

Technical Report on the Wisconsin School-Level Value-Added Model Academic Year 2016-2017

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INTRODUCTION

This report describes the value-added model used by the Value-Added Research Center (VARC) of the Wisconsin Center for Education Research at the University of Wisconsin to measure the productivity or effectiveness of Wisconsin public schools using Forward test score data. The report is in four parts. The first part describes the data set used to produce the value-added estimates. The second part describes the model used to estimate value-added for schools in Wisconsin. The third part presents some properties of the value-added results. Finally, the fourth part identifies areas for continuous improvement in the development and production of value-added measures.

Conceptually, value-added analysis is the use of statistical techniques to isolate the component of measured student knowledge that is attributable to schools from other factors such as prior knowledge and student characteristics associated with growth in student achievement. In practice, value-added models focus on the improvement students make on annual assessments from one year and grade to the next, taking account of differences in student characteristics. Value-added models often control for measurable student characteristics using available data such as economic disadvantage and disability, to help isolate the impact of schooling. The model used in Wisconsin uses the available set of student characteristics to identify the extent to which schools contribute to the improvement of student achievement outcomes.

ANALYSIS DATA SET

Before estimation can take place, a substantial amount of work is required to assemble the analysis data sets used to produce the value-added estimates. A separate analysis data set is produced for each grade, subject, and test. In total, 10 analysis data sets are produced, covering grades 4 through 8 for Forward English language arts (ELA) and math in 2016-17.

Each analysis data set includes students who have a posttest in the grade and subject being considered, pretests in both ELA and math, had full academic year (FAY) status in their school or district, and were tested in consecutive grades.

In previous years, the model was restricted to students in traditional public and charter schools. In the current year, this was expanded to include students in voucher school programs. In addition, privately run schools receiving voucher students were entitled to an optional value-added score that included all attending students, including those not receiving public funds.

Student-level variables

Posttest and pretest variables

The test scores used are from the 2015-16 and 2016-17 Forward assessments. The value-added system produces school-level measures for grades 4 through 8 in ELA and math based on

performance on the 2016-17 Forward assessment. Value-added in ELA and math is defined by its usage of an ELA or math test as a posttest. All value-added models include pretests in both ELA and math. All test scores were linearly transformed to the z-statistic scale with means equal to zero and standard deviations equal to 1 in each grade and subject. Thus, in the value-added analyses, all test scores were measured relative to the state means and in the units of the statewide standard deviation of test scores in given grades and subjects. The transformation is used to make it easier to interpret estimates of the value-added models, but it does not affect the statistical properties of the model or the ranking of estimated school effects.

Reliability of pretest variables

The reliability estimates of math and ELA pretest scores are available in the technical manual for the Forward exam prepared by the Wisconsin Department of Public Instruction. They range from 0.85 to 0.91 across grades and subjects. These reliability estimates are used for a correction for measurement error in the pretests.

Gender, race/ethnicity, economic disadvantage, and migrancy

Gender, race/ethnicity, economic disadvantage, and migrancy are drawn from the Forward student test score dataset. In the analysis data set, students are assigned the gender, race/ethnicity, low-income status, and migrant status reported in the posttest year. Gender categories are male and female. Race categories are American Indian/Alaskan Native, Asian, Native Hawaiian/Pacific Islander, Black/African American, Hispanic/Latino, White, and multi-racial. The analysis employs an indicator for economically disadvantaged students and an indicator for migrant students.

English as a second language (ESL) classification

There are six indicators for English-language proficiency included in the analysis dataset. Students with ESL classifications of 1 through 5 are considered to be English-language learners. Students with an ESL status of 6 are those that were formerly classified as having limited English proficiency. These students and students reported as proficient in English form the omitted group.

Disability

The analysis includes five indicators for students with disabilities according to their primary disability code. There are separate indicators for emotional/behavioral disability (EBD), learning or intellectual disability (LD/ID), autism (A), and speech/language disability (SL). All other disability codes are grouped into a single indicator for other disabilities.

School enrollment

Students that have FAY status at a single school are assigned to that school using the school enrollment data. Some students have FAY status in a single district but not at a single school because of mobility within the district. These students are included in the district model but not in the school model.

Voucher students

Beginning with the 2016-2017 school year, the analysis set includes test scores for voucher students attending private schools. All such schools receive a value added score based solely on voucher students.

In addition, private schools with voucher students were given an opportunity to receive a second value added score which could include non-voucher students as well as voucher students. Such schools are denoted as “opt-in” schools. To accommodate the additional students and calculations, an additional set of 10 analysis datasets is created, differing from the first only in that it adds in non-voucher students at private schools. Due to the small number of such students, the effect on the demographic composition, mean test scores, and model coefficients is minimal.

Counts of non-voucher students are reduced by the requirement that to be included in the growth analysis data set students must have test score data from both 2016 and 2017. As indicated in the table below, many non-voucher students did not have test scores from 2016.

Number of Non-Voucher Students in Forward 2017 Data and Growth Analysis Data Set

		Grade 4	Grade 5	Grade 6	Grade 7	Grade 8
Math	Forward 2017 Data	286	275	279	294	315
	Growth Analysis Set (2016 & 2017)	189	189	180	195	209
ELA	Forward 2017 Data	286	275	279	294	315
	Growth Analysis Set (2016 & 2017)	189	189	180	195	209

Descriptive statistics of analysis samples

The following tables describe the sample used for the 2017 year:

Forward Math Sample, Non-Voucher Students at Private Schools Omitted

Grade Level	4	5	6	7	8
Number of Students	60063	58957	58476	58899	58133
Number of Public School Students	57791	56825	56415	57067	56270
Number of Voucher Students	2272	2132	2061	1832	1863
Number of Non-Voucher Private School Students	0	0	0	0	0
Total Number of Private School Students	2272	2132	2061	1832	1863
Number of Public Schools	1104	1044	697	666	666
Number of Private Schools	121	122	120	110	114
Number of Public School District Codes	424	423	423	423	424
Number of Private School District Codes	34	32	31	27	28
Posttest Mean	575.973	601.235	614.695	629.343	643.244
Posttest Standard Deviation	54.059	50.176	54.007	57.795	58.191
Math Pretest Mean	555.501	574.960	600.941	614.408	629.428
ELA Pretest Mean	561.563	583.823	600.820	611.748	625.412
Math Pretest Standard Deviation	45.904	55.399	49.614	52.207	56.358
ELA Pretest Standard Deviation	47.001	49.059	50.558	51.580	54.191
Proportion in ESL Level 1	0.002	0.001	0.001	0.001	0.002
Proportion in ESL Level 2	0.005	0.003	0.006	0.006	0.005
Proportion in ESL Level 3	0.023	0.016	0.017	0.016	0.015
Proportion in ESL Level 4	0.030	0.024	0.014	0.013	0.013
Proportion in ESL Level 5	0.012	0.009	0.005	0.003	0.004
Proportion in ESL Level 6	0.025	0.043	0.046	0.046	0.046
Proportion Female	0.488	0.489	0.490	0.487	0.486
Proportion Asian	0.038	0.038	0.038	0.038	0.036
Proportion African American	0.103	0.101	0.098	0.093	0.092
Proportion Hispanic	0.132	0.134	0.125	0.122	0.121
Proportion Native American	0.011	0.011	0.011	0.011	0.011
Proportion Native Hawaiian or Other Pacific Islander	0.001	0.001	0.001	0.001	0.001
Proportion Two or More Races	0.038	0.036	0.033	0.031	0.028
Proportion Special Education LD, ID	0.037	0.043	0.046	0.049	0.050
Proportion Special Education EBD	0.014	0.016	0.017	0.016	0.016
Proportion Special Education A	0.011	0.012	0.011	0.011	0.011
Proportion Special Education SL	0.034	0.025	0.015	0.009	0.007
Proportion Special Education Other	0.033	0.032	0.033	0.035	0.034
Proportion with Economic Disadvantage	0.431	0.425	0.402	0.384	0.373
Proportion Migrant	0.000	0.000	0.000	0.000	0.000

Forward ELA Sample, Non-Voucher Students at Private Schools Omitted

Grade Level	4	5	6	7	8
Number of Students	60064	58943	58483	58912	58167
Number of Public School Students	57791	56830	56419	57078	56299
Number of Voucher Students	2273	2113	2064	1834	1868
Number of Non-Voucher Private School Students	0	0	0	0	0
Total Number of Private School Students	2273	2113	2064	1834	1868
Number of Public Schools	1104	1044	697	666	666
Number of Private Schools	121	122	120	110	114
Number of Public School District Codes	424	423	423	423	424
Number of Private School District Codes	34	32	31	27	28
Posttest Mean	586.563	604.538	615.989	628.518	639.560
Posttest Standard Deviation	51.730	50.289	49.042	58.286	60.789
Math Pretest Mean	555.517	574.982	600.946	614.396	629.385
ELA Pretest Mean	561.572	583.843	600.818	611.748	625.393
Math Pretest Standard Deviation	45.877	55.385	49.609	52.200	56.400
ELA Pretest Standard Deviation	46.988	49.051	50.563	51.574	54.188
Proportion in ESL Level 1	0.002	0.001	0.001	0.001	0.002
Proportion in ESL Level 2	0.005	0.003	0.006	0.006	0.005
Proportion in ESL Level 3	0.023	0.016	0.017	0.016	0.015
Proportion in ESL Level 4	0.030	0.024	0.014	0.013	0.013
Proportion in ESL Level 5	0.012	0.009	0.005	0.003	0.004
Proportion in ESL Level 6	0.025	0.043	0.046	0.046	0.046
Proportion Female	0.488	0.488	0.490	0.487	0.487
Proportion Asian	0.038	0.038	0.038	0.038	0.036
Proportion African American	0.103	0.101	0.098	0.093	0.093
Proportion Hispanic	0.132	0.134	0.125	0.122	0.121
Proportion Native American	0.011	0.011	0.011	0.011	0.011
Proportion Native Hawaiian or Other Pacific Islander	0.001	0.001	0.001	0.001	0.001
Proportion Two or More Races	0.038	0.036	0.033	0.031	0.028
Proportion Special Education LD, ID	0.037	0.043	0.046	0.049	0.050
Proportion Special Education EBD	0.014	0.016	0.017	0.016	0.016
Proportion Special Education A	0.011	0.012	0.011	0.011	0.011
Proportion Special Education SL	0.034	0.025	0.015	0.009	0.007
Proportion Special Education Other	0.033	0.032	0.033	0.035	0.034
Proportion with Economic Disadvantage	0.431	0.424	0.402	0.384	0.373
Proportion Migrant	0.000	0.000	0.000	0.000	0.000

Forward Math Sample, Including Non-Voucher Students at Private Schools

Grade Level	4	5	6	7	8
Number of Students	60252	59146	58656	59094	58342
Number of Public School Students	57791	56825	56415	57067	56270
Number of Voucher Students	2272	2132	2061	1832	1863
Number of Non-Voucher Private School Students	189	189	180	195	209
Total Number of Private School Students	2461	2321	2241	2027	2072
Number of Public Schools	1104	1044	697	666	666
Number of Private Schools	123	123	120	113	116
Number of Public School District Codes	424	423	423	423	424
Number of Private School District Codes	36	32	31	29	30
Posttest Mean	575.992	601.246	614.718	629.355	643.290
Posttest Standard Deviation	54.022	50.141	53.968	57.765	58.173
Math Pretest Mean	555.506	574.948	600.948	614.399	629.461
ELA Pretest Mean	561.574	583.826	600.849	611.763	625.454
Math Pretest Standard Deviation	45.900	55.371	49.573	52.178	56.343
ELA Pretest Standard Deviation	47.014	49.073	50.563	51.568	54.197
Proportion in ESL Level 1	0.002	0.001	0.001	0.001	0.002
Proportion in ESL Level 2	0.005	0.003	0.006	0.006	0.005
Proportion in ESL Level 3	0.023	0.016	0.017	0.016	0.015
Proportion in ESL Level 4	0.030	0.024	0.014	0.013	0.013
Proportion in ESL Level 5	0.012	0.009	0.005	0.003	0.004
Proportion in ESL Level 6	0.025	0.043	0.046	0.046	0.046
Proportion Female	0.488	0.489	0.490	0.487	0.486
Proportion Asian	0.038	0.038	0.038	0.038	0.036
Proportion African American	0.103	0.101	0.097	0.093	0.092
Proportion Hispanic	0.132	0.134	0.125	0.122	0.121
Proportion Native American	0.011	0.011	0.011	0.011	0.011
Proportion Native Hawaiian or Other Pacific Islander	0.001	0.001	0.001	0.001	0.001
Proportion Two or More Races	0.038	0.036	0.033	0.031	0.028
Proportion Special Education LD, ID	0.037	0.043	0.046	0.049	0.050
Proportion Special Education EBD	0.014	0.016	0.016	0.016	0.016
Proportion Special Education A	0.011	0.012	0.011	0.011	0.011
Proportion Special Education SL	0.034	0.025	0.015	0.009	0.007
Proportion Special Education Other	0.032	0.032	0.033	0.035	0.034
Proportion with Economic Disadvantage	0.430	0.424	0.401	0.383	0.372
Proportion Migrant	0.000	0.000	0.000	0.000	0.000

Forward ELA, Including Non-Voucher Students at Private Schools

Grade Level	4	5	6	7	8
Number of Students	60253	59132	58663	59107	58376
Number of Public School Students	57791	56830	56419	57078	56299
Number of Voucher Students	2273	2113	2064	1834	1868
Number of Non-Voucher Private School Students	189	189	180	195	209
Total Number of Private School Students	2462	2302	2244	2029	2077
Number of Public Schools	1104	1044	697	666	666
Number of Private Schools	123	123	120	113	116
Number of Public School District Codes	424	423	423	423	424
Number of Private School District Codes	36	32	31	29	30
Posttest Mean	586.618	604.563	616.031	628.566	639.658
Posttest Standard Deviation	51.720	50.290	49.023	58.262	60.790
Math Pretest Mean	555.523	574.969	600.953	614.388	629.418
ELA Pretest Mean	561.583	583.845	600.847	611.764	625.435
Math Pretest Standard Deviation	45.873	55.358	49.568	52.171	56.385
ELA Pretest Standard Deviation	47.001	49.065	50.568	51.562	54.194
Proportion in ESL Level 1	0.002	0.001	0.001	0.001	0.002
Proportion in ESL Level 2	0.005	0.003	0.006	0.006	0.005
Proportion in ESL Level 3	0.023	0.016	0.017	0.016	0.015
Proportion in ESL Level 4	0.030	0.024	0.014	0.013	0.013
Proportion in ESL Level 5	0.012	0.009	0.005	0.003	0.004
Proportion in ESL Level 6	0.025	0.043	0.046	0.046	0.046
Proportion Female	0.488	0.488	0.490	0.487	0.487
Proportion Asian	0.038	0.038	0.037	0.038	0.036
Proportion African American	0.103	0.101	0.098	0.093	0.092
Proportion Hispanic	0.132	0.134	0.125	0.122	0.121
Proportion Native American	0.011	0.011	0.011	0.011	0.011
Proportion Native Hawaiian or Other Pacific Islander	0.001	0.001	0.001	0.001	0.001
Proportion Two or More Races	0.038	0.036	0.033	0.031	0.028
Proportion Special Education LD, ID	0.036	0.043	0.045	0.049	0.050
Proportion Special Education EBD	0.014	0.016	0.016	0.016	0.016
Proportion Special Education A	0.011	0.012	0.011	0.011	0.011
Proportion Special Education SL	0.034	0.025	0.015	0.009	0.007
Proportion Special Education Other	0.032	0.032	0.033	0.035	0.034
Proportion with Economic Disadvantage	0.430	0.423	0.401	0.384	0.372
Proportion Migrant	0.000	0.000	0.000	0.000	0.000

VALUE-ADDED MODEL

For the Wisconsin school level model, value-added is measured in math and ELA in grades four through eight for the Forward assessment. Schools are assigned single-year value-added measures that reflect student growth from the Spring of 2016 to the Spring of 2017.

The model, in brief

The value-added model is defined by four equations: a "best linear predictor" value-added model defined in terms of true student post and prior achievement and three measurement error models for observed post and prior achievement:

$$\text{Student achievement: } y_{1i} = \zeta + \lambda y_{0i} + \lambda^{alt} y_{0i}^{alt} + \beta X_i + \alpha S_i + e_i \quad (1)$$

$$\text{Posttest measurement error: } Y_{1i} = y_{1i} + v_{1i} \quad (2)$$

$$\text{Same-subject pretest measurement error: } Y_{0i} = y_{0i} + v_{0i} \quad (3)$$

$$\text{Other-subject pretest measurement error: } Y_{0i}^{alt} = y_{0i}^{alt} + v_{0i}^{alt} \quad (4)$$

where:

- y_{1i} is true post achievement;
- y_{0i} and y_{0i}^{alt} are true prior achievement in the same subject and in the other subject (math in the ELA model, ELA in the math model), with slope parameters λ and λ^{alt} ;
- X_i is a vector of characteristics of student i , with slope parameter vector β ;
- S_i is a vector of indicators for school;
- α is a vector of school effects;
- e_i is the error in predicting post achievement given the explanatory variables included in the model;
- Y_{1i} is measured post achievement;
- v_{1i} is measurement error in post achievement;
- Y_{0i} and Y_{0i}^{alt} are measured prior achievement; and
- v_{0i} and v_{0i}^{alt} are measurement error in prior achievement.

Substituting the measurement error equations (2), (3), and (4) into the student achievement equation (1) yields an equation defined in terms of measured student achievement:

$$\text{Measured achievement: } Y_{1i} = \zeta + \lambda Y_{0i} + \lambda^{alt} Y_{0i}^{alt} + \beta X_i + \alpha S_i + \varepsilon_i \quad (5)$$

where the error term ε_i includes both the original error component and the measurement error components:

$$\text{Error in measured achievement: } \varepsilon_i = e_i + v_{1i} - \lambda v_{0i} - \lambda^{alt} v_{0i}^{alt} \quad (6)$$

Estimating the measured student achievement equation (5) without controlling for pretest measurement error yields biased estimates of all parameters, including the value-added effects. This bias stems from the fact that measurement error in prior achievement causes the error term (6), which includes the measurement error components v_{0i} and v_{0i}^{alt} , to be correlated with measured prior achievement. The desired parameters, as defined in equation (1), can be estimated consistently if external information is available on the variance of measurement error for prior achievement; approaches for consistent estimation in the presence of measurement error are described in detail in Wayne Fuller, *Measurement Error Models* (Wiley, 1987). Information about the variance of test measurement error is obtained from the reliability estimates reported in the technical manual for the 2014-15 Badger exam assessment.

A shrinkage approach is employed to ensure that schools with fewer students are not overrepresented among the highest- and lowest-value-added cases due to randomness. The approach, Empirical Bayes shrinkage, is described in J. N. K. Rao, *Small Area Estimation* (Wiley, 2003).

The variables in the model

In addition to posttest and pretest scores, the student-level variables included in the model (the X variables in equation 1) are gender, race/ethnicity, ESL category, economic disadvantage, disability code, and migrancy.

Value-added regression

The value-added model is estimated using a least-squares regression approach that corrects for measurement error in the pretest variables. It estimates the coefficients λ , β , and α by regressing posttest on same-subject pretest, other-subject pretest, other student-level variables, and a full set of school fixed effects. This regression is estimated using an approach that accounts for measurement error in the pretests Y_{0i} and Y_{0i}^{alt} . Recall from equation (6) above that the measurement error components of Y_{0i} and Y_{0i}^{alt} , v_{0i} and v_{0i}^{alt} , are part of the error term ε_i . As a result, estimating the regression using ordinary least squares (without control for pretest measurement error) will lead to biased estimates. The regression approach employed accounts for measurement error by removing the variance in the pretests that is attributable to measurement error. To illustrate the measurement error corrected regression, re-cast the above value-added regression equation into vector form:

$$Y_t = Y_{t-1}\lambda + W\delta + \varepsilon$$

where Y_t is an $N \times 1$ vector of post-test scores, Y_{t-1} is an $N \times 2$ vector of same-subject and other-subject pre-test scores Y_{t-1} and Y_{t-1}^{alt} , λ is a 2×1 vector made up of λ and λ^{alt} , W is an $N \times K$ vector of the X demographic variables, δ is a $K \times 1$ vector of the β and α^* coefficients, and ε is an $N \times 1$ vector of error terms. The biased ordinary-least-squares estimates of the coefficients in λ and δ are equal to:

$$\begin{bmatrix} \hat{\lambda}_{OLS} \\ \hat{\delta}_{OLS} \end{bmatrix} = \begin{bmatrix} Y'_{t-1}Y_{t-1} & Y'_{t-1}W \\ W'Y_{t-1} & W'W \end{bmatrix}^{-1} \begin{bmatrix} Y'_{t-1}Y_t \\ W'Y_t \end{bmatrix}$$

The measurement-error-corrected estimates of the coefficients in λ and δ are equal to:

$$\begin{bmatrix} \hat{\lambda}_{EIV} \\ \hat{\delta}_{EIV} \end{bmatrix} = \begin{bmatrix} Y'_{t-1}Y_{t-1} - \sum_i^N sem_{it-1} & Y'_{t-1}W \\ W'Y_{t-1} & W'W \end{bmatrix}^{-1} \begin{bmatrix} Y'_{t-1}Y_t \\ W'Y_t \end{bmatrix}$$

where sem_{it-1} is a 2×2 variance-covariance matrix of the errors of measurement of Y_{it-1} and Y_{it-1}^{alt} for student i . This model is described in section 2.2 of Wayne Fuller, *Measurement Error Models* (Wiley, 1987).

In 2017 we explored using a two-step method for estimating the value-added model that was designed to minimize the influence of test scores at the extremes of the distribution. This method was found to be useful for the mathematics model because in some grades the percent of students receiving the lowest observable scale score (LOSS) in mathematics is somewhat higher than in previous years – see the table below. In step one model parameters are estimated using all students other than those at the LOSS on the mathematics pretest. In step two the estimated parameters for the two pretest variables (prior math and ELA) are treated as known and the model is re-estimated using all students. This approach yields estimates of model parameters and value-added estimates that are comparable to those obtained in previous years.¹

¹ Since the data sets used in the estimation are very large, the pretest coefficients from step one are estimated with extremely high precision. Thus, estimates of standard errors for all parameters are obtained from step two, using the measurement error correction method described above.

Percentage of Students at Test Floor (Lowest Observable Scale Score, LOSS) for Pre- and Posttests

Non-Voucher Private School Students	Grade	Test Subject	Percent at Posttest Floor	Percent at Math Pretest Floor	Percent at ELA Pretest Floor	
Included in Growth Analysis Data Set	4	ELA	0.0%	0.6%	0.0%	
		Mathematics	2.8%	0.6%	0.0%	
	5	ELA	0.0%	3.3%	0.0%	
		Mathematics	1.7%	3.3%	0.0%	
	6	ELA	0.0%	1.1%	0.0%	
		Mathematics	1.9%	1.1%	0.0%	
	7	ELA	0.0%	1.2%	0.1%	
		Mathematics	2.8%	1.2%	0.1%	
	8	ELA	0.0%	2.8%	0.0%	
		Mathematics	4.2%	2.8%	0.0%	
	Omitted from Growth Analysis Data Set	4	ELA	0.0%	0.6%	0.0%
			Mathematics	2.8%	0.6%	0.0%
5		ELA	0.0%	3.3%	0.0%	
		Mathematics	1.7%	3.3%	0.0%	
6		ELA	0.0%	1.1%	0.0%	
		Mathematics	1.9%	1.1%	0.0%	
7		ELA	0.0%	1.2%	0.1%	
		Mathematics	2.8%	1.2%	0.1%	
8		ELA	0.0%	2.8%	0.0%	
		Mathematics	4.2%	2.8%	0.0%	

Aggregation to multiple-grade value-added

The value-added regression to obtain unshrunk school value-added is performed separately for each combination of grade and subject. For schools that have results for more than one grade level, these estimates are averaged across grades, using the number of students attributed to the school as weights, to produce unshrunk multiple-grade value-added estimates. Before aggregation, value-added measures by grade are normalized in order to be on similar scales (i.e. with a mean of 0 and a true standard deviation of 1) across grades. This normalization is made by dividing the measures by an estimate of the standard deviation of value-added within grade.

Shrinkage of value-added

At all levels, the unshrunk value-added estimates are shrunk using an Empirical Bayes univariate shrinkage technique described in J. N. K. Rao, *Small Area Estimation* (Wiley, 2003). This procedure is employed to bring value-added estimates based on smaller sample sizes closer to the state average, so that schools with fewer students are not overrepresented among the highest-

and lowest-value-added cases simply due to randomness. This is estimated by multiplying each value-added measure by its reliability:

$$\alpha_{\text{shrunk}} = (\omega^2 / (\omega^2 + \sigma^2))\alpha_{\text{unshrunk}}$$

where α_{unshrunk} is an unshrunk value-added estimate for a given school; σ^2 is the squared standard error of α_{unshrunk} ; and ω^2 is the variance of value-added across schools within subject, test, and grade(s). The standard error of the shrunk value-added estimate is equal to

$$\text{s.e.}(\alpha_{\text{shrunk}}) = \text{sqrt}[\omega^2\sigma^2 / (\omega^2 + \sigma^2)]$$

The variance measure ω^2 is estimated by computing the variance of the unshrunk value-added estimates, then subtracting from that the average squared standard error of the unshrunk value-added estimates. This variance measure is an estimate of the variance of the underlying value-added measures, excluding variance due to randomness in the value-added estimates. The square root of this variance measure is also used for normalizing value-added measures by grade before aggregation to multiple-grade measures.

Student group value-added

Value-added is also measured by student groups defined by certain demographic characteristics. Specifically, we calculated differential value-added effects for the seven race/ethnicity groups, for students with disabilities, for economically disadvantaged students, and for English-language learners.

To produce the group results by school, we regress the estimate of the sum of the school effects and the residual, $\alpha'S_i + \varepsilon_i$, on a vector of school indicators to produce a new residual, which we will refer to as ε_i^* . This residual is the component of student achievement that cannot be explained with the pretest scores, demographics, or any overall school effect. These residuals are then regressed on interactions between school indicators and the group variable demeaned within school:

$$\varepsilon_i^* = \sum_{j=1}^J \theta_j I_{ij} (x_i - x_j) + u_i$$

where I_{ij} is an indicator that equals 1 if student i is associated with school j , x_i is an indicator variable that indicates whether a student is part of the group, and x_j is the proportion of students in school j who are in the group. This yields a slope estimate θ_j for each school, which is shrunk using Empirical Bayes shrinkage. Value-added for students in the group for school j is set to $\alpha_j + \theta_j(1 - x_j)$, where α_j is overall value-added for school j . For students outside the group, value-added is set

to $\alpha_j - \theta_j x_j$. This description is for a case of a binary student group variable, such as disability (where X_i can only equal 0 or 1).

In the case of race/ethnicity groups, of which there are seven (American Indian/Alaskan Native, Asian, Native Hawaiian/Pacific Islander, Black/African American, Hispanic/Latino, White, and multi-racial), this approach is generalized to use six x variables, one for each race/ethnicity group with one of the groups excluded so that the effects represent differences in the effects of race/ethnicity across groups. Student group value-added measures that cover multiple grades are computed by averaging the unshrunk slopes θ_j across grades and years, shrinking them using Empirical Bayes shrinkage, and combining them with overall multiple-grade value-added measures in the same way as for single-grade measures.

Final stage for estimation of school and district value-added results

In order to enhance the accuracy and precision of the value-added estimates, final estimates of school value-added effects are obtained by combining estimates for 2016 and 2017, the first two years in which results using the Wisconsin Forward Exam are available. This applies only to schools with two years of value-added results, which requires that schools administered state assessments in 2015, 2016, and 2017. The weights used in combining the two shrunk estimates are composed of two factors: (1) the number of students in each school in each year and (2) a higher weight for the 2017 results (weight = 2/3) and a lower weight for the 2016 results (weight = 1/3).

Final estimates of district value-added effects are obtained by averaging the shrunk combined value-added estimates (as described above) for all of the schools in each district, with weights determined by the number of students in each school in 2017. As mentioned earlier, the district results include students if they were FAY at the district even if they were not FAY at any of the district's schools. Thus, students who moved from one school in a district to another school in the district are included.

PROPERTIES OF THE VALUE-ADDED RESULTS

Coefficients on student-level variables in the model

The coefficients estimated in the value-added model are presented on the next page. To interpret these coefficients, note that both pretest and posttest are measured using standardized scores; therefore all coefficients are measured in the posttest standard deviation scale. For example, note that the coefficient on female gender is -0.055 in grade 4 Math. The posttest standard deviation for grade 4 Math is 54.059. This implies that male students improved 0.083 standard deviations or about 2.973 scale score points more on the grade 4 Math test from spring to spring than otherwise similar female students.

It is important to keep in mind the standard errors of the coefficients when interpreting them. A span of 1.96 standard errors in both the positive and negative directions provides a 95 percent confidence range for a coefficient. Continuing with the example of the coefficient on female gender in grade 4 Math, note that the standard error of this coefficient estimate is 0.005 in posttest SD units or 0.270 in scale score points. This means that, while our best estimate of the difference in growth between female and male students is -2.973 scale score points, a 95 percent confidence interval for the difference ranges from -3.502 to -2.444 scale score points.

Coefficients on Student-Level Variables, 2016-17 Forward Math, Non-Voucher Students at Private Schools Omitted

Variable	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
	Coeff.	SE								
Math Pretest	0.788	0.007	0.843	0.008	0.673	0.006	0.861	0.008	0.770	0.008
ELA Pretest	0.068	0.007	0.135	0.007	0.223	0.007	0.069	0.008	0.164	0.007
ESL Level 1	-0.033	0.061	-0.041	0.084	-0.193	0.065	0.266	0.065	0.194	0.064
ESL Level 2	-0.194	0.034	0.054	0.046	-0.090	0.031	-0.001	0.031	-0.050	0.036
ESL Level 3	-0.073	0.018	0.169	0.022	-0.035	0.019	-0.038	0.021	0.013	0.023
ESL Level 4	0.031	0.016	0.162	0.018	-0.043	0.021	-0.025	0.022	0.115	0.024
ESL Level 5	0.056	0.023	0.127	0.028	0.085	0.032	0.094	0.043	0.070	0.041
ESL Level 6	0.072	0.017	0.038	0.015	0.052	0.013	-0.015	0.014	0.032	0.015
Female	-0.055	0.005	0.036	0.005	-0.060	0.005	-0.055	0.005	0.039	0.005
Asian	0.035	0.015	0.066	0.016	0.013	0.014	-0.006	0.015	0.075	0.016
African-American	-0.083	0.012	-0.002	0.013	-0.085	0.011	-0.084	0.012	-0.034	0.013
Hispanic	-0.048	0.010	0.004	0.010	-0.036	0.009	-0.049	0.010	-0.006	0.011
Indian	-0.041	0.026	0.023	0.027	0.003	0.024	-0.069	0.026	-0.038	0.027
Native Hawaiian or Other Pacific Islander	-0.058	0.093	0.069	0.093	0.038	0.074	-0.036	0.095	-0.128	0.101
Two or More Races	-0.026	0.013	-0.017	0.014	0.003	0.013	-0.034	0.014	0.003	0.016
Special Education LD/ID	-0.127	0.013	0.083	0.013	-0.243	0.011	0.031	0.011	-0.034	0.012
Special Education EBD	-0.131	0.020	-0.061	0.021	-0.208	0.018	-0.053	0.020	-0.126	0.020
Special Education A	-0.127	0.023	-0.055	0.024	-0.235	0.021	0.052	0.023	0.007	0.025
Special Education SL	-0.009	0.013	0.036	0.016	-0.038	0.019	-0.013	0.025	0.010	0.031
Special Education Other	-0.106	0.014	-0.008	0.015	-0.273	0.013	0.040	0.013	-0.036	0.014
Economic Disadvantage	-0.034	0.006	-0.022	0.006	-0.050	0.006	-0.012	0.006	-0.034	0.006
Migrancy Status	0.482	0.183	0.337	0.177	0.147	0.137	0.504	0.146	0.197	0.166

Coefficients on Student-Level Variables, 2016-17 Forward ELA, Non-Voucher Students at Private Schools Omitted

Variable	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
	Coeff.	SE								
Math Pretest	0.109	0.007	0.048	0.006	0.048	0.006	0.098	0.008	0.096	0.006
ELA Pretest	0.766	0.007	0.826	0.006	0.835	0.007	0.834	0.008	0.828	0.006
ESL Level 1	-0.100	0.061	-0.314	0.079	-0.279	0.070	-0.063	0.064	0.010	0.059
ESL Level 2	-0.163	0.034	-0.300	0.043	-0.048	0.033	0.059	0.032	0.034	0.033
ESL Level 3	-0.002	0.018	-0.046	0.021	0.008	0.021	0.052	0.021	0.071	0.021
ESL Level 4	0.085	0.016	0.040	0.017	-0.052	0.023	0.064	0.022	0.077	0.022
ESL Level 5	0.049	0.023	0.083	0.026	-0.031	0.035	0.068	0.043	0.037	0.038
ESL Level 6	0.066	0.017	0.050	0.014	0.005	0.014	0.094	0.014	0.003	0.014
Female	0.038	0.005	0.069	0.005	0.039	0.005	0.057	0.005	0.113	0.005
Asian	-0.054	0.015	-0.023	0.015	-0.003	0.015	0.086	0.015	0.011	0.015
African-American	-0.099	0.012	-0.073	0.012	-0.060	0.012	0.007	0.012	0.014	0.012
Hispanic	-0.038	0.010	-0.015	0.010	-0.041	0.010	0.019	0.010	-0.014	0.010
Indian	-0.081	0.026	-0.054	0.026	-0.032	0.026	0.044	0.025	-0.018	0.025
Native Hawaiian or Other Pacific Islander	-0.206	0.093	-0.081	0.087	0.031	0.080	0.165	0.094	-0.149	0.093
Two or More Races	-0.007	0.013	-0.025	0.013	-0.038	0.014	0.009	0.014	-0.001	0.015
Special Education LD/ID	-0.071	0.013	-0.187	0.013	-0.183	0.013	-0.019	0.012	-0.001	0.012
Special Education EBD	-0.122	0.020	-0.196	0.019	-0.256	0.020	-0.117	0.020	-0.080	0.019
Special Education A	-0.160	0.023	-0.228	0.022	-0.176	0.023	-0.006	0.023	0.039	0.023
Special Education SL	0.000	0.013	-0.064	0.015	-0.046	0.020	-0.001	0.025	-0.006	0.028
Special Education Other	-0.130	0.014	-0.196	0.014	-0.207	0.014	-0.055	0.014	-0.037	0.014
Economic Disadvantage	-0.049	0.006	-0.043	0.006	-0.055	0.006	-0.026	0.006	-0.041	0.006
Migrancy Status	-0.219	0.183	-0.018	0.165	0.190	0.149	0.114	0.145	-0.005	0.153

Coefficients on Student-Level Variables, 2016-17 Forward Math, Including Non-Voucher Students at Private Schools

Variable	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
	Coeff.	SE								
Math Pretest	0.788	0.007	0.842	0.008	0.672	0.006	0.860	0.008	0.770	0.008
ELA Pretest	0.068	0.007	0.136	0.007	0.224	0.007	0.070	0.008	0.165	0.007
ESL Level 1	-0.033	0.061	-0.041	0.084	-0.193	0.065	0.267	0.065	0.196	0.064
ESL Level 2	-0.195	0.034	0.055	0.046	-0.093	0.031	0.000	0.031	-0.050	0.036
ESL Level 3	-0.073	0.018	0.169	0.022	-0.035	0.019	-0.036	0.021	0.014	0.023
ESL Level 4	0.031	0.016	0.163	0.018	-0.042	0.021	-0.024	0.022	0.116	0.024
ESL Level 5	0.057	0.023	0.127	0.028	0.085	0.032	0.094	0.043	0.071	0.041
ESL Level 6	0.072	0.017	0.038	0.015	0.052	0.013	-0.014	0.014	0.033	0.015
Female	-0.055	0.005	0.036	0.005	-0.060	0.005	-0.055	0.005	0.039	0.005
Asian	0.036	0.015	0.067	0.016	0.014	0.014	-0.007	0.015	0.074	0.016
African-American	-0.083	0.012	-0.002	0.013	-0.085	0.011	-0.083	0.012	-0.032	0.013
Hispanic	-0.048	0.010	0.004	0.010	-0.036	0.009	-0.050	0.010	-0.007	0.011
Indian	-0.041	0.026	0.025	0.027	0.003	0.024	-0.070	0.026	-0.038	0.027
Native Hawaiian or Other Pacific Islander	-0.058	0.093	0.069	0.093	0.038	0.074	-0.036	0.095	-0.128	0.101
Two or More Races	-0.025	0.013	-0.016	0.014	0.003	0.013	-0.034	0.014	0.001	0.016
Special Education LD/ID	-0.128	0.013	0.083	0.013	-0.243	0.011	0.032	0.011	-0.034	0.012
Special Education EBD	-0.132	0.020	-0.061	0.021	-0.208	0.018	-0.053	0.020	-0.126	0.020
Special Education A	-0.127	0.023	-0.055	0.024	-0.237	0.021	0.052	0.023	0.007	0.025
Special Education SL	-0.009	0.013	0.036	0.016	-0.038	0.019	-0.013	0.025	0.008	0.031
Special Education Other	-0.106	0.014	-0.008	0.015	-0.274	0.013	0.040	0.013	-0.036	0.014
Economic Disadvantage	-0.034	0.006	-0.022	0.006	-0.051	0.006	-0.013	0.006	-0.034	0.006
Migrancy Status	0.482	0.183	0.338	0.177	0.147	0.137	0.504	0.146	0.198	0.166

Coefficients on Student-Level Variables, 2016-17 Forward ELA, Including Non-Voucher Students at Private Schools

Variable	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
	Coeff.	SE								
Math Pretest	0.108	0.007	0.048	0.006	0.048	0.006	0.099	0.008	0.095	0.006
ELA Pretest	0.767	0.007	0.827	0.006	0.834	0.007	0.833	0.008	0.829	0.006
ESL Level 1	-0.101	0.061	-0.315	0.079	-0.281	0.070	-0.064	0.064	0.010	0.059
ESL Level 2	-0.164	0.034	-0.298	0.043	-0.050	0.033	0.058	0.032	0.034	0.033
ESL Level 3	-0.002	0.018	-0.045	0.021	0.006	0.021	0.051	0.021	0.071	0.021
ESL Level 4	0.084	0.016	0.041	0.017	-0.053	0.023	0.063	0.022	0.077	0.022
ESL Level 5	0.049	0.023	0.083	0.026	-0.031	0.035	0.068	0.042	0.037	0.038
ESL Level 6	0.065	0.017	0.050	0.014	0.004	0.014	0.094	0.014	0.003	0.014
Female	0.038	0.005	0.069	0.005	0.038	0.005	0.057	0.005	0.113	0.005
Asian	-0.053	0.015	-0.023	0.015	-0.002	0.015	0.086	0.015	0.012	0.015
African-American	-0.099	0.012	-0.073	0.012	-0.060	0.012	0.009	0.012	0.014	0.012
Hispanic	-0.038	0.010	-0.014	0.010	-0.040	0.010	0.020	0.010	-0.015	0.010
Indian	-0.081	0.026	-0.052	0.026	-0.032	0.026	0.044	0.025	-0.019	0.025
Native Hawaiian or Other Pacific Islander	-0.206	0.093	-0.081	0.087	0.031	0.080	0.165	0.094	-0.149	0.093
Two or More Races	-0.007	0.013	-0.024	0.013	-0.039	0.014	0.010	0.014	-0.001	0.015
Special Education LD/ID	-0.071	0.013	-0.187	0.013	-0.184	0.013	-0.018	0.012	0.000	0.012
Special Education EBD	-0.122	0.020	-0.195	0.019	-0.256	0.020	-0.117	0.020	-0.080	0.019
Special Education A	-0.160	0.023	-0.227	0.022	-0.177	0.023	-0.006	0.023	0.040	0.023
Special Education SL	0.000	0.013	-0.064	0.015	-0.047	0.020	-0.001	0.025	-0.007	0.028
Special Education Other	-0.130	0.014	-0.195	0.014	-0.207	0.014	-0.055	0.014	-0.036	0.014
Economic Disadvantage	-0.049	0.006	-0.043	0.006	-0.055	0.006	-0.026	0.006	-0.041	0.006
Migrancy Status	-0.220	0.183	-0.019	0.165	0.190	0.149	0.114	0.145	-0.004	0.153

Test of model neutrality: Correlation with average prior proficiency

Results show a very low correlation between average prior proficiency--a measure of average performance in the previous year--and value-added. In general, schools were not more or less likely to have a low value-added score than a high score if their students began the year with low pretest scores rather than high scores.

Correlations between Prior Attainment and Value-Added

Subject	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	School
ELA	0.04	0.01	-0.08	-0.10	-0.16	-0.01
Math	0.17	-0.17	0.00	-0.17	0.08	0.02

Correlation between Math and ELA value-added

There were also substantive positive correlations between math and ELA value-added within each school. Schools that were high value-added in math were also more often than not high value-added in ELA.

Correlations between Subjects

	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	School
2017 Math and ELA	0.51	0.41	0.60	0.41	0.44	0.53

CONCLUSION

This technical report describes the value-added model used to estimate the productivity of Wisconsin public schools and developed by the Value-Added Research Center of the Wisconsin Center for Education Research at the University of Wisconsin. For more information, contact the Principal Investigator for this project, Dr. Robert Meyer, at Robert.Meyer@wisc.edu.