



Education Analytics INC.

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# TECHNICAL REPORT ON THE WISCONSIN SCHOOL-LEVEL VALUE-ADDED MODEL ACADEMIC YEAR 2017-2018

PREPARED BY

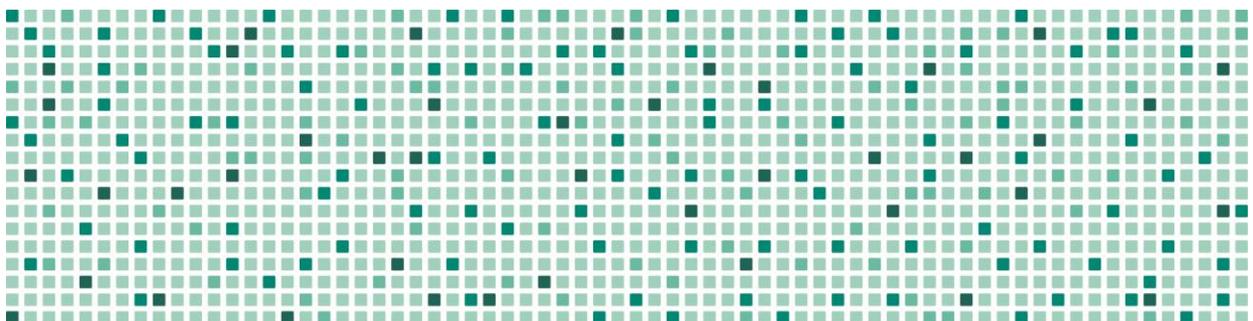
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# CONTENTS

INTRODUCTION.....	3
ANALYSIS DATA SET .....	3
Student-level variables .....	4
School enrollment .....	5
Voucher students.....	5
Descriptive statistics of analysis samples .....	6
VALUE-ADDED MODEL.....	9
The model, in brief .....	9
Value-added regression.....	10
The variables in the model.....	12
Aggregation to multiple-grade value-added .....	12
Shrinkage of value-added .....	12
Student group value-added .....	14
Final stage for estimation of school and district value-added results .....	14
PROPERTIES OF THE VALUE-ADDED RESULTS .....	15
Coefficients on student-level variables in the model .....	15
Test of model neutrality: Correlation with average prior proficiency.....	18
Correlation between Math and ELA value-added.....	18
CONTACT .....	18
REFERENCES.....	19





## INTRODUCTION

This report describes the value-added model used by Education Analytics to measure the productivity or effectiveness of Wisconsin public schools using Forward test score data. The report is divided into three sections. The first section describes the data set used to produce the value-added estimates. The second section describes the model used to estimate value-added for schools in Wisconsin. Finally, the third section presents some properties of the value-added results.

Conceptually, value-added analysis is the use of statistical techniques to isolate the component of measured student knowledge that is attributable to schools from other factors such as prior knowledge and student characteristics associated with growth in student achievement. In practice, value-added models focus on the improvement students make on annual assessments from one year and grade to the next, taking into account differences in student characteristics. Value-added models often control for measurable student characteristics using available data, such as economic disadvantage and disability, to help isolate the impact of schooling.

The model used in Wisconsin includes the available set of student characteristics to identify the extent to which schools contribute to the improvement of student achievement outcomes. Once the school-level value-added results are calculated, these are averaged to obtain district scores. In order to calculate the final scores, up to three years of results are combined: 2015-2016, 2016-2017, and 2017-2018.

## ANALYSIS DATA SET

Before estimation can take place, a substantial amount of work is required to assemble the analysis data sets used to produce the value-added estimates. A separate analysis data set is produced for each grade, subject, and test. In total, 10 analysis data sets are produced, covering grades 4 through 8 for Forward English language arts (ELA) and math in 2017-18.

Each analysis data set includes students who have a posttest in the grade and subject being considered, pretests in both ELA and math, had full academic year (FAY) status in their school or district, and were tested in consecutive grades.

The model has recently been expanded to include students in voucher school programs (referred to as Private School Choice Programs in Wisconsin). In addition, privately run schools receiving voucher students were entitled to an optional value-added score that included all attending students, including those not receiving public funds.





## STUDENT-LEVEL VARIABLES

### POSTTEST AND PRETEST VARIABLES

The test scores used are from the 2016-17 and 2017-18 Forward assessments. The value-added system produces school-level measures for grades 4 through 8 in ELA and math based on performance on the 2017-18 Forward assessment. Value-added in ELA and math is defined by its usage of an ELA or math test as a posttest. All value-added models include pretests in both ELA and math. All test scores were linearly transformed to the z-statistic scale with means equal to zero and standard deviations equal to 1 in each grade and subject. Thus, in the value-added analyses, all test scores were measured relative to the state means, and in the units of the statewide standard deviation of test scores in given grades and subjects. The transformation is used to make it easier to interpret estimates of the value-added models, but it does not affect the statistical properties of the model or the ranking of estimated school effects.

### RELIABILITY OF PRETEST VARIABLES

The reliability estimates of math and ELA pretest scores are available in the technical manual for the Forward exam prepared by the Wisconsin Department of Public Instruction. They range from 0.87 to 0.92 across grades and subjects. These reliability estimates are used for a correction for measurement error in the pretests.

### GENDER, RACE/ETHNICITY, ECONOMIC DISADVANTAGE, AND MIGRANCY

Gender, race/ethnicity, economic disadvantage, and migrancy are drawn from the Wisconsin Information System for Education data (WISEdata) elements. Specifically, the values for these variables are drawn from the Assessment Snapshot of WISEdata captured on August 31, 2018.<sup>1</sup> In the analysis data set, students are assigned the gender, race/ethnicity, low-income status, and migrant status reported in the post-test year. Gender categories are male and female. Race categories are American Indian/Alaskan Native, Asian, Native Hawaiian/Pacific Islander, Black/African American, Hispanic/Latino, White, and multi-racial. The analysis employs an indicator for [economically disadvantaged students](#) and an indicator for [migrant students](#).

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<sup>1</sup> WISEdata is a dynamic data delivery system. Snapshots capture a static version of the data as it was delivered to Wisconsin DPI on a given date. The Assessment Snapshot taken near the end of the school year was for the purpose of supplying demographic characteristics to associate with student assessment results.





## ENGLISH LANGUAGE PROFICIENCY CLASSIFICATION

There are seven indicators for [English-language proficiency](#) (ELP) included in the analysis dataset. Students with ELP classifications of 1 through 5 are considered to be English-language learners in ascending levels of proficiency. Students with an ELP classification of 6 are those that were formerly classified as having limited English proficiency. Students with an ELP classification of 7 are those who have been proficient in English for two years or more. ELP classification is drawn from the WISEdata Assessment Snapshot.

## DISABILITY

The analysis includes five indicators for students with disabilities according to their primary disability code. There are separate indicators for emotional/behavioral disability (EBD), learning or intellectual disability (LD/ID), autism (A), and speech/language disability (SL). All other disability codes are grouped into a single indicator for other disabilities. Disability status is drawn from the WISEdata Assessment Snapshot.

## SCHOOL ENROLLMENT

Students that have full academic year (FAY) status at a single school are assigned to that school using the school enrollment data. For the purpose of Wisconsin accountability systems and therefore value-added modeling, FAY is defined as being enrolled from the beginning of the year through completion of required statewide testing. Some students have FAY status in a single district but not at a single school because of mobility within the district. These students are included in the district growth measures but not in the school growth measures.

## VOUCHER STUDENTS

Beginning with the 2016-2017 school year, the analysis set includes test scores for voucher students attending private schools. As of the 2015-16 year, private schools that enroll voucher students were included in the Wisconsin accountability system. The 2017-18 year is the second year in which two years of data are available to calculate a value-added score for such schools. All such schools receive a value-added score based on voucher students only.

In addition, these private schools with voucher students are given the option to receive a second report card in the Wisconsin accountability system (including a value-added score) which includes non-voucher students as well as voucher students. Such schools are denoted as "opt-in" schools, because they opted to receive the second non-compulsory score. Growth measures for "opt-in" schools that include non-voucher students are computed using a parallel analysis that





applies the parameters of the estimated value-added model to a data set that includes both voucher and non-voucher students.

Counts of non-voucher students are reduced by the requirement that, to be included in the growth analysis data set, students must have test score data from both 2017 and 2018. As indicated in Table 1 below, many non-voucher students did not have test scores from 2017.

Table 1. Number of Non-Voucher Students in Forward 2018 Data and Growth Analysis Data Set

		GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8
Math	Forward 2018 Data	253	261	226	214	251
ELA	Forward 2018 Data	253	261	225	214	251

## DESCRIPTIVE STATISTICS OF ANALYSIS SAMPLES

Tables 2 and 3 describe the sample used for the 2018 year. Note that the sample includes students from public schools and private schools participating in one of the Private School Choice programs in Wisconsin. The private school students include non-voucher students attending schools that opted in to receive a score for all their students.





Table 2. Math Sample

GRADE LEVEL	4	5	6	7	8
Number of Students	60251	60780	59303	58923	59177
Number of Public School Students	57582	58226	56773	56681	57084
Number of Voucher Students	2416	2293	2304	2028	1842
Number of Non-Voucher Private School Students	253	261	226	214	251
Total Number of Private School Students	2669	2554	2530	2242	2093
Number of Public Schools	1090	1038	683	641	647
Number of Private Schools	124	123	129	119	111
Number of Public School District Codes	425	425	424	425	424
Posttest Mean	578.327	600.363	613.661	624.856	646.227
Posttest Standard Deviation	52.1477	55.8222	56.8032	64.684	59.933
Math Pretest Mean	556.348	575.932	601.267	614.820	629.510
ELA Pretest Mean	560.175	586.532	604.442	616.047	628.660
Math Pretest Standard Deviation	47.873	54.113	50.1572	53.849	57.722
ELA Pretest Standard Deviation	46.547	51.955	50.546	49.207	58.311
Proportion in ESL Level 1	0.005	0.002	0.002	0.003	0.003
Proportion in ESL Level 2	0.017	0.006	0.004	0.009	0.008
Proportion in ESL Level 3	0.038	0.027	0.019	0.021	0.018
Proportion in ESL Level 4	0.024	0.032	0.024	0.010	0.011
Proportion in ESL Level 5	0.004	0.005	0.004	0.002	0.002
Proportion in ESL Level 6 (former English learners)	0.011	0.027	0.045	0.048	0.048
Proportion Female	0.492	0.488	0.488	0.488	0.485
Proportion Asian	0.039	0.038	0.039	0.038	0.038
Proportion African American	0.104	0.102	0.099	0.095	0.092
Proportion Hispanic	0.135	0.133	0.134	0.125	0.122
Proportion Native American	0.010	0.011	0.011	0.011	0.011
Proportion Native Hawaiian or Other Pacific Islander	0.001	0.001	0.001	0.001	0.001
Proportion Two or More Races	0.041	0.040	0.037	0.035	0.032
Proportion Special Education : Learning/Intellectual	0.036	0.042	0.046	0.047	0.050
Proportion Special Education: Emotional Behavioral	0.015	0.016	0.016	0.017	0.016
Proportion Special Education Autism	0.012	0.011	0.012	0.012	0.011
Proportion Special Education: Speech/Language	0.033	0.023	0.015	0.009	0.006
Proportion Special Education: Other	0.032	0.034	0.034	0.034	0.036
Proportion with Economic Disadvantage	0.458	0.444	0.438	0.410	0.397
Proportion Migrant	0.000	0.000	0.000	0.000	0.000



Table 3. English Language Arts (ELA) Sample

GRADE LEVEL	4	5	6	7	8
Number of Students	60252	60790	59318	58922	59191
Number of Public School Students	57580	58236	56790	56679	57097
Number of Voucher Students	2419	2293	2303	2029	1843
Number of Non-Voucher Private School Students	253	261	225	214	251
Total Number of Private School Students	2672	2554	2528	2243	2094
Number of Public Schools	1090	1038	683	641	647
Number of Private Schools	124	123	129	119	111
Number of Public School District Codes	425	425	424	425	424
Posttest Mean	582.163	601.965	610.864	629.158	632.825
Posttest Standard Deviation	51.218	47.767	49.592	55.674	59.036
Math Pretest Mean	556.333	575.9216	601.261	614.837	629.516
ELA Pretest Mean	560.170	586.527	604.437	616.049	628.652
Math Pretest Standard Deviation	47.890	54.123	50.168	53.839	57.705
ELA Pretest Standard Deviation	46.557	51.964	50.559	49.195	58.307
Proportion in ESL Level 1	0.005	0.002	0.001	0.003	0.003
Proportion in ESL Level 2	0.017	0.006	0.004	0.009	0.008
Proportion in ESL Level 3	0.039	0.027	0.020	0.021	0.018
Proportion in ESL Level 4	0.024	0.032	0.024	0.010	0.011
Proportion in ESL Level 5	0.004	0.005	0.004	0.002	0.002
Proportion in ESL Level 6 (former English learners)	0.011	0.027	0.046	0.048	0.048
Proportion Female	0.492	0.488	0.488	0.489	0.486
Proportion Asian	0.039	0.039	0.039	0.038	0.038
Proportion African American	0.104	0.101	0.099	0.094	0.092
Proportion Hispanic	0.134	0.133	0.134	0.125	0.122
Proportion Native American	0.010	0.011	0.011	0.011	0.011
Proportion Native Hawaiian or Other Pacific Islander	0.000	0.000	0.000	0.000	0.000
Proportion Two or More Races	0.041	0.039	0.037	0.035	0.032
Proportion Special Education : Learning/Intellectual	0.036	0.042	0.046	0.047	0.050
Proportion Special Education: Emotional Behavioral	0.015	0.015	0.016	0.017	0.016
Proportion Special Education Autism	0.012	0.011	0.012	0.012	0.011
Proportion Special Education: Speech/Language	0.033	0.023	0.015	0.009	0.006
Proportion Special Education: Other	0.032	0.034	0.034	0.034	0.036
Proportion with Economic Disadvantage	0.456	0.444	0.438	0.410	0.397
Proportion Migrant	0.000	0.000	0.000	0.000	0.000



## VALUE-ADDED MODEL

For the Wisconsin school-level model, 2017-18 value-added is measured in mathematics and English language arts (ELA) in grades four through eight for the Forward assessment. Schools are assigned single-year value-added measures that reflect student growth from Spring 2017 to Spring 2018. Once the schools get a growth value, these values are averaged to obtain the district's score, using the number of students attributed to each school as weights. The single-year value-added measures for 2017-18 are averaged with value-added measures in previous years to smooth year-to-year variance in value-added measures.

### THE MODEL, IN BRIEF

The value-added model is defined by four equations: a "best linear predictor" value-added model defined in terms of true student post and prior achievement (i.e., student achievement in the absence of test measurement error) and three measurement error models for observed post and prior achievement:

$$\text{Student achievement: } y_{1i} = \zeta + \lambda y_{0i} + \lambda^{alt} y_{0i}^{alt} + \beta' X_i + \alpha' S_i + e_i \quad (1)$$

$$\text{Posttest measurement error: } Y_{1i} = y_{1i} + v_{1i} \quad (2)$$

$$\text{Same-subject pretest measurement error: } Y_{0i} = y_{0i} + v_{0i} \quad (3)$$

$$\text{Other-subject pretest measurement error: } Y_{0i}^{alt} = y_{0i}^{alt} + v_{0i}^{alt} \quad (4)$$

where:

- the subscript  $i$  denotes each individual student;
- $y_{1i}$  is true post achievement;
- $y_{0i}$  and  $y_{0i}^{alt}$  are true prior achievement in the same subject and in the other subject (math in the ELA model, ELA in the math model), with slope parameters  $\lambda$  and  $\lambda^{alt}$ ;
- $X_i$  is a vector of characteristics of student  $i$ , with slope parameter vector  $\beta$ ;
- $S_i$  is a vector of indicators for school;
- $\alpha$  is a vector of school effects;
- $e_i$  is the error in predicting post achievement given the explanatory variables included in the model;
- $Y_{1i}$  is measured post achievement;
- $v_{1i}$  is measurement error in post achievement;
- $Y_{0i}$  and  $Y_{0i}^{alt}$  are measured prior achievement for the same subject and alternate subject, respectively; and
- $v_{0i}$  and  $v_{0i}^{alt}$  are measurement error in prior achievement for the same subject and alternate subject, respectively.





Substituting the measurement error equations (2), (3), and (4) into the student achievement equation (1) yields an equation defined in terms of measured student achievement:

$$\text{Measured achievement: } Y_{1i} = \zeta + \lambda Y_{0i} + \lambda^{alt} Y_{0i}^{alt} + \beta X_i + \alpha' S_i + \varepsilon_i \quad (5)$$

where the error term  $\varepsilon_i$  includes both the original error component and the measurement error components:

$$\text{Error in measured achievement: } \varepsilon_i = e_i + v_{1i} - \lambda v_{0i} - \lambda^{alt} v_{0i}^{alt} \quad (6)$$

Estimating the measured student achievement equation (5) without controlling for pretest measurement error yields biased estimates of all parameters, including the value-added effects. This bias stems from the fact that measurement error in prior achievement causes the error term (6), which includes the measurement error components  $v_{0i}$  and  $v_{0i}^{alt}$ , to be correlated with measured prior achievement. The desired parameters, as defined in equation (1), can be estimated consistently if external information is available on the variance of measurement error for prior achievement; approaches for consistent estimation in the presence of measurement error are described in detail in Fuller (1987). Information about the variance of test measurement error is obtained from the reliability estimates reported in the technical manual for the 2016-17 Forward exam assessment.

## VALUE-ADDED REGRESSION

As mentioned, the value-added model is estimated using a least-squares regression approach that corrects for measurement error in the pretest variables. It estimates the coefficients  $\lambda$ ,  $\beta$ , and  $\alpha$  by regressing posttest on same-subject pretest, other-subject pretest, other student-level variables, and a full set of school fixed effects. This regression is estimated using an approach that accounts for measurement error in the pretests  $Y_{0i}$  and  $Y_{0i}^{alt}$ . Recall from equation (6) above that the measurement error components of  $Y_{0i}$  and  $Y_{0i}^{alt}$ ,  $v_{0i}$  and  $v_{0i}^{alt}$ , are part of the error term  $\varepsilon_i$ . As a result, estimating the regression using ordinary least squares (without control for pretest measurement error) will lead to biased estimates. The regression approach employed accounts for measurement error by removing the variance in the pretests that is attributable to measurement error. To illustrate the measurement error corrected regression, re-cast the above value-added regression equation into vector form:





$$Y_t = Y_{t-1}\lambda + W\delta + \varepsilon$$

where  $Y_t$  is an  $N \times 1$  vector of post-test scores,  $Y_{t-1}$  is an  $N \times 2$  vector of same-subject and other-subject pre-test scores  $Y_{t-1}$  and  $Y_{t-1}^{alt}$ ,  $\lambda$  is a  $2 \times 1$  vector made up of  $\lambda$  and  $\lambda^{alt}$ ,  $W$  is an  $N \times K$  vector of the  $X$  demographic variables,  $\delta$  is a  $K \times 1$  vector of the  $\beta$  and  $\alpha^*$  coefficients, and  $\varepsilon$  is an  $N \times 1$  vector of error terms. The biased ordinary-least-squares estimates of the coefficients in  $\lambda$  and  $\delta$  are equal to:

$$\begin{bmatrix} \hat{\lambda}_{OLS} \\ \hat{\delta}_{OLS} \end{bmatrix} = \begin{bmatrix} Y'_{t-1}Y_{t-1} & Y'_{t-1}W \\ W'Y_{t-1} & W'W \end{bmatrix}^{-1} \begin{bmatrix} Y'_{t-1}Y_t \\ W'Y_t \end{bmatrix}$$

The measurement-error-corrected estimates of the coefficients in  $\lambda$  and  $\delta$  are equal to:

$$\begin{bmatrix} \hat{\lambda}_{EIV} \\ \hat{\delta}_{EIV} \end{bmatrix} = \begin{bmatrix} Y'_{t-1}Y_{t-1} - \sum_i^N V_{it-1} & Y'_{t-1}W \\ W'Y_{t-1} & W'W \end{bmatrix}^{-1} \begin{bmatrix} Y'_{t-1}Y_t \\ W'Y_t \end{bmatrix}$$

where  $V_{it-1}$  is a  $2 \times 2$  variance-covariance matrix of the errors of measurement of  $Y_{it-1}$  and  $Y_{it-1}^{alt}$  for student  $i$ . This model is described in section 2.2 of Fuller (1987).

To minimize the influence of test scores at the extreme of the distribution on the estimates of the coefficients on the pretests  $\lambda$  and  $\lambda^{alt}$ , we estimated the value-added model in two steps in models of student growth in mathematics. This method was found to be useful for the mathematics model because in some grades the percent of students receiving the lowest observable scale score (LOSS) in mathematics is somewhat higher than in previous years (see the Table 4). In step one, model parameters are estimated using all students other than those at the LOSS on the mathematics pretest. In step two, the estimated parameters for the two pretest variables (prior math and ELA) are treated as known and the model is re-estimated using all students. This approach yields estimates of model parameters and value-added estimates that are comparable to those obtained in previous years.<sup>2</sup>

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<sup>2</sup> Since the data sets used in the estimation are very large, the pretest coefficients from step one are estimated with extremely high precision. Thus, estimates of standard errors for all parameters are obtained from step two, using the measurement error correction method described above.





## THE VARIABLES IN THE MODEL

In addition to posttest and pretest scores, the student-level variables included in the model (the *X* variables in equation 1) are gender, race/ethnicity, ELP category, economic disadvantage, disability status, and migrancy. No higher order terms or interactions of terms are used in the model. Refer to Section “1.1 Analysis Data Set – Student-level variables” on the categories that make up each student-level variable described here.

Table 4. Percentage of Students at Test Floor (Lowest Observable Scale Score, LOSS) for Pre- and Posttests

	GRADE	TEST SUBJECT	PERCENT AT POSTTEST FLOOR	PERCENT AT MATH PRETEST FLOOR	PERCENT AT ELA PRETEST FLOOR
Included in Growth Analysis Data Set	4	ELA	0.0%	0.7%	0.0%
		Mathematics	2.0%	0.7%	0.0%
	5	ELA	0.0%	2.9%	0.0%
		Mathematics	4.3%	2.9%	0.0%
	6	ELA	0.0%	1.7%	0.0%
		Mathematics	2.8%	1.7%	0.0%
	7	ELA	0.0%	1.9%	0.0%
		Mathematics	5.3%	1.9%	0.0%
	8	ELA	0.0%	2.8%	0.0%
		Mathematics	4.1%	2.8%	0.0%

## AGGREGATION TO MULTIPLE-GRADE VALUE-ADDED

The value-added regression to obtain unshrunk school value-added is performed separately for each grade and subject combination. For schools that have results for more than one grade level, these estimates are averaged across grades, using the number of students attributed to the school as weights, to produce unshrunk multiple-grade value-added estimates. Before aggregation, value-added measures are normalized by subject and grade so they are on a similar scale (i.e. with a mean of 0 and a true standard deviation of 1). This normalization is done by dividing the measures by an estimate of the standard deviation of within-grade value-added.

## SHRINKAGE OF VALUE-ADDED

At all levels, the unshrunk value-added estimates are shrunk using an Empirical Bayes multivariate shrinkage technique described in Longford (1999). This procedure is employed to bring value-added estimates based on smaller sample sizes closer to the state average, so that schools with fewer students are not overrepresented among the highest- and lowest-value-





added cases simply due to randomness. It is also employed to reduce year-by-year variation in value-added scores within schools.

To use this multivariate shrinkage approach, we first estimate single-year value-added for the 2016-17 school year using the same approach that was used to estimate single-year value-added for the 2017-18 school year. Let  $\hat{\alpha}_{kt}$  be the estimated value-added for school  $k$  in year  $t$ . We can group the value-added estimates for a given school  $k$  into a  $T \times 1$  column vector  $\hat{\alpha}_k$ , where  $T$  is the number of years in which value-added is measured for school  $k$ . (In this particular application,  $T$  will usually be 2, although it will equal 1 in schools in which value-added is measured in 2017-18 but not 2016-17 or vice versa.) Also let  $\alpha_{kt}$  be the true value-added (which is unmeasured, and equal to what estimated value-added would be in the absence of sampling error) for school  $k$  in year  $t$ , which can be grouped by school into a  $T \times 1$  column vector  $\alpha_k$ . Let the variance of  $\alpha_k$  be the  $T \times T$  matrix  $Var[\alpha_k] = \Omega$ , which reflects the within-year variance and across-year covariance of true value-added across schools. Also let the variance of  $\hat{\alpha}_k$  conditional on  $\alpha_k$  be the  $T \times T$  matrix  $Var[\hat{\alpha}_k|\alpha_k] = \Sigma_{kk}$ , which reflects the within-year variance and across-year covariance of sampling error in  $\hat{\alpha}_k$ . We produce shrunk estimates of value-added using the following equation:

$$\alpha_k^* = \Omega[\Omega + \Sigma_{kk}]^{-1}\hat{\alpha}_k$$

where  $\alpha_k^*$  is a  $T \times 1$  column vector of shrunk value-added measures for school  $k$  over the  $T$  years in which value-added is measured for school  $k$ . The expected mean squared error of the shrunk value-added estimates  $\alpha_k^*$  is equal to:

$$EMSE_k = \Omega - \Omega[\Omega + \Sigma_{kk}]^{-1}\Omega$$

In practice, we use estimates of  $\Omega$  and  $\Sigma_{kk}$  to estimate  $\alpha_k^*$  and its expected mean squared error. The estimate of the matrix  $\Sigma_{kk}$  is the estimated variance-covariance matrix of the value-added estimates in  $\hat{\alpha}_k$ . Let  $\hat{\sigma}_{t\tau kk}$  be the entry of this matrix in the row corresponding to  $\hat{\alpha}_{kt}$  and the column corresponding to  $\hat{\alpha}_{k\tau}$ . The diagonal entries of this matrix are the squares of the estimated standard errors of the value-added estimates in  $\hat{\alpha}_k$ .

The diagonal entries of  $\Omega$ , which are equal to the variance of  $\alpha_{kt}$  across schools in a given year  $t$  and which we denote  $\omega_{tt}$ , are estimated by computing the variance across schools  $k$  within year  $t$  of the unshrunk value-added estimates  $\hat{\alpha}_{kt}$ , then subtracting from that the average across schools  $k$  within year  $t$  of  $\hat{\sigma}_{ttkk}$ , the estimated squared standard error of  $\hat{\alpha}_{kt}$ . This estimates the variance of the true school value-added for each year  $t$ , excluding variance due to randomness in the value-added estimates. The square root of this variance measure is also used for normalizing value-added measures by grade before aggregation to multiple-grade measures. The off-diagonal entries of  $\Omega$ , which we denote  $\omega_{t\tau}$  and are equal to the covariance of  $\alpha_{kt}$  and  $\alpha_{k\tau}$  across schools between years  $t$  and  $\tau$ , is estimated by computing the covariance of the unshrunk value-added estimates  $\hat{\alpha}_{kt}$  and  $\hat{\alpha}_{k\tau}$ , and then subtracting from that the average error covariance estimate  $\hat{\sigma}_{t\tau kk}$ .





## STUDENT GROUP VALUE-ADDED

Value-added is also measured by student groups defined by certain demographic characteristics. Specifically, we calculated differential value-added effects for the seven race/ethnicity groups, for students with disabilities, for economically disadvantaged students, and for English-language learners.

To produce the group results by school, we produce unshrunk value-added effects for both 2016-17 and 2017-18 for each subgroup for each school. These are produced by computing the sum of the school effects and the residual,  $\alpha'S_i + \varepsilon_i$ , for each student, and then computing the average of this variable by year, school, and subgroup. These measures are then shrunk using a bivariate shrinkage approach that takes into account correlations in school- and subgroup-level value-added across subgroups and across years.

## FINAL STAGE FOR ESTIMATION OF SCHOOL AND DISTRICT VALUE-ADDED RESULTS

In order to enhance the accuracy and precision of the value-added estimates, final estimates of school value-added effects are obtained by combining estimates for 2016, 2017, and 2018, the years in which results using the Wisconsin Forward Exam are available. This is approached by computing an average of the single-year value-added measures estimated for 2017-18 and the reported value-added measures for 2016-17, using the number of students associated with each school in 2017-18 and 2016-17 as weights. The reported value-added measures for 2016-17 are themselves weighted averages of single-year value-added measures for 2015-16 and 2016-17, rescaled to have a variance similar to that of a single-year value-added measure, with the number of students associated with the school in each year as a weight, and with the 2016-17 single-year measures weighted double. This produces value-added measures that implicitly weight growth in 2017-18, 2016-17, and 2015-16 by 1/2, 1/3, and 1/6 in a way that maintains consistency with value-added measures reported in the past. The averaged value-added measure includes the reported 2016-17 value-added measure only if there are at least twenty (in the case of subgroup measures, ten) students associated with that value-added measure in 2016-17.

Final estimates of district value-added effects are obtained by averaging the shrunk combined value-added estimates (as described above) for all of the schools in each district, with weights determined by the number of students in each school in 2018. As mentioned earlier, the district results include students if they were FAY at the district even if they were not FAY at any of the district's schools. Thus, students who moved from one school in a district to another school in the district are included. These students are incorporated into the estimation of the model using





a fixed effect estimate for a placeholder school for each district for students who were FAY in the district but not FAY in any school in the district.

## PROPERTIES OF THE VALUE-ADDED RESULTS

### COEFFICIENTS ON STUDENT-LEVEL VARIABLES IN THE MODEL

The coefficients estimated in the value-added model that includes non-voucher students are presented in Tables 7 and 8. To interpret these coefficients, note that both pretest and posttest are measured using standardized scores; therefore, all coefficients are measured in the posttest standard deviation scale. For example, note that the coefficient on female gender is  $-0.055$  in grade 4 Math. The posttest standard deviation for grade 4 Math is  $54.059$ . This implies that male students improved  $0.083$  standard deviations or about  $2.973$  scale score points more on the grade 4 Math test from spring to spring than otherwise similar female students.

It is important to keep in mind the standard errors of the coefficients when interpreting them. A span of  $1.96$  standard errors in both the positive and negative directions provides a 95 percent confidence range for a coefficient. Continuing with the example of the coefficient on female gender in grade 4 Math, note that the standard error of this coefficient estimate is  $0.005$  in posttest SD units or  $0.270$  in scale score points. This means that, while our best estimate of the difference in growth between female and male students is  $-2.973$  scale score points, a 95 percent confidence interval for the difference ranges from  $-3.502$  to  $-2.444$  scale score points.





Table 7. Coefficients on Student-Level Variables, 2017-18 Forward Math, Including Non-Voucher Students at Private Schools

Variable	GRADE 4		GRADE 5		GRADE 6		GRADE 7		GRADE 8	
	Coeff.	SE								
Math Pretest	0.737	0.007	0.755	0.007	0.707	0.006	0.831	0.009	0.756	0.007
ELA Pretest	0.124	0.007	0.158	0.006	0.192	0.006	0.093	0.008	0.178	0.007
ESL Level 1	-0.245	0.034	-0.228	0.059	-0.160	0.060	0.011	0.053	-0.129	0.053
ESL Level 2	-0.044	0.020	-0.202	0.035	-0.123	0.037	-0.065	0.029	-0.024	0.030
ESL Level 3	-0.005	0.014	0.014	0.017	-0.103	0.018	-0.135	0.020	0.019	0.020
ESL Level 4	0.084	0.016	0.082	0.015	0.004	0.016	-0.054	0.026	0.090	0.024
ESL Level 5	0.120	0.038	0.107	0.034	0.022	0.034	0.013	0.055	0.044	0.062
ESL Level 6	0.061	0.023	0.058	0.016	0.065	0.013	0.023	0.014	0.050	0.013
Female	-0.049	0.005	0.022	0.005	0.011	0.005	-0.042	0.006	0.027	0.005
Asian	0.023	0.014	0.067	0.015	0.076	0.014	0.005	0.015	0.075	0.015
African-American	-0.102	0.012	-0.058	0.012	-0.086	0.011	-0.100	0.013	-0.047	0.013
Hispanic	-0.050	0.009	-0.007	0.010	-0.032	0.009	-0.035	0.010	-0.019	0.010
Indian	-0.010	0.026	-0.018	0.026	-0.089	0.024	-0.035	0.027	-0.090	0.026
Native Hawaiian or Other Pacific Islander	-0.090	0.082	0.093	0.098	-0.083	0.082	-0.097	0.083	-0.056	0.088
Two or More Races	-0.036	0.012	-0.003	0.012	-0.016	0.012	-0.006	0.014	0.012	0.014
Special Education LD/ID	-0.098	0.019	-0.184	0.020	-0.285	0.018	-0.061	0.020	-0.149	0.020
Special Education EBD	-0.108	0.013	-0.169	0.013	-0.202	0.012	-0.069	0.013	-0.090	0.012
Special Education A	-0.123	0.021	-0.134	0.023	-0.150	0.021	0.076	0.024	-0.035	0.023
Special Education SL	0.001	0.013	-0.035	0.016	-0.016	0.018	-0.002	0.027	0.010	0.032
Special Education Other	-0.130	0.014	-0.213	0.014	-0.240	0.013	-0.026	0.015	-0.125	0.014
Economic Disadvantage	-0.044	0.006	-0.046	0.006	-0.049	0.005	-0.022	0.006	-0.042	0.006
Migrancy Status	-0.074	0.160	0.044	0.157	-0.001	0.178	-0.147	0.138	-0.174	0.173





Table 8. Coefficients on Student-Level Variables, 2017-18 Forward ELA, Including Non-Voucher Students at Private Schools

Variable	GRADE 4		GRADE 5		GRADE 6		GRADE 7		GRADE 8	
	Coeff.	SE								
Math Pretest	0.107	0.007	0.078	0.006	0.080	0.006	0.093	0.008	0.129	0.006
ELA Pretest	0.783	0.007	0.822	0.006	0.824	0.007	0.852	0.008	0.805	0.006
ESL Level 1	-0.090	0.034	-0.006	0.050	0.222	0.058	0.262	0.046	-0.021	0.044
ESL Level 2	-0.085	0.020	-0.011	0.030	0.060	0.037	-0.012	0.026	0.016	0.026
ESL Level 3	0.012	0.015	-0.100	0.016	-0.033	0.019	0.000	0.018	-0.022	0.018
ESL Level 4	0.059	0.017	-0.050	0.015	-0.017	0.017	0.097	0.024	0.025	0.022
ESL Level 5	0.059	0.040	0.017	0.033	-0.015	0.036	0.072	0.052	-0.062	0.059
ESL Level 6	0.069	0.024	0.041	0.016	0.060	0.013	0.061	0.013	0.012	0.013
Female	0.052	0.005	0.045	0.005	0.049	0.005	0.040	0.005	0.063	0.005
Asian	0.001	0.015	0.016	0.014	0.057	0.015	0.076	0.015	0.013	0.014
African-American	-0.079	0.012	-0.030	0.011	-0.065	0.012	-0.042	0.012	-0.075	0.011
Hispanic	-0.019	0.010	0.020	0.009	-0.042	0.010	0.008	0.010	-0.034	0.009
Indian	0.002	0.027	-0.027	0.025	-0.050	0.025	-0.011	0.025	-0.032	0.024
Native Hawaiian or Other Pacific Islander	-0.048	0.085	0.086	0.097	0.025	0.086	-0.050	0.079	-0.081	0.082
Two or More Races	-0.013	0.012	-0.007	0.012	-0.010	0.013	0.002	0.013	-0.021	0.013
Special Education LD/ID	-0.081	0.020	-0.116	0.019	-0.127	0.019	-0.012	0.019	-0.009	0.018
Special Education EBD	-0.089	0.013	-0.189	0.012	-0.086	0.012	0.003	0.012	-0.024	0.011
Special Education A	-0.093	0.022	-0.118	0.021	-0.042	0.022	0.124	0.022	0.112	0.021
Special Education SL	-0.009	0.013	-0.084	0.015	-0.016	0.019	0.025	0.025	0.017	0.029
Special Education Other	-0.120	0.014	-0.160	0.013	-0.127	0.014	-0.018	0.014	-0.027	0.013
Economic Disadvantage	-0.056	0.006	-0.054	0.006	-0.054	0.006	-0.033	0.006	-0.032	0.006
Migrancy Status	-0.014	0.160	0.246	0.155	-0.040	0.191	-0.243	0.129	0.138	0.156





## TEST OF MODEL NEUTRALITY: CORRELATION WITH AVERAGE PRIOR PROFICIENCY

In this test, we calculate correlations between growth estimates and school-level pretest/demographic covariates. This is a method for validating whether the variables included on the right-hand side of our regression adequately control for school-level factors influencing growth estimates. The higher the correlation magnitude, the higher the level of “non-neutrality”.

Our results show a very low correlation between average prior proficiency--a measure of average performance in the previous year--and value-added. In general, schools were not more or less likely to have a low value-added score than a high score if their students began the year with low pretest scores rather than high scores.

Table 9. Correlations between Prior Attainment and Value-Added

SUBJECT	GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8	SCHOOL
ELA	0.13	0.03	-0.02	-0.09	-0.03	0.21
Math	0.32	-.005	0.08	-0.16	0.14	0.31

## CORRELATION BETWEEN MATH AND ELA VALUE-ADDED

There were also substantive positive correlations between math and ELA value-added within each school. Schools that were high value-added in math were also more often than not high value-added in ELA. This implies that schools with a higher-than-average impact in mathematics also had a higher-than-average impact in English language arts.

Table 10. Correlations between Subjects

	GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8	SCHOOL
2018 Math and ELA	0.56	0.49	0.64	0.45	0.39	0.54

## CONTACT

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