voice and shared identity in mathematical conversations.
- Consciously give voice and authority to students from marginalized populations (Smith, Steele, and Raith 2017, 95). For example, intentionally teach students about mathematicians from diverse backgrounds throughout the year and provide students with opportunities to share their mathematics thinking by speaking, drawing, using text-to-speech, etc.
- Recognize and value students' primary languages, developing proficiency in English, integrated use of multiple languages (i.e., translanguaging), and mode of communication (e.g., aided and/or unaided augmentative and alternative communication [AAC]) including the use of devices, gestures, images, and/or objects while learning.[18]
- Support students in creating self-awareness of their personal effort toward learning through goal-setting, progress monitoring, and self-reflection.
- Develop the background knowledge of students and access their prior knowledge in order to provide all students regular access to high level tasks.

- Launch tasks so that students understand what is expected of them and provide students with appropriate resources that will support their entry into the task. For example, students turn and talk about their ideas for representing a problem. This allows students to draw on each other as resources in understanding and clarifying task expectations, as well as draw from their own experiences, as they consider how to begin (Smith, Steele, and Raith 2017, 51).

- When possible, provide choice in the way students represent their mathematical thinking.

- Provide students opportunities to express mathematical understanding by inviting students to publicly share their math thinking. For example, students demonstrate the ways they used manipulatives, recreate their math drawings for others, or share their reasoning so that they see themselves as mathematical authorities in the classroom (Huinker and Bill 2017, 34).

- Focus on going deep with the mathematics. Provide opportunities for deep learning to foster the development of a classroom's collective mathematical agency as students understand mathematics and expect it to make sense. When a student understands what an x-intercept is, they are able to apply that understanding to their own strategy and strategies of their peers.

- Position mistakes made as a mathematician as opportunities for growth and learning. Highlight mistakes as contributions to classroom understanding.

- Invite students to share real world examples of mathematical and statistical concepts based on personal experiences and observations.

Anchor Statement 5: Responsive Teaching

The mathematics students are learning must be relevant to their life experiences and learning environment.

"Equity-based teaching depends on the capacity to recognize and intentionally tap students' knowledge and experiences - mathematical, cultural, linguistic, peer, family, and community - as resources for mathematics teaching and learning. Drawing on this knowledge and experience includes helping students bridge everyday experiences to learn mathematics, capitalizing on linguistic resources to support mathematics learning, recognizing family or community mathematical practices to support mathematics learning, and finding ways to help students learn and use mathematics to solve authentic problems that affect their lives" (Aguirre 2013).

Sample Equity-based Instructional Practices:

- Take a community walk. Look for and document evidence of mathematics within students' local neighborhoods. This could include people that are experienced using mathematics or are enacting mathematical concepts. Talk to individuals who work, play, or shop in the community about how they use mathematics or problem solve.[19]

- Write a series of questions about the contexts experienced on the community walk that could be investigated mathematically. Brainstorm a list of possible questions and data sources that students could use to answer those questions (Bartell, et al. 2017).

- Use community experiences to connect mathematics to community contexts and enhance high quality instructional materials. For example, students use mathematical modeling to find solutions to problems relevant and important to the student.

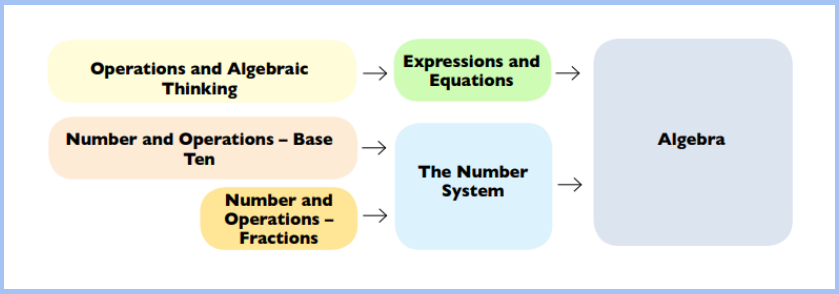
- Use interdisciplinary connections to bridge mathematics and other content areas. For example, provide opportunities for students to think about how to create a plan to find and purchase a car. This might include components of the modeling cycle as they identify and specify what criteria will be used, analyze the model, and plan to make the purchase.

- Talk to students about activities and experiences outside of school. For example, provide opportunities to pose questions about places in the community that students know about and find interesting and listen for and discuss social justice topics that are important to students.

Vision Statement 2: Coherent and Connected

Coherent and Connected: Mathematics is a coherent and connected discipline.

The Wisconsin Standards for Mathematics are built upon coherence, one of the design principles of the Standards. The intentional progression and sequencing of topics lays the foundation for the mathematics that is developed from kindergarten through high school. The diagram below depicts how domains at the elementary and middle school levels converge toward algebra at the high school. It is important that educators are knowledgeable about these progressions so that students learn mathematics with understanding and new content can build upon prior learning.[20]



Across high school mathematics, the domains and conceptual categories have strong connections to each other. For example, students understand that the distance formula is a restatement of the Pythagorean theorem. In addition, right triangle trigonometry provides a basis for developing unit circle trigonometry. Within the context of the unit circle, the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$ is yet another restatement of the Pythagorean Theorem.

Anchor Statement 6: Coherent and Connected

Establish math goals to focus learning.
(NCTM Mathematics Teaching Practice)
Effective teaching of mathematics establishes clear goals for the mathematics students are learning, situates goals within developmental learning progressions, and uses goals to guide instructional decisions (NCTM 2014, 10).

Anchor Statement 7: Coherent and Connected

Use and connect mathematical representations.
(NCTM Mathematics Teaching Practice)
Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving (NCTM 2014, 10).

Anchor Statement 6: Coherent and Connected

Establish math goals to focus learning.
(NCTM Mathematics Teaching Practice)

As illustrated in The Teaching Framework for Mathematics (p. 7 of this guide), the mathematics goal serves as the starting point for all instructional decisions and focuses and frames the teaching and learning as it unfolds throughout a lesson or over the course of a unit.

"Effective mathematics teaching begins with a shared understanding among teachers of the mathematics that students are learning and how this mathematics develops along learning progressions" (NCTM 2014, 12). Mathematics goals serve to both identify the mathematical concepts, ideas, or methods students will understand as a result of instruction, and the mathematical practices students will be employing as they work. Situating goals within learning progressions informs real-time instructional decisions resulting in instructional scaffolds that preserve the mathematical goal of the lesson and honor students' developmental milestones and markers. Given the traditional compartmentation of high school mathematics within courses, it is important that this shared understanding occurs among an entire high school staff and not merely among teachers of one single course.

In responsive mathematics classrooms, teachers regularly ensure that each and every student has the opportunity to meet the mathematical goal of each lesson. To that end, they establish learning environments that promote learning mathematics as just, equitable, and inclusive and intentionally establish norms for participation that position students as competent mathematics thinkers (Turner, Dominguez, Maldonado, & Empon 2013). Teachers utilize research-based, developmental learning trajectories and progressions to build students' mathematical understanding, increase student confidence, and support student mathematical identities as doers of mathematics.

Sample Equity-based Instructional Practices

- Ensure that goals match course expectations and address the design principles of focus, coherence, and rigor.

- Differentiate the mathematical goal from the mathematical activity. This clarity helps plan what questions, related to the important mathematics, will be asked as feedback is provided during the lesson. For example, the mathematical goal is to recognize functional relationships in different representations, whereas the mathematical activity is to represent real-life situations through tables, graphs, and equations.

- Articulate the goal to students in appropriate, student-friendly language. The goal includes success criteria to ensure that students understand the lesson expectations. For example: Functional relationships can be represented in different ways. I am successful when, given a representation, I can tell if a relationship is linear, quadratic, exponential or none of these. I can also explain how I know.

- Connect learning goals to the broader mathematical landscape and help students understand the relevance of the mathematical goal.

- Post the learning target, including the goal and success criteria, and revisit the learning target throughout the lesson.

- Ensure that all learners have access to the goal. The teacher anticipates, plans and monitors how each student will work to reach the goal (NCTM 2014, 13).

- Support students in monitoring their own progress toward the learning goal by analyzing their work and citing evidence to support their conclusions.

- Employ formative assessment practices to gauge student growth in meeting the learning goal.

Anchor Statement 7: Coherent and Connected

Use and connect mathematical representations.
(NCTM Mathematics Teaching Practice)

Effective teaching emphasizes using and making connections among mathematical representations to deepen student understanding of concepts and procedures, support mathematical discourse among students, and serve as tools for problem solving. As students use and make connections among contextual, physical, visual, verbal, and symbolic representations, they grow their appreciation of mathematics as a unified, coherent discipline (NCTM 2014, 24). Teachers support students' use of representations when they select mathematical tasks that can be solved utilizing a variety of representations. Once students generate their representations, teachers deepen students' understanding of mathematics as they intentionally engage students in describing and discussing the connections among representations.

In responsive mathematics classrooms, teachers develop learning environments rich in the use of multiple representations-contextual, physical, visual, verbal, and symbolic. They intentionally engage students in seeing and connecting multiple representations for the same mathematical idea, ensuring students develop deep and connected knowledge of mathematics. When teachers allow student choice in visual representations and mathematical tools during problem solving, student thinking is validated, they intentionally foster an interest in their own and others' thinking, and position students as mathematically competent.

Sample Equity-based Instructional Practices:

- Provide time for students to use multiple representations as tools for problem-solving. By examining concepts through a variety of representations, and therefore lenses to provide different perspectives, the mathematical concept is understood more richly and deeply. [21]

- Plan for instruction and practice that includes multiple ways for children to experience mathematics and guide students in attending to each of the representations used. The five modes of mathematical representation include contextual, visual, verbal, physical, and symbolic (Smith, Steele, and Raith 2017, 100).

- Over time, encourage students to explore concepts concretely, visually and abstractly and move fluidly among these representations.

- Use representations to compare and contrast student solution strategies and to reveal the underlying mathematics.

- Use representations to help students advance their understanding of mathematical concepts and highlight the structure of the mathematics.[22]

- Engage students in dialogue to make explicit connections within and among representations.

- Encourage students to use meaningful representations of their choosing. Ask students to justify or consider the appropriateness of the selected representation to develop a knowledge of when and why to make a specific choice (Huinker 2015).

- Examine materials to look for which representations are used most often and be aware of which are privileged and why. For example, all five modes of representation should be equally validated and part of mathematical discussions.

- Plan experiences so that students can practice moving flexibly between representations once they have conceptual understanding. For example, explicitly ask for more than one representation from students as they engage in a task.

- Understand the progression of learning that comes before and after grade level content being taught in order to connect representations over time. For example, students think about multiplication through arrays, numerical expressions, and area models. Later, these representations are connected to the solving of algebraic equations through the use of factoring.

- Use developmentally appropriate math manipulatives and tools to provide opportunities for students to visualize mathematics. For example, students use algebra tiles as they factor a polynomial based on an area model.

Vision Statement 3: Access and Engagement

Access and Engagement: Every student must have access to and engage in meaningful, challenging, and rigorous mathematics.

Students are successful when they are engaged in doing mathematics. As mathematicians "students are active learners, constructing their knowledge through exploration, discussion, and reflection. The tasks in which students engage are both challenging and interesting and cannot be answered quickly by applying a known rule or procedure. Students must reason about and make sense of a situation and persevere when a pathway is not immediately evident" (NCTM 2014).

Anchor Statement 8: Access and Engagement

Elicit and use evidence of student thinking.
(NCTM Mathematics Teaching Practice)
Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning (NCTM 2014, 10).

Anchor Statement 9: Access and Engagement

Support productive struggle in learning mathematics.
(NCTM Mathematics Teaching Practice)
Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships (NCTM 2014, 10).

Anchor Statement 8: Access and Engagement

Elicit and use evidence of student thinking.
(NCTM Mathematics Teaching Practice)

"Effective mathematics teaching elicits evidence of students' current mathematical understanding and uses it as a basis for making instructional decisions" (NCTM 2014, 53). Effective teaching includes the skills of surfacing and identifying student thinking, interpreting student thinking during instruction, and then deciding how to respond on the basis of student thinking in order to advance learning. Teachers may gather information regarding what students know and understand about the mathematics they are learning by listening to what students say, observing their actions, and analyzing their written work. Intentionally planning for how to elicit students' thinking during mathematics lessons is a key aspect of effective lesson planning (NCTM 2017).

In responsive mathematics classrooms, eliciting and using evidence of student thinking is central to informing the daily, in-the-moment instructional decisions of teachers. By carefully listening to and thoughtfully responding to student thinking, teachers position students' contributions as mathematically valuable and contributing to a broader collective understanding of the mathematical ideas at hand. Whose work gets selected and discussed during a lesson sends important messages to students about the solution paths that are valued and valid in the mathematics classroom. Listening to what students say, observing their actions, and analyzing their written work are all ways to gather information on what students currently know and understand about key mathematical ideas. Therefore, eliciting and using evidence of student thinking is an aspect of formative assessment. Establishing a classroom culture in which mistakes are viewed as important reasoning opportunities promotes a wider range of students to engage in mathematical discussions with their peers and the teacher. Correct answers matter but not as a sole indicator of who is able to do mathematics. Eliciting student thinking and "engaging in mathematical discourse is essential for developing mathematical identity and should be recognized as an indicator of mathematical competence" (Barry 2018).

Sample Equity-based Instructional Practices:

- Build a safe, judgement free community of learners so that students can share successes, questions, mistakes, and challenges.
- Encourage, make public, and value student observations, conjectures, and initial reasoning.
- Consider how and why to elicit mathematical thinking prior to a lesson.
- Use tasks that have the potential to elicit student thinking. These include high-level tasks that may have multiple solutions or paths to the solution.
- Hone the skills of noticing students' mathematical thinking, interpreting student understandings, and deciding how to respond to what is found. Professional learning communities can provide an important place for teachers to look at and analyze student evidence alongside each other.
- Ask permission before sharing student work with the class as a whole in order to honor the student's authorship and ownership of the work. After talking about a student's work, ask if what was shared accurately portrays the student's thinking.
- Sequence student work for discussion in a variety of purposeful ways to shine light on the important thinking that each student brings to the mathematics. For example, student work can be shared by most common strategy to less common strategy, as a series of misconceptions, or as a comparison and contrast of two work samples.
- Validate student mathematical thinking even when they do not get to a final answer. Position students' ideas as worthy of exploring, adjusting instruction to incorporate student thinking that supports learning.
- Use writing as a powerful way to elicit student thinking. Through writing, students clarify and organize their mathematical ideas. Writing can help students self-identify misunderstandings or questions that they have. The writing process, including the use of a scribe or voice-to-text, continues the learning as students merge, connect, and consolidate their ideas (Huinker and Bill 2017, 196).

Anchor Statement 9: Access and Engagement

Support productive struggle in learning mathematics.
(NCTM Mathematics Teaching Practice)

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships (NCTM 2014, 10).

In responsive mathematics classrooms, teachers engage students in cognitively demanding tasks with the intent of giving students positive math experiences that encourage students to see themselves as competent problem solvers (Aguirre, et al. 2014). Strengthening mathematics learning and cultivating positive mathematical identities begins with teachers knowing and believing in their students' abilities as competent and capable of engaging in cognitively demanding tasks. These types of tasks may include those that have more than one answer, are more complex or have multiple steps, and have multiple entry points.

Relevant and challenging tasks provide for differing solution strategies and can support a culture of curiosity in mathematics laying the foundation for motivation and persistence. Responsive mathematics classrooms value struggle and allow time for students to engage with mathematical ideas. These actions support student perseverance and belief in themselves as mathematicians. In such environments, teachers maintain high expectations, while offering just enough support and scaffolding to facilitate student progress on challenging work, to communicate caring and confidence in students.

When teachers "carefully monitor and support students in ways that acknowledge and build on their ideas, they work to ensure the struggle with the task is productive in moving students toward the intended learning goals while maintaining the cognitive demand and ownership of actions" (Huinker & Bill 2017, 238).

Engaging with mathematics as a subject of learning and sense making rather than a subject of performance (Boaler 2014) is empowering for students and teachers. In order to set the stage for this, Boaler asserts that students "need tasks and questions in math class that have space to learn built in" (p. 2). To that end, in responsive classrooms, students see mistakes as opportunities for learning and as a natural part of learning mathematics.

Sample Equity-based Instructional Practices:

- Intentionally shift from teacher prompted thinking to independent student thinking and reasoning.

- Support students as they learn to be comfortable being uncomfortable. Teachers create a classroom environment that expects productive struggle, makes it safe to fail, and provides support when needed. For example, My Favorite No[23] can be used as an assessment strategy that turns a student mistake into a learning opportunity.

- Recognize and name moments of struggle with students. Talk about what it looks like, sounds like, and feels like in order for students to understand their feelings and persevere.

- Understand the line between productive and unproductive struggle. Based on knowledge of students and anticipated needs, know when to use questions to focus student thinking and when to provide scaffolds while maintaining the integrity of the task.

- Continually examine unconscious prejudices as to student abilities based on cultural grouping. A Hidden Bias Test [24] is one way to measure unconscious, or automatic, biases.

- Focus on the mathematical process rather than answers when providing feedback to students, including the risks students take during the process. For example, highlight the process, not the answer. Say, "You tried another strategy when you noticed this one didn't work!"

- Provide opportunities for students to work on tasks that extend over time or have multiple answers.

Vision Statement 4: Problem Solving, Understanding, Reasoning, and Sense-Making

Problem Solving, Understanding, Reasoning, and Sense-Making: Problem solving, understanding, reasoning, and sense-making are at the heart of mathematics teaching and learning and are central to mathematical proficiency.

The Mathematics Teaching Framework (p. 7 of this Guide) is a reminder of the relationships among the NCTM Mathematical Practices. Depending on the goal for instruction, a teacher might select tasks that promote reasoning and problem solving or engage students in developing procedural fluency from conceptual understanding. Tasks that promote reasoning and problem solving provide the conceptual base on which fluency can be developed. Thus these two teaching practices strongly support one another.

Anchor Statement 10: Problem solving, understanding, reasoning, and sense-making

Implement tasks that promote reasoning and problem solving.
(NCTM Mathematics Teaching Practice)
Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies (NCTM 2014, 10).

Anchor Statement 11: Problem solving, understanding, reasoning, and sense-making

Build procedural fluency from conceptual understanding.
(NCTM Mathematics Teaching Practice)
Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibility as they solve contextual and mathematical problems (NCTM 2014, 10).

Anchor Statement 10: Problem Solving, Understanding, Reasoning, and Sense-Making

Implement tasks that promote reasoning and problem solving.
(NCTM Mathematics Teaching Practice)

"Effective mathematics teaching uses tasks as one way to motivate student learning and help students build new mathematical knowledge through problem solving" (NCTM 2014, 17). Mathematics tasks are intellectually challenging yet achievable and are relevant to students' lives and contexts. Intentionally selected and thoughtfully implemented tasks: (1) provide access to the mathematical goal of the lesson, (2) offer multiple entry points and solution paths through the use of tools and varied representations, and (3) build on and extend students' current mathematical understanding.

In responsive mathematics classrooms, teachers provide regular opportunities for each and every student to engage in high-level tasks which promote reasoning, sense making, and problem solving, feature multiple entry points (low threshold), and encourage the use and discussion of varied solution strategies (high ceiling). Tasks are crafted around contexts that students find relatable, meaningful, and interesting to their everyday lives thus enhancing each student's interest in mathematics, mathematical identity, and agency (Aguirre, et al., 2013; Cross, Hudson, Adefope, Lee, Rapacki, & Perea 2012; Moschkovic 1999; 2011). Tools and manipulatives are readily available for student use and students have agency over which tools they will use. Scaffolded teacher-supports lay the foundation for successful experiences with mathematical thinking and problem solving without reducing the cognitive demand of the tasks.

Sample Equity-based Instructional Practices:

1. *Prepare for the lesson prior to student engagement with the math task(s).*

    - Use tasks from instructional materials that are at grade level and standards aligned, including both Practice and Content Standards.
    - Plan for thoughtful scaffolds as needed that engage and empower students.
    - Plan for opportunities for students to engage in all levels of tasks, keeping in mind that experience with high-demand tasks in particular is critical in developing thinking, reasoning, and problem solving skills (Huinker and Bill 2017, 62).

- Connect relevant tasks to student interests. Connections can be made to home, school and community contexts. If tasks from current instructional materials are not relevant, adjustments are made. For example, students use relevant data to interpret differences in shape, center and spread in the context of the data sets.

- Provide tasks that have multiple solutions or solution paths so that students have choice in how they engage in the task. Putting the decision of a pathway in students' hands empowers them to be mathematical problem solvers. For example, students are asked to determine the temperature of a cooling cup of coffee at a specific time. Students might create an algebraic model, use technology to create a regression, or use an accepted formula like Newton's Law of Cooling.

- Personally engage in math tasks. This provides an opportunity to predict possible solution strategies, anticipate misconceptions that may arise during the lesson, and plan for ways to maximize accessibility and elicit mathematical discourse for all students.

- Plan for student use of math tools during engagement with the task. Tools are made available to students in a variety of ways. This may look like shelves with organized tools available to all students, personal bins of supplies, or table caddies.

2. *Guide student learning during the implementation of the math task(s).*

    - Choose problems that students will feel a need to solve, or build a student need at the beginning of the lesson.

    - Make explicit connections within, between and among concepts and solution strategies throughout the task.

    - Maintain the integrity of the task throughout implementation. Refrain from being too helpful and lowering the cognitive demand. Instead of offering ideas for how to begin or continue the task, pose questions to help students reflect on what they already know and what they are trying to figure out.

Anchor Statement 11: Problem Solving, Understanding, Reasoning, and Sense-Making

Build procedural fluency from conceptual understanding.
(NCTM Mathematics Teaching Practice)

"Effective mathematics teaching focuses on the development of both conceptual and procedural fluency" (NCTM 2014, 42). Being fluent means students understand and explain the mathematical basis for the procedure they are using. They are able to demonstrate a flexible use of strategies and methods and can justify their choice of procedure for specific types of problems. Over time, this strong conceptual understanding leads to the meaningful use of general methods and algorithms. Building procedural fluency takes time and in no way identifies or implies memorization as a way to reach fluency. In fact, procedural fluency extends from a deep understanding of how numbers and operations work together and is grounded in an understanding of the meanings and properties of the operations (e.g., commutative and associative). If fluency is achieved through an emphasis on memorization, this may convey the message that mathematics is not about knowing and doing, but about memorizing.

In responsive mathematics classrooms, teachers build procedural fluency from conceptual understanding helping students make sense of the mathematics, and in turn supporting the development of a positive disposition toward mathematics (NCTM 2018). Learning mathematical procedures and facts with understanding provides students with a wider range of options for entering a task and building understanding supports the development of students' agency and ownership of mathematical knowledge. Ensuring students understand the mathematics behind the procedures they are employing helps them see mathematics as coherent and connected and not solely grounded in rote memorization and speed. Moreover, mathematics instruction that "focuses solely on remembering and applying procedures advantages students who are strong in memorization skills and disadvantages students who are not" (NCTM 2018, 31). When teachers intentionally build procedural fluency from conceptual understanding they reduce mathematical anxiety in their students and help position them as confident knowers and doers of mathematics (Ashcraft 2002; Ramirez, Gunderson, Levine, & Beilock 2013).

Sample Equity-based Instructional Practices:

1. *Support the development of deep conceptual understanding of mathematical ideas, relationships, and operations as a foundation for all mathematics.*

    - Encourage students to use a variety of strategies when solving problems.

    - Use intentionally-focused problem strings to promote critical strategies through finding patterns and reasoning. For example, the following problem string of equations highlights the different structural types of quadratic equations.

      $X^2 - 4 = 0$
      $(x - 4)^2 - 6 = 0$
      $X^2 - 4x = 0$
      $X^2 - 4x - 5 = 0$
      $X^2 - 4x - 6 = 0$

    - Expect students to explain and justify their thinking and allow time and space for those explanations to be formed and communicated. Doing math is not simply about getting an answer.

    - Support students in connecting new ideas to previous learning. For example, connect rigid transformations to triangle congruence.

2. *Teachers guide students in developing conceptual understanding with an eye toward the eventual connections to procedures* (Huinker and Bill 2017, 68).

    - Consider continuing conversations around fluency and number sense when the situation arises. In a geometric sequence with a common ratio of 1.5 finding the next term provides an opportunity for students to engage in flexible strategies, in this case, using the distributive property.

    - Help students move among concrete models, representational drawings, and abstract ways of engaging with mathematical concepts.

    - Promote the writing of symbolic equations to connect with contextual situations and ask learners to write contextual situations that stem from symbolic equations.

    - Connect student-generated strategies to more abstract procedures during instruction to help students identify the underlying mathematics that is many times shrouded in generalized strategies.

- Pair procedures with visual models offering students another critical way to see the mathematics in the procedures they are learning. For example, using a right triangle to pair the visual model of the Pythagorean Theorem to the distance formula.

- Ensure that all students engage in challenging mathematics while developing procedural fluency. For example using abstract symbolic manipulation to find key features of a graph is not a prerequisite for modeling applications that involve analyzing graphs.

- Measure progress for both conceptual understanding and procedural fluency.

- Intentionally use the analysis of solutions as the learning task.

Vision Statement 5: Collaboration, Discourse, and Reflection

Collaboration, Discourse, and Reflection: Effective mathematics learning leverages collaboration, discourse and reflection to engage students.

The heart of any lesson is mathematical discourse. This is represented by the large rectangle in The Mathematics Teaching Framework (p. 7 of this Guide). As students are provided with opportunities to communicate their mathematical understanding to others, either orally, visually, or in writing, they are engaged in discourse. Discourse is mediated by the use of the four mathematics teaching practices situated within the discourse rectangle in the Framework, that is, pose purposeful questions, use and connect mathematical representations, elicit and use evidence of students' thinking, and support productive struggle. Together, these teaching practices interact (in service of the goals and reliant on the tasks) to engage students in meaningful discourse (NCTM 2017).

To ensure these practices are both effective and equitable, classroom environments must be intentionally structured to ensure each and every student is supported to actively contribute their mathematical ideas and reasoning and that all student voices are heard and valued.

Anchor Statement 12: Collaboration, Discourse and Reflection

Facilitate meaningful mathematical discourse.
(NCTM Mathematics Teaching Practice)
Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments (NCTM 2014, 10).

Anchor Statement 13: Collaboration, Discourse, and Reflection

Pose purposeful questions.
(NCTM Mathematics Teaching Practice)
Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships (NCTM 2014, 10).

Anchor Statement 12: Collaboration, Discourse, and Reflection

Facilitate meaningful mathematical discourse.
(NCTM Mathematics Teaching Practice)

"Effective mathematics teaching engages students in discourse to advance the mathematical learning of the whole class" (NCTM 2014, 29). Mathematical discourse is viewed as central to meaningful learning of mathematics and includes all forms of communication students use to share their ideas, including verbal, visual (e.g., drawings and gestures), and written formats. Discourse provides a platform for students to share ideas and clarify understandings, construct mathematical arguments and justifications, develop mathematical vocabulary in meaningful and contextually rich settings, and learn to see ideas from other perspectives (NCTM 2000). Teachers support and encourage meaningful discourse as they create and sustain a classroom environment and establish discourse norms that prioritize student to student collaboration and justification of solution strategies. This focus on meaningful discourse is one way that teachers of mathematics are called to teach the language mathematics.

In responsive mathematics classrooms, students' approaches and reasoning are the starting point for mathematical discussions. Responsive teachers utilize mathematical discourse as a way to elicit students' ideas and strategies providing stronger access to the mathematics for each and every student, including those who have immediate ideas or solution strategies, those who need more time to grapple with ideas and develop a reasonable approach, as well as those who have faulty reasoning or misconceptions (Huinker & Bill 2017). Effective classroom discourse engages teachers in providing opportunity and time for students to analyze, compare, justify, and prove their own solution strategies and representations, and to engage with the thinking of others. When discourse is facilitated in collaborative and supportive environments each and every student knows and believes that their explanations, ideas, and solution strategies are valued and worthy of being heard (Berry 2018). Therefore, in responsive mathematics classrooms teachers purposefully ensure diversity and equity in student voice and ideas.

Sample Equity-based Instructional Practices:

- Identify a learning goal to anchor the mathematical discussion.
- Plan for accountability during classroom discourse. Routines are established for techniques like Notice and Wonder, number talks, gallery walks, and other forms of student discourse that position all students as valuable resources.
- Anticipate accurate and inaccurate solution strategies that students might use in order to prepare for opportunities for discourse (Huinker and Bill 2017, 160).
- Use mathematically accurate language while being careful to nurture discourse by beginning with and scaffolding from the language generated by students.

- Provide opportunities for students to talk with, respond to, and question one another in ways that support and advance learning for the whole class. Students come to mathematical understandings without relying on the teacher as the authority.

- Provide language support to allow all students to engage in mathematical discourse. Possible supports include the use of object visuals, sentence frames, sentence stems, word banks, modeling of language, familiar contexts, and the use of cognates.

- Ensure that all students have access to linguistic resources, considering their English mastery. Refer to WIDA's English Language Development standards[25], specifically standard 3 Language of Mathematics, to make connections between language development and math content.

- Use classroom discourse to position all students as competent and capable. Be intentional about ensuring there is diversity in voice, and that every student has a voice.

- Connect visual models to the conversations that support the mathematics. Students can use algebra tiles to illustrate the distributive property, for example, $3(2x+2) = 6x+6$.

- Allow multiple and varied opportunities for discourse which appreciate different cultural speaking norms:
    - Talk moves like wait-time, revoicing, repeating, rephrasing, adding on, etc.
    - Turn and talk
    - Written response
    - Think-Pair-Share
    - Gallery Walk to analyze the work of others
    - Notice and Wonder
    - Strategic student groupings: whole group, small group, partner, individual, or interview

- Structure conversations to help students make mathematical connections and apply precise mathematical language. For example, the teacher monitors students while working and purposefully chooses three students to share their group's thinking during the class reflection. These three students are selected and ordered specifically to explain and justify the underlying mathematical ideas.

Anchor Statement 13: Collaboration, Discourse, and Reflection

Pose purposeful questions.
(NCTM Mathematics Teaching Practice)

"Effective mathematics teaching relies on questions that encourage students to explain and reflect on their thinking as an essential component of meaningful mathematics discourse" (NCTM 2014, 35). Teachers use a variety of question types to assess and gather evidence of student thinking, including questions that gather information, probe understanding, make the mathematics visible, and ask students to reflect on and justify their reasoning. Questions posed in the mathematics classroom should both build on student thinking and advance student thinking.

In responsive mathematics classrooms, teachers pose purposeful questions to ensure that each and every student not only progresses in their learning of important and challenging mathematical ideas but also develops a strong mathematical identity (Aguirre, et al., 2013). Purposeful questions reveal insight into students' understanding and strategies and orient students to each other's reasoning. Teacher questioning and positioning of students and student responses influence how students view themselves as members of the mathematics learning community in the classroom. When teachers pose purposeful questions and then listen in order to understand students' thinking they signal to their students that their thinking is valued. Teachers carefully consider what questions to ask their students and they are also conscious of which students are given voice and authority in those mathematical conversations. Teachers follow up on the students' responses in such a way that supports the students' development of a positive mathematical identity and sense of agency as a thinker and doer of mathematics.

Sample Equity-based Instructional Practices:

- Ask a variety of questions of students in order to elicit a range of responses (Smith, Steele, and Raith 2017, 83 & Huinker and Bill 2017, 102).
    - questions to gather information
    - questions to probe thinking
    - questions to make the mathematics visible
    - questions to encourage reflection and justification
    - questions to engage with the reasoning of others

- Encourage students to ask questions about mathematical content and concepts as well as about other student thinking. Asking questions helps to build students' mathematical identities and gives them voice and authority. Student questions also build an interest in the content and acknowledge that each student is a part of a community of learners.

- Monitor student interactions related to questions. Consciously consider question patterns to ensure that all students have an opportunity to pose questions and respond to them (Huinker and Bill 2017, 105).

- Understand the coherence within and across grade level learning so that in the moment, meaningful questions are asked and critical responses are built upon. Anticipate student needs and adjust questions as the lesson unfolds to support student mathematical agency.

- Pose purposeful questions in order to maintain the students as doers of mathematics and therefore maintain the cognitive rigor of the task.

- Model mathematically rich language and accurate vocabulary within the questions posed.

- Use a focusing pattern of questioning to have students make connections and build their understanding. Teachers capitalize on student thinking as they press students to communicate their thinking clearly and reflect on their own thoughts and those of their mathematical community. Leading questions are avoided (Smith, Steele, and Raith 2017, 82 & NCTM 2014, 37).

- Display key questions publicly to support students as they generate ideas and to keep the lesson focused.

Glossary

culturally responsive practice	"An approach to teaching that recognizes the value of learners' cultural beliefs and practices and draws upon them to inform instruction, enhance learner self-advocacy, and bridge learners' home and school experiences." (p. 2)[26]
culture	"Culture describes how we live on a daily basis in terms of our language, ancestry, religion, food, dress, musical tastes, traditions, values, political and social affiliations, recreation, and so on." (p.16)[27]
diversity	"Diversity is inclusive of individual differences (e.g., personality, interests, learning modalities, and life experiences), and group differences (e.g., race, ethnicity, ability, gender identity, gender expression, sexual orientation, nationality, language, religion, political affiliation, and socio-economic background)" (p.49).[28]
intervention	"The systematic use of technique, practice, or program designed and shown to improve learning in specific areas of student need." (p.4)[29]
mathematical agency	"The ability to participate and perform effectively in mathematical contexts." (Aguirre, Mayfield-Ingram, Martin 2013, 16)
mathematical authority	Who or what determines the ideas and reasoning that are the focus of learning. Is it the teacher, the materials, or the students? [30]
mathematics identity	"The dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives." (Aguirre, Mayfield-Ingram, Martin 2013, 13)
number line	"A length model in which numbers are represented as a length from zero along a line segmented into equal lengths." (NCTM 2010, 38)
positionality	"The tension between individual representations of self and the ascriptions made of the individual by wider society. How individuals are positioned in a context depends on both the manner in which they wish to be represented and their perceptions of how others view them." (Berry 2020)

translanguaging	"Translanguaging is the act performed by bilinguals of accessing different linguistic features or various modes of what are described as autonomous languages, in order to maximize communicative potential."[31]
universal instruction	"The academic and behavioral curriculum and instruction deemed critical, delivered to all students, and expected to meet the needs of most students in a school. Also referred to as Core instruction, Primary Level of Intervention, and Tier One Instruction." (p. 3)[32]

Contributing Educators

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