

An Introduction

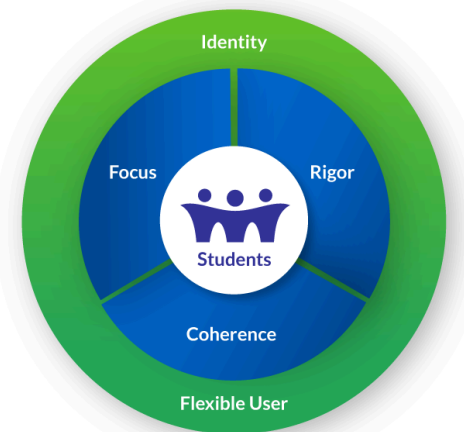
To experience agency students must feel that their identity is valued, have the ability to build knowledge and demonstrate their understanding as a learner, feel that they belong, and have a sense of efficacy - set an intention and produce a desired result (Safir and Dugan 2021, 100-104).

Centering students in educational systems requires attending to each of Wisconsin's [five shifts](#) both individually and collectively. When the shifts impact learning and teaching, students gain a deep understanding of mathematics with which to see, interact, and appreciate the world around them. Students also develop a deep understanding of the content by experiencing a balanced and connected approach among conceptual understanding, procedural flexibility and efficiency, and application to authentic contexts. These five shifts serve to move the conversation from “teaching mathematics to students” to “teaching students mathematics” at state, regional, district, school, and classroom levels.

The five shifts graphic at the right begins with students at the very center. The three national shifts of focus, coherence, and rigor are represented by the attached blue wedges since these shifts are interconnected and rely on one another to support connected and balanced mathematics based on sense-making. Finally, the underlying circle includes the two new Wisconsin shifts of identity and flexible users in gradient green, showing the interdependence of these shifts on each other and their impact on student learning in mathematics.

This Wisconsin *Leveraging Focus* document is designed to support thinking about the role focus plays within educational systems. While all five shifts are important and none of these function on their own, the shifts must be considered as we think about how to create opportunities for our students to be engaged as confident and legitimate thinkers and doers.

This document aims to support educators in taking the shift of focus and determining how one can apply this shift within various educational systems in support of instructional coherence. Cultivating student agency is the goal of successful mathematics learning and therefore must be at the center of all systems.



It is the intent that this Wisconsin *Leveraging Focus* document will support educators while collaborating and making planning decisions within a variety of systems with students at the center of the work. Several components of the national focus documents have been maintained in *Leveraging Focus*, with more attention given to the development of mathematical agency, why a focus as described at each level is important, and sample systems descriptions with connections to focus.

While *Leveraging Focus* does more specifically identify the work in each grade level for Wisconsin educators due to the content standard revisions found in [Wisconsin Standards for Mathematics, 2021](#), the Student Achievement Partners (SAP) focus documents are still predominantly applicable and surely valued.

The shift of focus as a design feature of mathematics standards can be a powerful lever within educational systems as it supports students in meeting the full intent of grade-level standards. Students develop a strong foundational knowledge and deep conceptual understanding and are able to transfer mathematical skills and understanding across concepts and grades (National Mathematics Advisory Panel 2008, 15-20). Until now, Wisconsin educators have relied on the [focus documents at the national level from SAP](#) for grades K-8. These critical documents show where students and teachers should spend the large majority of their time in order to meet the expectations of the Common Core State Standards for Mathematics (CCSSM) on their mathematical journey to be college and career ready. In high school grades, the [Widely Applicable Prerequisite](#) document shows the content that is a prerequisite and applicable to a range of post-secondary work.

These national documents have been instrumental in setting benchmarks for what it looks like when instructional materials attend to the shift of focus as one criteria for being considered high-quality, standards-aligned materials. Now, almost a decade after the publishing of the national focus documents, educators can rely on their high-quality instructional materials, selected and adopted through a thoughtful process such as the [Instructional Materials and Professional Learning \(IMPL\) project](#), to attend to the shift of focus. Ongoing professional learning, connected to the instructional materials, supports implementation with integrity, and therefore attention to the shift of focus. With high-quality materials now doing the heavy lifting related to where the large majority of time should be spent, educators can go beyond and consider how focus can be used as a lever within a variety of educational systems.

The following pages contain a table of examples of educational systems that educators can use to examine how focus can be leveraged within their own context. This table is followed by grade band tables that describe the content that provide a strong mathematics foundation.

Leveraging Focus Within Educational Systems

<p>Instructional Materials High-quality instructional materials will honor the shift of focus and therefore will support educators in spending time and energy on all standards in <i>WI Standards for Mathematics</i> (2021), with careful attention to the major work of the grade.</p>	<p>Some clusters require greater emphasis than others based on the depth of the ideas, the time they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade (DPI 2021, 29).</p>
<p>Professional Learning Professional learning is opportunities designed to improve teacher classroom practice and effectiveness with students (Research Partnership for Professional Learning, 2).</p> <p>Curriculum-Based Professional Learning Curriculum-based professional learning supports teachers in building a deeper understanding of the student academic standards based upon the standards-aligned instructional materials with which they are teaching.</p>	<p>Schools can choose high-quality professional learning to aid in adoption, implementation, and ongoing support with high-quality instructional materials.</p> <p>High-quality professional learning is:</p> <ul style="list-style-type: none"> ● specific as it deepens educators’ understanding of what to teach and how to teach it within the context of the instructional materials teachers are expected to use ● relevant as it is customized to teachers’ subject area, grade level, and cultural experiences ● equitable as it equips teachers to ensure that all of their students have the opportunity to thrive academically and personally ● ongoing as it provides teachers and school leaders with regular support on their curriculum ● engaging as it creates a space in which educators can develop and discuss ideas, practice new skills, receive feedback and reflect on their learning
<p>Intervention/Co-Requisite Model As part of a continuum of supports within an equitable multi-level system of support (EMLSS), schools provide varying types of supports at differing levels of intensity to proactively and responsively adjust to the needs of the whole child.</p>	<p>The clear goal of interventions is to prepare students to successfully participate in grade level instruction. No matter the intensity, students receive supplemental, “just-in-time” instruction that supports current or upcoming core instruction. This connection between intervention and core instruction is crucial in order to create an environment in which students can make sense of their educational experiences throughout the day, the week, and the year. In high school, systems should make use of a co-requisite model for students to receive “just-in-time” support while having access to high school mathematics.</p>
<p>Student Assessment The formative assessment process plays a critical role in strategic assessment systems. This process is designed to quickly inform instruction by providing specific and immediate feedback through daily, ongoing instructional strategies that are student- and classroom-centered, and that answer "what comes next for student learning?"</p>	<p>The shift of focus calls for assessment practices that support all students in developing deep understanding of and connections among mathematical concepts. The formative assessment process provides the information educators need to know about students and their strengths in order to cultivate student agency, supporting a strong mathematical foundation. When success is defined as the cultivation of student agency, realignment of our measures of success to this goal is necessary (Safir and Dugan 2021, 100). What would it look like to measure student agency in terms of identity, belonging, mastery, and efficacy (Safir and Dugan 2021, 101) ?</p>

System Assessment

Schools use a variety of assessments – formative, interim, and summative – in order to drive the teaching and learning process and gauge program effectiveness.

Healthy educational programming depends on gathering information that can determine how systems are working. Engaging in deep listening supports the principle of seeking root causes over quick fixes. By listening to the voices, narratives, and perspectives at the margins, we begin to humanize the process of data-gathering (Safir and Dugan 2021, 71). “What if we invested in the observation and analysis skills of trained educators rather than depending on test developers and policymakers to tell us who’s successful? What if reflective review teams comprised of educators, parents, and students developed lines of inquiry around a school-based asset or opportunity versus a perceived ‘gap’ (Safir and Dugan 2021, 53)?”

Gathering data around the major work of the grade leverages focus in support of strong learning and teaching in mathematics. In an assessment system, the types of assessments and their purposes must have balance. These balanced components must assess everything that is valued, not just those things that are easy to test, quantify, and grade. For example, the Standards for Mathematical Practice describe ways in which developing students of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise through their K-12 years. Assessments should attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

Questions for reflection:

What educational systems in your district/school/classroom would benefit from an examination with a lens of focus?

What other systems, that are not listed, might benefit from an examination with a lens of focus?

LEVERAGING FOCUS IN
KINDERGARTEN - GRADE 2
MATHEMATICS



The Counting and Cardinality domain lays the foundation from which all four operations grow. The whole number work of the Operations and Algebraic Thinking domain includes concepts, properties, and representations that eventually extend to other number systems and to algebra. Deep understanding of representing and comparing whole numbers as well as describing shapes and space is foundational for future success (CCSS Initiative 2022).

Children deserve mathematically powerful learning spaces that emphasize reasoning and sense making on a daily basis, including more opportunities to (NCTM 2020):

- Develop flexibility in reasoning with number and operation relationships
- Use subitizing activities to develop quantitative relationships
- Learn basic number combinations through sense making, not memorization

“Developing deep understanding of mathematics is a major goal of equity-based mathematics teaching practices” (Aguirre, Martin, and Mayfield-Ingram 2013, 43). Educators provide opportunities for deep learning to foster the development of a classroom’s collective mathematical agency as students understand mathematics and expect it to make sense (DPI 2020, 20).

Shift 3 from *WI Standards for Mathematics* (2021) continues to call for a focus in mathematics content that significantly narrows and deepens the way time and energy are spent. All successful systems are built on students meeting the full intent of grade-level standards.

“The collaborative focus on mathematics learning among the adults in the educational setting can then serve as a catalyst for creating a school culture of deeper mathematics learning and mathematically powerful learning spaces for each and every child” (NCTM 2020, 121).

HIGHLIGHTS OF MAJOR WORK
IN GRADES K-8

- K-2 Addition and subtraction - concepts, skills, and problem solving; place value
- 3-5 Multiplication and division of whole numbers and fractions-concepts, skills, & problem solving
- 6 Ratios and proportional relationships; early expressions and equations
- 7 Ratios and proportional relationships; arithmetic of rational numbers
- 8 Linear equations and linear functions

CLUSTER STATEMENTS

Refer to the [Student Achievement Partners Focus by Grade Level documents](#) for details about the specific cluster statements that support focus in K-2 mathematics

FLUENCIES FOR KINDERGARTEN

FLUENCIES FOR GRADE 1

FLUENCIES FOR GRADE 2

M.K.OA.A.5
Flexibly and efficiently add and subtract within 5

M.1.OA.C.6a
Flexibly and efficiently add and subtract within 10

M.2.OA.B.2
Flexibly and efficiently add and subtract within 20

M.2.NBT.B.5
Flexibly and efficiently add and subtract within 100

LEVERAGING FOCUS IN
GRADE 3 - GRADE 5
MATHEMATICS

3-5

The whole number work of the Operations and Algebraic Thinking domain and the fraction work of the Number and Operations - Fractions domain include concepts, properties, and representations that extend to other number systems and to algebra. Deep understanding of representing whole numbers and fractions as well as describing shapes and space is foundational for future success.

Children deserve mathematically powerful learning spaces that emphasize reasoning and sense making on a daily basis, including more opportunities to (NCTM 2020):

- Transition successfully and intentionally from additive to multiplicative thinking
- Use unit fractions as the building blocks for developing fraction knowledge
- See fractions as numbers whose magnitude can be represented on a number line and used along with real-world contexts for understanding fraction operations conceptually

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Shift 3 from *WI Standards for Mathematics (2021)* continues to call for a focus in mathematics content that significantly narrows and deepens the way time and energy are spent. This means ensuring that all students learn mathematics focused deeply on the major work of the grade as described on the right.

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HIGHLIGHTS OF MAJOR WORK
IN GRADES K-8

- K-2 Addition and subtraction - concepts, skills, and problem solving; place value
- 3-5 Multiplication and division of whole numbers and fractions-concepts, skills, & problem solving
- 6 Ratios and proportional relationships; early expressions and equations
- 7 Ratios and proportional relationships; arithmetic of rational numbers
- 8 Linear equations and linear functions

CLUSTER STATEMENTS

Refer to the [Student Achievement Partners Focus by Grade Level documents](#) for details about the specific cluster statements that support focus in 3-5 mathematics.

FLUENCIES FOR GRADE 3

FLUENCIES FOR GRADE 4

FLUENCIES FOR GRADE 5

M.3.OA.C.6
Flexibly and efficiently find products and quotients with multiples of 0, 1, 2, 5, & 10 within 100

M.3.NBT.A.2 Flexibly and efficiently add and subtract within 1,000

M.4.OA.D.6
Flexibly and efficiently multiply and divide within 100

M.4.NBT.B.4
Flexibly and efficiently add and subtract multi-digit whole numbers

M.5.NBT.B.5
Flexibly and efficiently multiply multi-digit whole numbers

LEVERAGING FOCUS IN
GRADE 6 - GRADE 8
MATHEMATICS

6-8

The middle school work of the Number System and the Expressions and Equations domains are important in preparing students for Algebra in high school and are foundational for future success. Ratios and proportional relationships are foundational for further study in mathematics and science and useful in everyday life. In addition, middle school mathematics widens the types of numbers that students can conceptualize on the number line, as students begin to include irrational numbers. All of high school mathematics depends on the fundamental assumption that properties of rational numbers extend to irrational numbers.

Students deserve mathematically powerful learning spaces that prepare them for college and career and also develop a deep mathematical understanding while cultivating a positive mathematical identity. Students (NCTM 2020):

- Reason mathematically, including conjecturing, generalizing, and justifying
- Use technologies to investigate mathematics and push forward thinking across the content domains
- Use mathematical and statistical modeling as a link between making informed decisions in everyday life and developing a deeper and more coherent understanding of mathematics and statistics

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HIGHLIGHTS OF MAJOR WORK
IN GRADES K-8

- K-2 Addition and subtraction - concepts, skills, and problem solving; place value
- 3-5 Multiplication and division of whole numbers and fractions-concepts, skills, & problem solving
- 6 Ratios and proportional relationships; early expressions and equations
- 7 Ratios and proportional relationships; arithmetic of rational numbers
- 8 Linear equations and linear functions

CLUSTER STATEMENTS

Refer to the [Student Achievement Partners Focus by Grade Level documents](#) for details about the specific cluster statements that support focus in 6-8 mathematics.

FLUENCIES FOR GRADE 6

FLUENCIES FOR GRADE 7

FLUENCIES FOR GRADE 8

M.6.NS.B.2
Flexibly and efficiently divide multi-digit whole numbers

M.6.NS.B.3
Flexibly and efficiently add, subtract, multiply, and divide multi-digit decimals

M.7.EE.B.4
Flexibly and efficiently apply the properties of operations and equality to solve equations of the following forms: $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers

While no specific eighth grade content standards use the phrase “flexibly and efficiently”, there are many opportunities within the work of the eighth grade standards for students to select, understand, and evaluate strategies while developing flexibility and reasonable solutions when problem solving.

LEVERAGING FOCUS IN
HIGH SCHOOL
MATHEMATICS



FIRST TWO YEARS (F2Y)

The first two years of high school mathematics prepare students for a number of options for a third and fourth year course that will support a transition to post-secondary options. There are many mathematical big ideas that are key for students to develop skills and confidence in to be successful in college, career, and citizenry. These big ideas can be summarized into four categories. Students should have access to numerical, algebraic, geometric, and statistical ideas along with understanding through instructional materials, instructional practices and assessments.

All successful systems in high school mathematics education rely on students meeting the full intent of grade-level standards in K-8th grade. It is strongly recommended that each and every student have access and opportunity to engage and be successful learning the complete set of K-8 standards through materials and instruction that attends to the shifts of the standards.

It is important for all students to have immediate access to high school mathematics starting in grade 9 when high school can leverage more complex problem solving while still attending to using important middle school mathematics. "...some of the highest priority content for college and career readiness comes from Grades 6-8...Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8" (CCSSM 2010, 84). Students should have opportunities to re-engage in key takeaways (listed in table at the right) as they move through high school standards.

In all courses in high school mathematics, "students should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder and beauty of mathematics" (NCTM 2018, 9). Courses should focus on: key content progressions from K-8 to grades 11-12, connections among mathematical topics, attention to context by incorporating mathematical modeling throughout, and incorporating technology (Leinwand and Milou 2021).

High School math standards were created for districts to have flexibility in creating opportunities for students. This document provides some ideas for thinking about the standards in terms of broad goals or outcomes at the end of a two year sequence. Conversations "about access and equity, a reasonable and teachable scope (focusing on key content progressions from K-8 to grades 11 and 12)", can inform choices of instructional materials, instructional practices, and assessment.(Leinwand and Milou 2021, 60). For more information see Chapter 4: Grades 9-10: A Common Curriculum for All in Leinwand and Milou's *Invigorating High School Math*.

KEY TAKEAWAYS FROM GRADES 6-8

- Applying ratios and proportional relationships
- Applying percentages and unit conversion
- Applying basic function concepts
- Applying concepts and skills of geometric measurement
- Applying concepts and skills of basic statistics and probability
- Performing rational number arithmetic flexibly and efficiently

Refer to the [Widely Applicable Prerequisites for High School](#) for details about how these takeaways progress to high school standards.

Big Ideas moving from Grade 8 to 9&10

Essential Concepts should support students in knowing how to (NCTM 2018):

- Solve routine problems involving algorithms and procedures
- Solve non-routine problems using problem solving strategies
- Knowing how procedures work, why they work, and their limitations
- Knowing when and how to apply mathematics in real-world problems

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