Responsive Teaching: Schools and classrooms are responsive to and value the students they serve, conveying the message that all students are capable doers of mathematics.

NCTM's Principles to Actions: Ensuring Mathematical Success (2014) states that effective mathematics instruction must be "responsive to students' backgrounds, experiences, and knowledge" (p. 60). A major part of the daily work that takes place in mathematics classrooms cannot be singled out as one or more isolated strategies. Much of this work is directly related to educators' understanding of their students and responsiveness while teaching. Responsive teachers of mathematics consider and respect the experiences and knowledge that their students bring to the classroom. They work to validate students for whom they are and engage them in the study of challenging and relevant mathematics.

This first vision statement represents some of the work that can be done to place an explicit focus on equitable access to engaging, relevant, and challenging mathematics content for all learners. The learning based on this content will build on the strengths students bring to the classroom and expand students' mathematical identities. Learners will see themselves and each other as capable and confident doers of mathematics.

K-2	3-5	6-8	9-12
1 (K-2) Math proficiency is defined for everyone involved in a mathematical learning community. Students and teachers as well as family and community members must understand that mathematical proficiency is more than developing computation skills and memorizing algorithms. Mathematical proficiency is a balance of developing	**1 (3-5) Math proficiency is defined for everyone involved in a mathematical learning community.** Students and teachers as well as family and community members must understand that mathematical proficiency is more than developing computation skills and memorizing algorithms. Mathematical proficiency is a balance of developing	**1 (6-8) Math proficiency is defined for everyone involved in a mathematical learning community.** Students and teachers as well as family and community members must understand that mathematical proficiency is more than developing computation skills and memorizing algorithms. Mathematical proficiency is a balance of developing	**1 (9-12) Math proficiency is defined for everyone involved in a mathematical learning community.** Students and teachers as well as family and community members must understand that mathematical proficiency is more than developing computation skills and memorizing algorithms. Mathematical proficiency is a balance of developing

conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Adding It Up 2001). By expanding the idea of what counts as mathematics competency beyond speed and accuracy, mathematics ability is viewed as a function of opportunity and experience - not of innate intelligence. Therefore, mathematics lessons and learning opportunities are intentionally structured and facilitated around meaningful, challenging, and rigorous mathematics. Adjustments in how mathematical proficiency is defined can also support the development of positive identification with mathematics among students. • (K-2 1A) Publically draw attention to competencies aligned with the Wisconsin State Standards for Mathematics, including	conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Adding It Up 2001). By expanding the idea of what counts as mathematics competency beyond speed and accuracy, mathematics ability is viewed as a function of opportunity and experience - not of innate intelligence. Therefore, mathematics lessons and learning opportunities are intentionally structured and facilitated around meaningful, challenging, and rigorous mathematics. Adjustments in how mathematical proficiency is defined can also support the development of positive identification with mathematics among students. • (3-5 1A) Publically draw attention to competencies aligned with the Wisconsin State Standards for Mathematics, including	conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Adding It Up 2001). By expanding the idea of what counts as mathematics competency beyond speed and accuracy, mathematics ability is viewed as a function of opportunity and experience - not of innate intelligence. Therefore, mathematics lessons and learning opportunities are intentionally structured and facilitated around meaningful, challenging, and rigorous mathematics. Adjustments in how mathematical proficiency is defined can also support the development of positive identification with mathematics among students. • (6-8 1A) Publically draw attention to competencies aligned with the Wisconsin State Standards for Mathematics, including	conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Adding It Up 2001). By expanding the idea of what counts as mathematics competency beyond speed and accuracy, mathematics ability is viewed as a function of opportunity and experience - not of innate intelligence. Therefore, mathematics lessons and learning opportunities are intentionally structured and facilitated around meaningful, challenging, and rigorous mathematics. Adjustments in how mathematical proficiency is defined can also support the development of positive identification with mathematics among students. • (9-12 1A) Publically draw attention to competencies aligned with the Wisconsin State Standards for Mathematics, including

both the Content Standards and the Standards for Mathematical Practice. • (K-2 1B) Ensure that mathematical discussions are driven by ideas and concepts rather than by those who arrive at a solution first. • (K-2 1C) Explicitly tell students that fast work is not what is valued. Mathematical thinking is about depth, not speed. • (K-2 1D) Encourage creativity, different ways of thinking, and different explanations when working toward solutions and reflecting on math tasks. • (K-2 1E) Include examples of student thinking that illustrate this broader definition of math proficiency in	both the Content Standards and the Standards for Mathematical Practice. • (3-5 1B) Ensure that mathematical discussions are driven by ideas and concepts rather than by those who arrive at a solution first. • (3-5 1C) Explicitly tell students that fast work is not what is valued. Mathematical thinking is about depth, not speed. • (3-5 1D) Encourage creativity, different ways of thinking, and different explanations when working toward solutions and reflecting on math tasks. • (3-5 1E) Include examples of student thinking that illustrate this broader definition of math proficiency in	both the Content Standards and the Standards for Mathematical Practice. • (6-8 1B) Ensure that mathematical discussions are driven by ideas and concepts rather than by those who arrive at a solution first. • (6-8 1C) Explicitly tell students that fast work is not what is valued. Mathematical thinking is about depth, not speed. • (6-8 1D) Encourage creativity, different ways of thinking, and different explanations when working toward solutions and reflecting on math tasks. • (6-8 1E) Include examples of student thinking that illustrate this broader definition of math proficiency in	both the Content Standards and the Standards for Mathematical Practice. • (9-12 1B) Ensure that mathematical discussions are driven by ideas and concepts rather than by those who arrive at a solution first. • (9-12 1C) Explicitly tell students that fast work is not what is valued. Mathematical thinking is about depth, not speed. • (9-12 1D) Encourage creativity, different ways of thinking, and different explanations when working toward solutions and reflecting on math tasks. • (9-12 1E) Include examples of student thinking that illustrate this broader definition of math proficiency in

family communications. • (K-2 1F) Provide families with questions that can promote true mathematical proficiency when engaging in math at home. For example, ask "How did you start solving the problem? Why was that a good strategy for this problem? Will your strategy always work? What else did or could you try?" • (K-2 1G) Help families understand the value of think time when engaging in math at home. For example, share a video clip of a class discussion that highlights how important it is to give children space to think deeply. • (K-2 1H) Support families as they partner with school to shape a	family communications. • (3-5 1F) Provide families with questions that can promote true mathematical proficiency when engaging in math at home. For example, ask "How did you start solving the problem? Why was that a good strategy for this problem? Will your strategy always work? What else did or could you try?" • (3-5 1G) Help families understand the value of think time when engaging in math at home. For example, share a video clip of a class discussion that highlights how important it is to give children space to think deeply. • (3-5 1H) Support families as they partner with school to shape a	family communications. • (6-8 1F) Provide families with questions that can promote true mathematical proficiency when engaging in math at home. For example, ask "How did you start solving the problem? Why was that a good strategy for this problem? Will your strategy always work? What else did or could you try?" • (6-8 1G) Help families understand the value of think time when engaging in math at home. For example, share a video clip of a class discussion that highlights how important it is to give children space to think deeply. • (6-8 1H) Support families as they partner with school to shape a	family communications. • (9-12 1F) Provide families with questions that can promote true mathematical proficiency when engaging in math at home. For example, ask "How did you start solving the problem? Why was that a good strategy for this problem? Will your strategy always work? What else did or could you try?" • (9-12 1G) Help families understand the value of think time when engaging in math at home. For example, share a video clip of a class discussion that highlights how important it is to give children space to think deeply. • (9-12 1H) Support families as they partner with school to shape a

positive mathematics identity and foster a strong sense of mathematical agency in their children. For example, parents can be encouraged to regularly try new things with their children, share instances when they themselves have needed perseverance to accomplish a difficult task, and recognize their child's efforts during math problem solving as well as their achievements.	positive mathematics identity and foster a strong sense of mathematical agency in their children. For example, parents can be encouraged to regularly try new things with their children, share instances when they themselves have needed perseverance to accomplish a difficult task, and recognize their child's efforts during math problem solving as well as their achievements.	positive mathematics identity and foster a strong sense of mathematical agency in their children. For example, parents can be encouraged to regularly try new things with their children, share instances when they themselves have needed perseverance to accomplish a difficult task, and recognize their child's efforts during math problem solving as well as their achievements.	positive mathematics identity and foster a strong sense of mathematical agency in their children. For example, parents can be encouraged to regularly try new things with their children, share instances when they themselves have needed perseverance to accomplish a difficult task, and recognize their child's efforts during math problem solving as well as their achievements.
2 (K-2) The student and teacher relationship is viewed as a partnership (Hammond, 2015) and as part of a classroom community in which all students feel safe and valued. Responsive teachers act as guides, mediators, consultants, instructors, and advocates for students, helping to effectively connect their culturally- and community-based knowledge	**2 (3-5) The student and teacher relationship is viewed as a partnership (Hammond, 2015) and as part of a classroom community in which all students feel safe and valued.** Responsive teachers act as guides, mediators, consultants, instructors, and advocates for students, helping to effectively connect their culturally- and community-based knowledge	**2 (6-8) The student and teacher relationship is viewed as a partnership (Hammond, 2015) and as part of a classroom community in which all students feel safe and valued.** Responsive teachers act as guides, mediators, consultants, instructors, and advocates for students, helping to effectively connect their culturally- and community-based knowledge	**2 (9-12) The student and teacher relationship is viewed as a partnership (Hammond, 2015) and as part of a classroom community in which all students feel safe and valued.** Responsive teachers act as guides, mediators, consultants, instructors, and advocates for students, helping to effectively connect their culturally- and community-based knowledge

to classroom learning experiences. Responsive teachers care for and demonstrate a commitment to the mathematical learning and growth of every student, and exhibit through visible action an awareness and belief in the capacity of every student to learn. They take responsibility for obstacles to student success and work intentionally to ensure equitable access to high-quality learning opportunities. This includes a teacher's willingness to identify and acknowledge unproductive beliefs regarding students' abilities to learn significant mathematics at high levels. • (K-2 2A) Place student thinking at the center of classroom activity. • (K-2 2B) Establish norms for community first or group first work and discussions. For	to classroom learning experiences. Responsive teachers care for and demonstrate a commitment to the mathematical learning and growth of every student, and exhibit through visible action an awareness and belief in the capacity of every student to learn. They take responsibility for obstacles to student success and work intentionally to ensure equitable access to high-quality learning opportunities. This includes a teacher's willingness to identify and acknowledge unproductive beliefs regarding students' abilities to learn significant mathematics at high levels. • (3-5 2A) Place student thinking at the center of classroom activity. • (3-5 2B) Establish norms for community first or group first work and discussions. For	to classroom learning experiences. Responsive teachers care for and demonstrate a commitment to the mathematical learning and growth of every student, and exhibit through visible action an awareness and belief in the capacity of every student to learn. They take responsibility for obstacles to student success and work intentionally to ensure equitable access to high-quality learning opportunities. This includes a teacher's willingness to identify and acknowledge unproductive beliefs regarding students' abilities to learn significant mathematics at high levels. • (6-8 2A) Place student thinking at the center of classroom activity. • (6-8 2B) Establish norms for community first or group first work and discussions. For	to classroom learning experiences. Responsive teachers care for and demonstrate a commitment to the mathematical learning and growth of every student, and exhibit through visible action an awareness and belief in the capacity of every student to learn. They take responsibility for obstacles to student success and work intentionally to ensure equitable access to high-quality learning opportunities. This includes a teacher's willingness to identify and acknowledge unproductive beliefs regarding students' abilities to learn significant mathematics at high levels. • (9-12 2A) Place student thinking at the center of classroom activity. • (9-12 2B) Establish norms for community first or group first work and discussions. For

example, a math talk routine is established that fosters a community mindset and ensures that all in the group are engaged and understand the mathematics involved. • (K-2 2C) Name students as authors and owners of mathematical ideas. For example, the teacher facilitates a class discussion comparing and contrasting Beth's use of counting back with Keisha's use of the relationship between addition and subtraction to solve 10-4.	example, students are told that one student from their group will be responsible for reporting about the group's work. Students do not know ahead of time who will be chosen. The group, therefore, has a community mindset and ensures that all in the group are engaged and understand the mathematics involved. • (3-5 2C) Name students as authors and owners of mathematical ideas. For example, the teacher facilitates a class discussion comparing and contrasting Beth's use of drawing circles and stars with Keisha's use of the relationship between multiplication and division to solve 28÷4.	example, students are told that one student from their group will be responsible for reporting about the group's work. Students do not know ahead of time who will be chosen. The group, therefore, has a community mindset and ensures that all in the group are engaged and understand the mathematics involved. • (6-8 2C) Name students as authors and owners of mathematical ideas. For example, the teacher facilitates a class discussion comparing and contrasting Beth's use of a table of equivalent ratios with Keisha's use of a double number line diagram to understand unit rate in order to solve a mathematical problem.	example, students are told that one student from their group will be responsible for reporting about the group's work. Students do not know ahead of time who will be chosen. The group, therefore, has a community mindset and ensures that all in the group are engaged and understand the mathematics involved. • (9-12 2C) Name students as authors and owners of mathematical ideas. For example, the teacher facilitates a class discussion comparing and contrasting Beth's use of a table of values with Keisha's algebraic solution to solve a system of equations.

• (K-2 2D) Provide opportunities for students to share their own mathematical thinking, receive feedback and critique from others, and evolve their thinking in a visible, public space (Huinker and Bill 2017, p. 177). • (K-2 2E) Invite students to add on or paraphrase a peer's idea in order to experience diverse views and strategies as being mathematically valuable (Huinker and Bill 2017, p. 210). • (K-2 2F) Build an alliance with students. This alliance balances rigor and high expectations with support. (Hammond, 2015)	• (3-5 2D) Provide opportunities for students to share their own mathematical thinking, receive feedback and critique from others, and evolve their thinking in a visible, public space (Huinker and Bill 2017, p. 177). • (3-5 2E) Invite students to add on or paraphrase a peer's idea in order to experience diverse views and strategies as being mathematically valuable (Huinker and Bill 2017, p. 210). • (3-5 2F) Build an alliance with students. This alliance balances rigor and high expectations with support. (Hammond, 2015)	• (6-8 2D) Provide opportunities for students to share their own mathematical thinking, receive feedback and critique from others, and evolve their thinking in a visible, public space. • (6-8 2E) Invite students to add on or paraphrase a peer's idea in order to experience diverse views and strategies as being mathematically valuable (Huinker and Bill 2017, p. 210). • (6-8 2F) Build an alliance with students. This alliance balances rigor and high expectations with support. (Hammond, 2015)	• (9-12 2D) Provide opportunities for students to share their own mathematical thinking, receive feedback and critique from others, and evolve their thinking in a visible, public space. • (9-12 2E) Invite students to add on or paraphrase a peer's idea in order to experience diverse views and strategies as being mathematically valuable (Huinker and Bill 2017, p. 210). • (9-12 2F) Build an alliance with students. This alliance balances rigor and high expectations with support. (Hammond, 2015)

3 (K-2) Meaningful relationships between teachers, students, and their families & caregivers lay the foundation for responsive environments that support equitable teaching and learning of mathematics. Developing meaningful relationships with students is how teachers come to learn about and respect students' experiences and knowledge (Gay, 2018; Hammond, 2015; Kitchens, 2018; Nieto, 2010). Meaningful teacher-student relationships play a critical role in nurturing students' mathematical identity and agency (Aguirre et al, 2013; Boaler, 2002; NCTM, 2014). Understanding mathematical learners includes knowing about their backgrounds, interests, strengths, and personalities as well as knowing how students think about and learn mathematics. Responsive teachers learn about and demonstrate an awareness of and appreciation	**3 (3-5) Meaningful relationships between teachers, students, and their families & caregivers lay the foundation for responsive environments that support equitable teaching and learning of mathematics.** Developing meaningful relationships with students is how teachers come to learn about and respect students' experiences and knowledge (Gay, 2018; Hammond, 2015; Kitchens, 2018; Nieto, 2010). Meaningful teacher-student relationships play a critical role in nurturing students' mathematical identity and agency (Aguirre et al, 2013; Boaler, 2002; NCTM, 2014). Understanding mathematical learners includes knowing about their backgrounds, interests, strengths, and personalities as well as knowing how students think about and learn mathematics. Responsive teachers learn about and demonstrate an awareness of and appreciation	**3 (6-8) Meaningful relationships between teachers, students, and their families & caregivers lay the foundation for responsive environments that support equitable teaching and learning of mathematics.** Developing meaningful relationships with students is how teachers come to learn about and respect students' experiences and knowledge (Gay, 2018; Hammond, 2015; Kitchens, 2018; Nieto, 2010). Meaningful teacher-student relationships play a critical role in nurturing students' mathematical identity and agency (Aguirre et al, 2013; Boaler, 2002; NCTM, 2014). Understanding mathematical learners includes knowing about their backgrounds, interests, strengths, and personalities as well as knowing how students think about and learn mathematics. Responsive teachers learn about and demonstrate an awareness of and appreciation	**3 (9-12) Meaningful relationships between teachers, students, and their families & caregivers lay the foundation for responsive environments that support equitable teaching and learning of mathematics.** Developing meaningful relationships with students is how teachers come to learn about and respect students' experiences and knowledge (Gay, 2018; Hammond, 2015; Kitchens, 2018; Nieto, 2010). Meaningful teacher-student relationships play a critical role in nurturing students' mathematical identity and agency (Aguirre et al, 2013; Boaler, 2002; NCTM, 2014). Understanding mathematical learners includes knowing about their backgrounds, interests, strengths, and personalities as well as knowing how students think about and learn mathematics. Responsive teachers learn about and demonstrate an awareness of and appreciation

for cultural identities and social diversity, particularly as they are present in one's classroom, and draw on diversity as a resource in instruction. Families and caregivers are engaged as critical partners in their child's mathematics learning. Responsive teaching honors the mathematics of the home and helps families gain competence and comfort in the mathematics their children are learning. Productive communications between school and families are attentive to considerations of language and culture and designed to support parents and guardians in fostering their child's success with mathematics in and out of school. Responsive teaching honors and builds upon students' existing mathematical ideas and ways of knowing and learning to support and enhance mathematics learning and engagement, including attending to each student's	for cultural identities and social diversity, particularly as they are present in one's classroom, and draw on diversity as a resource in instruction. Families and caregivers are engaged as critical partners in their child's mathematics learning. Responsive teaching honors the mathematics of the home and helps families gain competence and comfort in the mathematics their children are learning. Productive communications between school and families are attentive to considerations of language and culture and designed to support parents and guardians in fostering their child's success with mathematics in and out of school. Responsive teaching honors and builds upon students' existing mathematical ideas and ways of knowing and learning to support and enhance mathematics learning and engagement, including attending to each student's	for cultural identities and social diversity, particularly as they are present in one's classroom, and draw on diversity as a resource in instruction. Families and caregivers are engaged as critical partners in their child's mathematics learning. Responsive teaching honors the mathematics of the home and helps families gain competence and comfort in the mathematics their children are learning. Productive communications between school and families are attentive to considerations of language and culture and designed to support parents and guardians in fostering their child's success with mathematics in and out of school. Responsive teaching honors and builds upon students' existing mathematical ideas and ways of knowing and learning to support and enhance mathematics learning and engagement, including attending to each student's	for cultural identities and social diversity, particularly as they are present in one's classroom, and draw on diversity as a resource in instruction. Families and caregivers are engaged as critical partners in their child's mathematics learning. Responsive teaching honors the mathematics of the home and helps families gain competence and comfort in the mathematics their children are learning. Productive communications between school and families are attentive to considerations of language and culture and designed to support parents and guardians in fostering their child's success with mathematics in and out of school. Responsive teaching honors and builds upon students' existing mathematical ideas and ways of knowing and learning to support and enhance mathematics learning and engagement, including attending to each student's

culture, race/ethnicity, language, gender, socioeconomic status, cognitive and physical abilities, and personal interests. ● (K-2 3A) Use family communications to highlight the mathematics being learned, explaining the significance of strategies, visual models, or student work so that families can engage in the mathematics with their children. ● (K-2 3B) Use home visits, one to one interviews, phone calls, and surveys to get to know students and their families. ● (K-2 3C) Invite families into the mathematics classroom in a variety of ways. Families may come in-person, engage through reading information	culture, race/ethnicity, language, gender, socioeconomic status, cognitive and physical abilities, and personal interests. ● (3-5 3A) Use family communications to highlight the mathematics being learned, explaining the significance of strategies, visual models, or student work so that families can engage in the mathematics with their children. ● (3-5 3B) Use home visits, one to one interviews, phone calls, and surveys to get to know students and their families. ● (3-5 3C) Invite families into the mathematics classroom in a variety of ways. Families may come in-person, engage through reading information	culture, race/ethnicity, language, gender, socioeconomic status, cognitive and physical abilities, and personal interests. ● (6-8 3A) Use family communications to highlight the mathematics being learned, explaining the significance of strategies, visual models, or student work so that families can engage in the mathematics with their children. ● (6-8 3B) Use home visits, one to one interviews, phone calls, and surveys to get to know students and their families. ● (6-8 3C) Invite families into the mathematics classroom in a variety of ways. Families may come in-person, engage through reading information	culture, race/ethnicity, language, gender, socioeconomic status, cognitive and physical abilities, and personal interests. ● (9-12 3A) Use family communications to highlight the mathematics being learned, explaining the significance of strategies, visual models, or student work so that families can engage in the mathematics with their children. ● (9-12 3B) Use home visits, one to one interviews, phone calls, and surveys to get to know students and their families. ● (9-12 3C) Invite families into the mathematics classroom in a variety of ways. Families may come in-person, engage through reading information

purposefully shared by the teacher, or complete take-home activities with their children and share feedback about the activity with the teacher. For example, families can be provided with directions and materials for making modeling dough. Families make the dough and together, play with it. Families share thoughts about the experience with school. • (K-2 3D) Connect with students and families to learn about their language and communication style preferences. Use these preferences to promote two-way communication about the math classroom and student learning. • (K-2 3E) Ask students to share their math story.	purposefully shared by the teacher, or complete take-home activities with their children and share feedback about the activity with the teacher. For example, families can be provided with information to foster math-based conversations or game play at home. • (3-5 3D) Connect with students and families to learn about their language and communication style preferences. Use these preferences to promote two-way communication about the math classroom and student learning. • (3-5 3E) Ask students to share their math story.	purposefully shared by the teacher, or complete take-home activities with their children and share feedback about the activity with the teacher. For example, families can be provided with information to foster math-based conversations or explorations at home that would share what is being learned. • (6-8 3D) Connect with students and families to learn about their language and communication style preferences. Use these preferences to promote two-way communication about the math classroom and student learning. • (6-8 3E) Ask students to share their math story.	purposefully shared by the teacher, or complete take-home activities with their children and share feedback about the activity with the teacher. For example, families can be provided with information to foster math-based conversations or explorations at home that would share what is being learned. • (9-12 3D) Connect with students and families to learn about their language and communication style preferences. Use these preferences to promote two-way communication about the math classroom and student learning. • (9-12 3E) Ask students to share their math

This may include drawings, narratives, or verbal recordings that allow the student to share their thoughts and feelings about learning mathematics. Keep the stories and refer back to them throughout the year. • (K-2 3F) Be aware of different counting strategies, gestures, or other mathematical strategies based on cultural practices. Honor and build new learning from these forms of mathematical thinking. For example, when using fingers to count, some children may start with the thumb, index, or pinkie finger and there is more than one way to show a specific quantity when using a finger pattern.	This may include drawings, narratives, or verbal recordings that allow the student to share their thoughts and feelings about learning mathematics. Keep the stories and refer back to them throughout the year. • (3-5 3F) Be aware of different mathematical strategies or gestures based on cultural practices. Honor and build new learning from these forms of mathematical thinking. For example, families may use non-standard measurements when cooking (e.g., fistful), use different grouping practices, or different algorithms for math computation.	This may include drawings, narratives, or verbal recordings that allow the student to share their thoughts and feelings about learning mathematics. Keep the stories and refer back to them throughout the year. • (6-8 3F) Be aware of different mathematical strategies or gestures based on cultural practices. Honor and build new learning from these forms of mathematical thinking. For example, families may use non-standard measurements when cooking (e.g., fistful), use different grouping practices, or different algorithms for math computation.	story. This may include drawings, narratives, or verbal recordings that allow the student to share their thoughts and feelings about learning mathematics. Keep the stories and refer back to them throughout the year. • (9-12 3F) Be aware of different mathematical strategies or gestures based on cultural practices. Honor and build new learning from these forms of mathematical thinking. For example, families may use non-standard measurements when cooking (e.g., fistful), use different grouping practices, or different algorithms for math computation.

4 (K-2) Students view themselves as competent learners and doers of mathematics (Aguirre et al., 2013). In order to ensure that each and every student not only understands and can make use of foundational mathematics concepts and relationships but also comes to experience the joy, wonder, and beauty of mathematics, each and every student must be positioned as mathematically competent. “This requires creating classrooms--structures and norms--that support students to take risks to engage in discourse and see themselves as capable and worthy of being heard.” Mathematical identity plays a critical role in how students see themselves in relation to mathematics and their ability to engage in mathematics; therefore, responsive teaching aims to increase each and every students’ confidence as learners of mathematics and	**4 (3-5) Students view themselves as competent learners and doers of mathematics (Aguirre et al., 2013).** In order to ensure that each and every student not only understands and can make use of foundational mathematics concepts and relationships but also comes to experience the joy, wonder, and beauty of mathematics, each and every student must be positioned as mathematically competent. “This requires creating classrooms--structures and norms--that support students to take risks to engage in discourse and see themselves as capable and worthy of being heard.” Mathematical identity plays a critical role in how students see themselves in relation to mathematics and their ability to engage in mathematics; therefore, responsive teaching aims to increase each and every students’ confidence as learners of mathematics and	**4 (6-8) Students view themselves as competent learners and doers of mathematics (Aguirre et al., 2013).** In order to ensure that each and every student not only understands and can make use of foundational mathematics concepts and relationships but also comes to experience the joy, wonder, and beauty of mathematics, each and every student must be positioned as mathematically competent. “This requires creating classrooms--structures and norms--that support students to take risks to engage in discourse and see themselves as capable and worthy of being heard.” Mathematical identity plays a critical role in how students see themselves in relation to mathematics and their ability to engage in mathematics; therefore, responsive teaching aims to increase each and every students’ confidence as learners of mathematics and	**4 (9-12) Students view themselves as competent learners and doers of mathematics (Aguirre et al., 2013).** In order to ensure that each and every student not only understands and can make use of foundational mathematics concepts and relationships but also comes to experience the joy, wonder, and beauty of mathematics, each and every student must be positioned as mathematically competent. “This requires creating classrooms--structures and norms--that support students to take risks to engage in discourse and see themselves as capable and worthy of being heard.” Mathematical identity plays a critical role in how students see themselves in relation to mathematics and their ability to engage in mathematics; therefore, responsive teaching aims to increase each and every students’ confidence as learners of mathematics and

nurture robust and powerful mathematical identities. • (K-2 4A) Co-create classroom norms to orchestrate participation and engagement in daily mathematics that give all students a voice and shared identity in mathematical conversations. • (K-2 4B) Consciously give voice and authority to students from marginalized populations. (Huinker and Bill, 2017, p. 116) For example, intentionally teach students about mathematicians from diverse backgrounds throughout the year and provide students with opportunities to share their mathematics thinking by speaking, drawing, using text-to-speech,	nurture robust and powerful mathematical identities. • (3-5 4A) Co-create classroom norms to orchestrate participation and engagement in daily mathematics that give all students a voice and shared identity in mathematical conversations. • (3-5 4B) Consciously give voice and authority to students from marginalized populations. (Huinker and Bill, 2017, p. 116) For example, intentionally teach students about mathematicians from diverse backgrounds throughout the year and provide students with opportunities to share their mathematics thinking by speaking, drawing, using text-to-speech,	nurture robust and powerful mathematical identities. • (6-8 4A) Co-create classroom norms to orchestrate participation and engagement in daily mathematics that give all students a voice and shared identity in mathematical conversations. • (6-8 4B) Consciously give voice and authority to students from marginalized populations (Smith, Steele, and Raith, p. 95). For example, intentionally teach students about mathematicians from diverse backgrounds throughout the year and provide students with opportunities to share their mathematics thinking by speaking, drawing, using text-to-speech,	nurture robust and powerful mathematical identities. • (9-12 4A) Co-create classroom norms to orchestrate participation and engagement in daily mathematics that give all students a voice and shared identity in mathematical conversations. • (9-12 4B) Consciously give voice and authority to students from marginalized populations (Smith, Steele, and Ratih 2017, p.95). For example, intentionally teach students about mathematicians from diverse backgrounds throughout the year and provide students with opportunities to share their mathematics thinking by speaking, drawing, using text-to-speech,

etc. • (K-2 4C) Recognize and value students' primary languages, developing proficiency in English, integrated use of multiple languages (i.e., translanguaging), and mode of communication (e.g., aided and/or unaided augmentative and alternative communication [AAC]) including the use of devices, gestures, images, and/or objects while learning. • (K-2 4D) Support students in creating self-awareness of their personal effort toward learning. For example, create and use a personal effort chart developed by the class to articulate that effective learners practice, focus, ask questions, and give best effort.	etc. • (3-5 4C) Recognize and value students' primary languages, developing proficiency in English, integrated use of multiple languages (i.e., translanguaging), and mode of communication (e.g., aided and/or unaided augmentative and alternative communication [AAC]) including the use of devices, gestures, images, and/or objects while learning. • (3-5 4D) Support students in creating self-awareness of their personal effort toward learning through goal-setting, progress monitoring, and self-reflection.	etc. • (6-8 4C) Recognize and value students' primary languages, developing proficiency in English, integrated use of multiple languages (i.e., translanguaging), and mode of communication (e.g., aided and/or unaided augmentative and alternative communication [AAC]) including the use of devices, gestures, images, and/or objects while learning. • (6-8 4D) Support students in creating self-awareness of their personal effort toward learning through goal-setting, progress monitoring, and self-reflection.	etc. • (9-12 4C) Recognize and value students' primary languages, developing proficiency in English, integrated use of multiple languages (i.e., translanguaging), and mode of communication (e.g., aided and/or unaided augmentative and alternative communication [AAC]) including the use of devices, gestures, images, and/or objects while learning. • (9-12 4D) Support students in creating self-awareness of their personal effort toward learning through goal-setting, progress monitoring, and self-reflection.

(Celedón-Pattichis, White, and Civil 2017, pp. 64-70) • (K-2 4E) Develop the background knowledge of students and access their prior knowledge in order to provide all students regular access to high level tasks. • (K-2 4F) Launch tasks so that students understand what is expected of them and provide students with appropriate resources that will support their entry into the task. For example, students turn and talk about their ideas for representing a problem. This allows students to draw on each other as resources in understanding and clarifying task expectations, as well as draw from their own experiences, as they consider how to begin. (Huinker and Bill 2017,	• (3-5 4E) Develop the background knowledge of students and access their prior knowledge in order to provide all students regular access to high level tasks. • (3-5 4F) Launch tasks so that students understand what is expected of them and provide students with appropriate resources that will support their entry into the task. For example, students turn and talk about their ideas for representing a problem. This allows students to draw on each other as resources in understanding and clarifying task expectations, as well as draw from their own experiences, as they consider how to begin. (Huinker and Bill 2017,	• (6-8 4E) Develop the background knowledge of students and access their prior knowledge in order to provide all students regular access to high level tasks. • (6-8 4F) Launch tasks so that students understand what is expected of them and provide students with appropriate resources that will support their entry into the task. For example, students turn and talk about their ideas for representing a problem. This allows students to draw on each other as resources in understanding and clarifying task expectations, as well as draw from their own experiences, as they consider how to begin (Smith, Steele, and	• (9-12 4E) Develop the background knowledge of students and access their prior knowledge in order to provide all students regular access to high level tasks. • (9-12 4F) Launch tasks so that students understand what is expected of them and provide students with appropriate resources that will support their entry into the task. For example, students turn and talk about their ideas for representing a problem. This allows students to draw on each other as resources in understanding and clarifying task expectations, as well as draw from their own experiences, as they consider how to begin (Smith, Steele, and

p. 64) • (K-2 4G) When possible, provide choice in the way students represent their mathematical thinking. • (K-2 4H) Provide students opportunities to express mathematical understanding by inviting students to publicly share their math thinking. For example, students demonstrate the ways they used manipulatives, recreate their math drawings for others, or share their reasoning so that they see themselves as mathematical authorities in the classroom. (Huinker and Bill 2017, p. 34) • (K-2 4I) Focus on going deep with the mathematics. Provide opportunities for deep	p. 64) • (3-5 4G) When possible, provide choice in the way students represent their mathematical thinking. • (3-5 4H) Provide students opportunities to express mathematical understanding by inviting students to publicly share their math thinking. For example, students demonstrate the ways they used manipulatives, recreate their math drawings for others, or share their reasoning so that they see themselves as mathematical authorities in the classroom. (Huinker and Bill 2017, p. 34) • (3-5 4I) Focus on going deep with the mathematics. Provide opportunities for deep	Raith 2017, p. 51). • (6-8 4G) When possible, provide choice in the way students represent their mathematical thinking. • (6-8 4H) Provide students opportunities to express mathematical understanding by inviting students to publicly share their math thinking. For example, students demonstrate the ways they used manipulatives, recreate their math drawings for others, or share their reasoning so that they see themselves as mathematical authorities in the classroom. (Huinker and Bill 2017, p. 34) • (6-8 4I) Focus on going deep with the mathematics. Provide opportunities for deep	Raith 2017, p. 51). • (9-124G) When possible, provide choice in the way students represent their mathematical thinking. • (9-12 4H) Provide students opportunities to express mathematical understanding by inviting students to publicly share their math thinking. For example, students demonstrate the ways they used manipulatives, recreate their math drawings for others, or share their reasoning so that they see themselves as mathematical authorities in the classroom. (Huinker and Bill 2017, p. 34) • (9-12 4I) Focus on going deep with the mathematics. Provide opportunities for deep

learning to foster the development of a classroom's collective mathematical agency as students understand mathematics and expect it to make sense. When a student understands addition and subtraction as operations, they are able to apply those understandings to their own strategy and strategies of their peers. • (K-2 4J) Position mistakes made as a mathematician as opportunities for growth and learning. Highlight mistakes as contributions to classroom understanding. • (K-2 4K) Invite students to write word problems based on their own personal experiences in order to share real world	learning to foster the development of a classroom's collective mathematical agency as students understand mathematics and expect it to make sense. When a student understands addition and subtraction as operations, they are able to apply those understandings to their own strategy and strategies of their peers. • (3-5 4J) Position mistakes made as a mathematician as opportunities for growth and learning. Highlight mistakes as contributions to classroom understanding. • (3-5 4K) Invite students to write word problems based on their own personal experiences in order to share real world examples of	learning to foster the development of a classroom's collective mathematical agency as students understand mathematics and expect it to make sense. When a student understands addition and subtraction as operations, they are able to apply those understandings to their own strategy and strategies of their peers. • (6-8 4J) Position mistakes made as a mathematician as opportunities for growth and learning. Highlight mistakes as contributions to classroom understanding. • (6-8 4K) Invite students to share real world examples of mathematical and statistical concepts based on personal	learning to foster the development of a classroom's collective mathematical agency as students understand mathematics and expect it to make sense. When a student understands addition and subtraction as operations, they are able to apply those understandings to their own strategy and strategies of their peers. • (9-12 4J) Position mistakes made as a mathematician as opportunities for growth and learning. Highlight mistakes as contributions to classroom understanding. • (9-12 4K) Invite students to share real world examples of mathematical and statistical concepts based on personal

examples of mathematical concepts.	mathematical concepts.	experiences and observations.	experiences and observations.
5 (K-2) The mathematics students are learning must be relevant to their life experiences and learning environment. "Equity-based teaching depends on the capacity to recognize and intentionally tap students' knowledge and experiences - mathematical, cultural, linguistic, peer, family, and community - as resources for mathematics teaching and learning. Drawing on this knowledge and experience includes helping students bridge everyday experiences to learn mathematics, capitalizing on linguistic resources to support mathematics learning, recognizing family or community mathematical practices to support mathematics learning, and finding ways to help students learn and use mathematics to solve authentic problems that affect their lives."(Aguirre 2013)	**5 (3-5) The mathematics students are learning must be relevant to their life experiences and learning environment.** "Equity-based teaching depends on the capacity to recognize and intentionally tap students' knowledge and experiences - mathematical, cultural, linguistic, peer, family, and community - as resources for mathematics teaching and learning. Drawing on this knowledge and experience includes helping students bridge everyday experiences to learn mathematics, capitalizing on linguistic resources to support mathematics learning, recognizing family or community mathematical practices to support mathematics learning, and finding ways to help students learn and use mathematics to solve authentic problems that affect their lives."(Aguirre 2013)	**5 (6-8) The mathematics students are learning must be relevant to their life experiences and learning environment.** "Equity-based teaching depends on the capacity to recognize and intentionally tap students' knowledge and experiences - mathematical, cultural, linguistic, peer, family, and community - as resources for mathematics teaching and learning. Drawing on this knowledge and experience includes helping students bridge everyday experiences to learn mathematics, capitalizing on linguistic resources to support mathematics learning, recognizing family or community mathematical practices to support mathematics learning, and finding ways to help students learn and use mathematics to solve authentic problems that affect their lives."(Aguirre 2013)	**5 (9-12) The mathematics students are learning must be relevant to their life experiences and learning environment.** "Equity-based teaching depends on the capacity to recognize and intentionally tap students' knowledge and experiences - mathematical, cultural, linguistic, peer, family, and community - as resources for mathematics teaching and learning. Drawing on this knowledge and experience includes helping students bridge everyday experiences to learn mathematics, capitalizing on linguistic resources to support mathematics learning, recognizing family or community mathematical practices to support mathematics learning, and finding ways to help students learn and use mathematics to solve authentic problems that affect their lives."(Aguirre 2013)

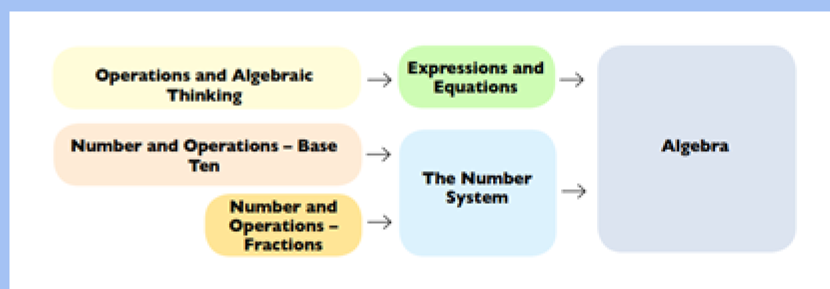
• (K-2 5A) Take a community walk. Look for and document evidence of mathematics within students' local neighborhoods. This could include people that are experienced using mathematics or are enacting mathematical concepts. Talk to individuals who work, play, or shop in the community about how they use mathematics or problem solve. • (K-2 5B) Write a series of questions about the contexts experienced on the community walk that could be investigated mathematically. Brainstorm a list of possible questions and data sources that students could use to answer those questions	• (3-5 5A) Take a community walk. Look for and document evidence of mathematics within students' local neighborhoods. This could include people that are experienced using mathematics or are enacting mathematical concepts. Talk to individuals who work, play, or shop in the community about how they use mathematics or problem solve. • (3-5 5B) Write a series of questions about the contexts experienced on the community walk that could be investigated mathematically. Brainstorm a list of possible questions and data sources that students could use to answer those questions	• (6-8 5A) Take a community walk. Look for and document evidence of mathematics within students' local neighborhoods. This could include people who have experience using mathematics or enacting mathematical concepts. Talk to individuals who work, play, or shop in the community about how they use mathematics or problem solve. • (6-8 5B) Write a series of questions about the contexts experienced on the community walk that could be investigated mathematically. Brainstorm a list of possible questions and data sources that students could use to answer those questions	• (9-12 5A) Take a community walk. Look for and document evidence of mathematics within students' local neighborhoods. This could include people who have experience using mathematics or enacting mathematical concepts. Talk to individuals who work, play, or shop in the community about how they use mathematics or problem solve. • (9-12 5B) Write a series of questions about the contexts experienced on the community walk that could be investigated mathematically. Brainstorm a list of possible questions and data sources that students could use to answer those questions

(Bartell et al., 2017). • (K-2 5C) Use storytelling about student after-school or weekend experiences to promote problem posing. Invite students to pose questions related to the story, shared by a peer, that can be explored mathematically. (Celedón-Pattichis, White, and Civil 2017, p. 61) • (K-2 5D) Use community experiences to connect mathematics to community contexts and enhance high quality instructional materials. For example, a teacher recognizes that many students walk to school and like to take shortcuts. Comparing different routes to school is an authentic activity, and one that involves mathematical ideas, like measuring and	(Bartell et al., 2017.) • (3-5 5C) Use storytelling about student after-school or weekend experiences to promote problem posing. Invite students to pose questions related to the story, shared by a peer, that can be explored mathematically. (Celedón-Pattichis, White, and Civil 2017, p. 61) • (3-5 5D) Use community experiences to connect mathematics to community contexts and enhance high quality instructional materials. For example, a teacher recognizes that many students walk to school and like to take shortcuts. Comparing different routes to school is an authentic activity, and one that involves mathematical ideas, like measuring and	(Bartell et al., 2017). • (6-8 5C) Use community experiences to connect mathematics to community contexts and enhance high quality instructional materials. For example, students use mathematical modeling to find solutions to problems relevant and important to the student.	(Bartell et al., 2017). • (9-12 5C) Use community experiences to connect mathematics to community contexts and enhance high quality instructional materials. For example, students use mathematical modeling to find solutions to problems relevant and important to the student.

comparing distances. (Bartell et al., 2017) • (K-2 5E) Use interdisciplinary connections to bridge mathematics and other content areas. For example, use a class' favorite children's book to connect to data collection about how many students "like" or "don't like" green eggs and ham. Students will naturally interpret data in a playful way. (Celedón-Pattichis, White, and Civil 2017, pp. 21-22) • (K-2 5F) Talk to students about activities and experiences outside of school. For example, provide opportunities to pose questions about places in the community that students know about and find interesting,	comparing distances. (Bartell et al., 2017) • (3-5 5E) Use interdisciplinary connections to bridge mathematics and other content areas. For example, while encouraging students to be thoughtful and inquiring readers, students might be asked to use mathematical modeling to answer questions related to the context of a book they are reading. • (3-5 5F) Talk to students about activities and experiences outside of school. For example, provide opportunities to pose questions about places in the community that students know about and find interesting,	• 6-8 5D) Use interdisciplinary connections to bridge mathematics and other content areas. For example, students consider the chance of rain, the chance that it will not rain, and how these probabilities are connected to weather systems in science. • (6-8 5E) Talk to students about activities and experiences outside of school. For example, provide opportunities to pose questions about places in the community that students know about and find interesting and	• 9-12 5D) Use interdisciplinary connections to bridge mathematics and other content areas. For example, provide opportunities for students to think about how to create a plan to find and purchase a car. This might include components of the modeling cycle as they identify and specify what criteria will be used, analyze the model, and plan to make the purchase. • (9-12 5E) Talk to students about activities and experiences outside of school. For example, provide opportunities to pose questions about places in the community that students know about and find interesting and

<table>
<tr><td>invite children to share photos of family activities, and invite family members into the classroom to share their work or hobbies. (Celedón-Pattichis, White, and Civil 2017, pp. 59-60)</td><td>invite children to share photos of family activities, and invite family members into the classroom to share their work or hobbies. (Celedón-Pattichis, White, and Civil 2017, pp. 59-60)</td><td>listen for and discuss social justice topics that are important to students.</td><td>listen for and discuss social justice topics that are important to students.</td></tr>
<tr><td colspan="4">Coherent and Connected: Mathematics is a coherent and connected discipline.

The Wisconsin Standards for Mathematics are built upon coherence, one of the design principles of the Standards. The intentional progression and sequencing of topics lays the foundation for the mathematics that is developed from kindergarten through high school. The diagram below depicts how domains at the elementary and middle school levels converge toward algebra at the high school. It is important that educators are knowledgeable about these progressions so that students learn mathematics with understanding and new content can build upon prior learning.


</td></tr>
<tr><td>Within each grade level, the domains themselves have strong connections to each</td><td>Within each grade level, the domains themselves have strong connections to each</td><td>Within each grade level, the domains themselves have strong connections to each</td><td>Across high school mathematics, the domains and conceptual categories have strong connections to each other.</td></tr>
</table>

other. "Instead of allowing additional or supporting topics to detract from the focus of the grade, these concepts serve the grade-level focus."	other. "Instead of allowing additional or supporting topics to detract from the focus of the grade, these concepts serve the grade-level focus."	other. "Instead of allowing additional or supporting topics to detract from the focus of the grade, these concepts serve the grade-level focus."	
For example, in the Grade 2 Measurement and Data domain, students understand whole numbers as measures on a number line. This understanding is strongly connected to the place value understanding of the Number and Operations in Base Ten domain.	For example, in the Grade 3 Measurement and Data domain, students understand concepts of area and relate area to multiplication and addition. This measurement understanding is strongly connected to the multiplication and division learning of the Operations and Algebraic Thinking domain which is a large part of the major work of the grade.	For example, in the Grade 7 Expressions and Equations domain, students use properties of operations to simplify expressions with rational coefficients . This understanding is strongly connected to the entire Number System domain and understanding operations with rational numbers.	For example, students understand that the distance formula is a restatement of the Pythagorean theorem. In addition, right triangle trigonometry provides a basis for developing unit circle trigonometry. Within the context of the unit circle, the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$ is yet another restatement of the Pythagorean Theorem.
6 (K-2) Establish math goals to focus learning. As illustrated in The Teaching Framework for Mathematics (p. 7 of this guide), the mathematics goal serves as the starting point for all instructional decisions and focuses and frames the teaching and learning as it unfolds throughout a lesson or over the course of a unit.	**6 (3-5) Establish math goals to focus learning.** As illustrated in The Teaching Framework for Mathematics (p. 7 of this guide), the mathematics goal serves as the starting point for all instructional decisions and focuses and frames the teaching and learning as it unfolds throughout a lesson or over the course of a unit.	**6 (6-8) Establish math goals to focus learning.** As illustrated in The Teaching Framework for Mathematics (p. 7 of this guide), the mathematics goal serves as the starting point for all instructional decisions and focuses and frames the teaching and learning as it unfolds throughout a lesson or over the course of a unit.	**6 (9-12) Establish math goals to focus learning.** As illustrated in The Teaching Framework for Mathematics (p. 7 of this guide), the mathematics goal serves as the starting point for all instructional decisions and focuses and frames the teaching and learning as it unfolds throughout a lesson or over the course of a unit.

"Effective mathematics teaching begins with a shared understanding among teachers of the mathematics that students are learning and how this mathematics develops along learning progressions "(NCTM, 2014, p. 12). Mathematics goals serve to both identify the mathematical concepts, ideas, or methods students will understand as a result of instruction, and the mathematical practices students will be employing as they work. Situating goals within learning progressions informs real-time instructional decisions resulting in instructional scaffolds that preserve the mathematical goal of the lesson and honor students' developmental milestones and markers. In responsive mathematics classrooms, teachers regularly ensure that each and every student has the opportunity to meet the mathematical goal of each lesson. To that end, they establish learning environments that promote	"Effective mathematics teaching begins with a shared understanding among teachers of the mathematics that students are learning and how this mathematics develops along learning progressions "(NCTM, 2014, p. 12). Mathematics goals serve to both identify the mathematical concepts, ideas, or methods students will understand as a result of instruction, and the mathematical practices students will be employing as they work. Situating goals within learning progressions informs real-time instructional decisions resulting in instructional scaffolds that preserve the mathematical goal of the lesson and honor students' developmental milestones and markers. In responsive mathematics classrooms, teachers regularly ensure that each and every student has the opportunity to meet the mathematical goal of each lesson. To that end, they establish learning environments that promote	"Effective mathematics teaching begins with a shared understanding among teachers of the mathematics that students are learning and how this mathematics develops along learning progressions "(NCTM, 2014, p. 12). Mathematics goals serve to both identify the mathematical concepts, ideas, or methods students will understand as a result of instruction, and the mathematical practices students will be employing as they work. Situating goals within learning progressions informs real-time instructional decisions resulting in instructional scaffolds that preserve the mathematical goal of the lesson and honor students' developmental milestones and markers. In responsive mathematics classrooms, teachers regularly ensure that each and every student has the opportunity to meet the mathematical goal of each lesson. To that end, they establish learning environments that promote	"Effective mathematics teaching begins with a shared understanding among teachers of the mathematics that students are learning and how this mathematics develops along learning progressions "(NCTM, 2014, p. 12). Mathematics goals serve to both identify the mathematical concepts, ideas, or methods students will understand as a result of instruction, and the mathematical practices students will be employing as they work. Situating goals within learning progressions informs real-time instructional decisions resulting in instructional scaffolds that preserve the mathematical goal of the lesson and honor students' developmental milestones and markers. Given the traditional compartmentation of high school mathematics within courses, it is important that this shared understanding occurs among an entire high school staff and not merely among teachers of one single course.

learning mathematics as just, equitable, and inclusive and intentionally establish norms for participation that position each and every student as a competent mathematics thinker (Turner, Dominguez, Maldonado, & Empon, 2013). Teachers utilize research-based, developmental learning trajectories and progressions to build students' mathematical understanding, increase student confidence, and support student mathematical identities as doers of mathematics. ● (K-2 6A) Ensure that goals match grade level expectations and address the design principles of focus, coherence, and rigor.	learning mathematics as just, equitable, and inclusive and intentionally establish norms for participation that position each and every student as a competent mathematics thinker (Turner, Dominguez, Maldonado, & Empon, 2013). Teachers utilize research-based, developmental learning trajectories and progressions to build students' mathematical understanding, increase student confidence, and support student mathematical identities as doers of mathematics. ● (3-5 6A) Ensure that goals match grade level expectations and address the design principles of focus, coherence, and rigor.	learning mathematics as just, equitable, and inclusive and intentionally establish norms for participation that position each and every student as a competent mathematics thinker (Turner, Dominguez, Maldonado, & Empon, 2013). Teachers utilize research-based, developmental learning trajectories and progressions to build students' mathematical understanding, increase student confidence, and support student mathematical identities as doers of mathematics. ● (6-8 6A) Ensure that goals match grade level expectations and address the design principles of focus, coherence, and rigor.	In responsive mathematics classrooms, teachers regularly ensure that each and every student has the opportunity to meet the mathematical goal of each lesson. To that end, they establish learning environments that promote learning mathematics as just, equitable, and inclusive and intentionally establish norms for participation that position each and every student as a competent mathematics thinker (Turner, Dominguez, Maldonado, & Empon, 2013). Teachers utilize research-based, developmental learning trajectories and progressions to build students' mathematical understanding, increase student confidence, and support student mathematical identities as doers of mathematics. ● (9-12 6A) Ensure that goals match course expectations and address the design principles of focus, coherence, and rigor.

• (K-2 6B) Differentiate the mathematical goal from the mathematical activity. This clarity helps plan what questions, related to the important mathematics, will be asked as feedback is provided during the lesson. For example, the mathematical goal is to understand that numbers can be decomposed in different ways, whereas the mathematical activity is to play Shake and Spill to practice decomposing numbers and representing those decompositions. • (K-2 6C) Articulate the goal to students in appropriate, student-friendly language. The goal includes success criteria to ensure that students understand the lesson expectations. For example: Numbers can	• (3-5 6B) Differentiate the mathematical goal from the mathematical activity. This clarity helps plan what questions, related to the important mathematics, will be asked as feedback is provided during the lesson. For example, the mathematical goal is to understand that numbers can be decomposed in different ways to support problem solving, whereas the mathematical activity is to solve a problem and compare the strategies used. • (3-5 6C) Articulate the goal to students in appropriate, student-friendly language. The goal includes success criteria to ensure that students understand the lesson expectations. For example: Numbers can	• (6-8 6B) Differentiate the mathematical goal from the mathematical activity. This clarity helps plan what questions, related to the important mathematics, will be asked as feedback is provided during the lesson. For example, the mathematical goal is to recognize proportional relationships in different representations, whereas the mathematical activity is to represent real-life situations through tables, graphs, and equations. • (6-8 6C) Articulate the goal to students in appropriate, student-friendly language. The goal includes success criteria to ensure that students understand the lesson expectations. For example: Proportional	• (9-12 6B) Differentiate the mathematical goal from the mathematical activity. This clarity helps plan what questions, related to the important mathematics, will be asked as feedback is provided during the lesson. For example, the mathematical goal is to recognize functional relationships in different representations, whereas the mathematical activity is to represent real-life situations through tables, graphs, and equations. • (9-12 6C) Articulate the goal to students in appropriate, student-friendly language. The goal includes success criteria to ensure that students understand the lesson expectations. For example: Functional

be broken apart in different ways. I am successful when I can break apart a number in different ways and explain my thinking in words, pictures or using objects. • (K-2 6D) Connect learning goals to the broader mathematical landscape and help students understand the relevance of the mathematical goal. • (K-2 6E) Post the learning target, including the goal and success criteria, and revisit the learning target throughout the lesson. • (K-2 6F) Ensure that all learners have access to the goal. The teacher anticipates, plans and monitors how each student will work to	be broken apart in different ways. I am successful when I can break apart a number in different ways and explain my thinking in words, pictures or using objects. • (3-5 6D) Connect learning goals to the broader mathematical landscape and help students understand the relevance of the mathematical goal. • (3-5 6E) Post the learning target, including the goal and success criteria, and revisit the learning target throughout the lesson. • (3-5 6F) Ensure that all learners have access to the goal. The teacher anticipates, plans and monitors how each student will work to reach the goal (NCTM	relationships can be represented in different ways. I am successful when, given a representation, I can tell if it is proportional. I can also explain how I know that it is or is not proportional. • (6-8 6D) Connect learning goals to the broader mathematical landscape and help students understand the relevance of the mathematical goal. • (6-8 6E) Post the learning target, including the goal and success criteria, and revisit the learning target throughout the lesson. • (6-8 6F) Ensure that all learners have access to the goal. The teacher anticipates, plans and monitors how each student will work to reach the goal (NCTM	relationships can be represented in different ways. I am successful when, given a representation, I can tell if it is linear, quadratic, exponential or none of these. I can also explain how I know. • (9-12 6D) Connect learning goals to the broader mathematical landscape and help students understand the relevance of the mathematical goal. • (9-12 6E) Post the learning target, including the goal and success criteria, and revisit the learning target throughout the lesson. • (9-12 6F) Ensure that all learners have access to the goal. The teacher anticipates, plans and monitors how each student will work to

reach the goal (NCTM 2014, p. 13). • (K-2 6G) Support students in monitoring their own progress toward the learning goal by analyzing their work and citing evidence to support their conclusions. • (K-2 6H) Employ formative assessment practices to gauge student growth in meeting the learning goal.	2014, p. 13). • (3-5 6G) Support students in monitoring their own progress toward the learning goal by analyzing their work and citing evidence to support their conclusions. • (3-5 6H) Employ formative assessment practices to gauge student growth in meeting the learning goal.	2014, p. 13). • (6-8 6G) Support students in monitoring their own progress toward the learning goal by analyzing their work and citing evidence to support their conclusions. • (6-8 6H) Employ formative assessment practices to gauge student growth in meeting the learning goal.	reach the goal (NCTM 2014, p. 13). • (9-12 6G) Support students in monitoring their own progress toward the learning goal by analyzing their work and citing evidence to support their conclusions. • (9-12 6H) Employ formative assessment practices to gauge student growth in meeting the learning goal.
7 (K-2) Use and connect mathematical representations. Effective teaching emphasizes using and making connections among mathematical representations to deepen student understanding of concepts and procedures, support mathematical discourse among students, and serve as tools for problem	**7 (3-5) Use and connect mathematical representations.** Effective teaching emphasizes using and making connections among mathematical representations to deepen student understanding of concepts and procedures, support mathematical discourse among students, and serve as tools for problem	**7 (6-8) Use and connect mathematical representations.** Effective teaching emphasizes using and making connections among mathematical representations to deepen student understanding of concepts and procedures, support mathematical discourse among students, and serve as tools for problem	**7 (9-12) Use and connect mathematical representations.** Effective teaching emphasizes using and making connections among mathematical representations to deepen student understanding of concepts and procedures, support mathematical discourse among students, and serve as tools for problem

solving. As students use and make connections among contextual, physical, visual, verbal, and symbolic representations, they grow their appreciation of mathematics as a unified, coherent discipline (NCTM, 2014, p. 24). Teachers support students' use of representations when they select mathematical tasks that can be solved utilizing a variety of representations. Once students generate their representations, teachers deepen students' understanding of mathematics as they intentionally engage students in describing and discussing the connections among representations. In responsive mathematics classrooms, teachers develop learning environments rich in the use of multiple representations-contextual, physical, visual, verbal, and symbolic. They intentionally	solving. As students use and make connections among contextual, physical, visual, verbal, and symbolic representations, they grow their appreciation of mathematics as a unified, coherent discipline (NCTM, 2014, p. 24). Teachers support students' use of representations when they select mathematical tasks that can be solved utilizing a variety of representations. Once students generate their representations, teachers deepen students' understanding of mathematics as they intentionally engage students in describing and discussing the connections among representations. In responsive mathematics classrooms, teachers develop learning environments rich in the use of multiple representations-contextual, physical, visual, verbal, and symbolic. They intentionally	solving. As students use and make connections among contextual, physical, visual, verbal, and symbolic representations, they grow their appreciation of mathematics as a unified, coherent discipline (NCTM, 2014, p. 24). Teachers support students' use of representations when they select mathematical tasks that can be solved utilizing a variety of representations. Once students generate their representations, teachers deepen students' understanding of mathematics as they intentionally engage students in describing and discussing the connections among representations. In responsive mathematics classrooms, teachers develop learning environments rich in the use of multiple representations-contextual, physical, visual, verbal, and symbolic. They intentionally	solving. As students use and make connections among contextual, physical, visual, verbal, and symbolic representations, they grow their appreciation of mathematics as a unified, coherent discipline (NCTM, 2014, p. 24). Teachers support students' use of representations when they select mathematical tasks that can be solved utilizing a variety of representations. Once students generate their representations, teachers deepen students' understanding of mathematics as they intentionally engage students in describing and discussing the connections among representations. In responsive mathematics classrooms, teachers develop learning environments rich in the use of multiple representations-contextual, physical, visual, verbal, and symbolic. They intentionally

engage students in seeing and connecting multiple representations for the same mathematical idea, ensuring students develop deep and connected knowledge of mathematics. When teachers allow student choice in visual representations and mathematical tools during problem solving, student thinking is validated, they intentionally foster an interest in their own and others' thinking, and position students as mathematically competent. ● (K-2 7A) Provide time for students to use multiple representations as tools for problem-solving. By examining concepts through a variety of representations, and therefore lenses to provide different perspectives, the mathematical concept is understood more	engage students in seeing and connecting multiple representations for the same mathematical idea, ensuring students develop deep and connected knowledge of mathematics. When teachers allow student choice in visual representations and mathematical tools during problem solving, student thinking is validated, they intentionally foster an interest in their own and others' thinking, and position students as mathematically competent. ● (3-5 7A) Provide time for students to use multiple representations as tools for problem-solving. By examining concepts through a variety of representations, and therefore lenses to provide different perspectives, the mathematical concept is understood more	engage students in seeing and connecting multiple representations for the same mathematical idea, ensuring students develop deep and connected knowledge of mathematics. When teachers allow student choice in visual representations and mathematical tools during problem solving, student thinking is validated, they intentionally foster an interest in their own and others' thinking, and position students as mathematically competent. ● (6-8 7A) Provide time for students to use multiple representations as tools for problem-solving. By examining concepts through a variety of representations, and therefore lenses to provide different perspectives, the mathematical concept is understood more	engage students in seeing and connecting multiple representations for the same mathematical idea, ensuring students develop deep and connected knowledge of mathematics. When teachers allow student choice in visual representations and mathematical tools during problem solving, student thinking is validated, they intentionally foster an interest in their own and others' thinking, and position students as mathematically competent. ● (9-12 7A) Provide time for students to use multiple representations as tools for problem-solving. By examining concepts through a variety of representations, and therefore lenses to provide different perspectives, the mathematical concept is understood more

richly and deeper. • (K-2 7B) Plan for instruction and practice that includes multiple ways for students to experience mathematics and guide students in attending to each of the representations used. The five modes of mathematical representation include contextual, visual, verbal, physical, and symbolic (Huinker and Bill 2017, p. 119). • (K-2 7C) Over time, encourage students to explore concepts concretely, visually and abstractly and move fluidly among these representations. • (K-2 7D) Use representations to compare and contrast student solution strategies and to reveal	richly and deeper. • (3-5 7B) Plan for instruction and practice that includes multiple ways for students to experience mathematics and guide students in attending to each of the representations used. The five modes of mathematical representation include contextual, visual, verbal, physical, and symbolic (Huinker and Bill 2017, p. 119). • (3-5 7C) Over time, encourage students to explore concepts concretely, visually and abstractly and move fluidly among these representations. • (3-5 7D) Use representations to compare and contrast student solution strategies and to reveal	richly and deeper. • (6-8 7B) Plan for instruction and practice that includes multiple ways for students to experience mathematics and guide students in attending to each of the representations used. The five modes of mathematical representation include contextual, visual, verbal, physical, and symbolic (Smith, Steele, and Raith 2017, p. 100). • (6-8 7C) Over time, encourage students to explore concepts concretely, visually and abstractly and move fluidly among these representations. • (6-8 7D) Use representations to compare and contrast student solution strategies and to reveal	richly and deeply. • (9-12 7B) Plan for instruction and practice that includes multiple ways for students to experience mathematics and guide students in attending to each of the representations used. The five modes of mathematical representation include contextual, visual, verbal, physical, and symbolic (Smith, Steele, and Raith 2017, p. 100). • (9-12 7C) Over time, encourage students to explore concepts concretely, visually and abstractly and move fluidly among these representations. • (9-12 7D) Use representations to compare and contrast student solution strategies and to reveal

the underlying mathematics. • (K-2 7E) Use representations to help students advance their understanding of mathematical concepts and highlight the structure of the mathematics. • (K-2 7F) Engage students in dialogue to make explicit connections within and among representations. • (K-2 7G) Encourage students to use meaningful representations of their choosing. Ask students to justify or consider the appropriateness of the selected representation to develop a knowledge of when and why to make a specific choice. (Huinker 2015) • (K-2 7H) Examine	the underlying mathematics. • (3-5 7E) Use representations to help students advance their understanding of mathematical concepts and highlight the structure of the mathematics. • (3-5 7F) Engage students in dialogue to make explicit connections within and among representations. • (3-5 7G) Encourage students to use meaningful representations of their choosing. Ask students to justify or consider the appropriateness of the selected representation to develop a knowledge of when and why to make a specific choice. (Huinker 2015) • (3-5 7H) Examine	the underlying mathematics. • (6-8 7E) Use representations to help students advance their understanding of mathematical concepts and highlight the structure of the mathematics. • (6-8 7F) Engage students in dialogue to make explicit connections within and among representations. • (6-8 G) Encourage students to use meaningful representations of their choosing. Ask students to justify or consider the appropriateness of the selected representation to develop a knowledge of when and why to make a specific choice. (Huinker 2015) • (6-8 7H) Examine	the underlying mathematics. • (9-12 7E) Use representations to help students advance their understanding of mathematical concepts and highlight the structure of the mathematics. • (9-12 7F) Engage students in dialogue to make explicit connections within and among representations. • (9-12 G) Encourage students to use meaningful representations of their choosing. Ask students to justify or consider the appropriateness of the selected representation to develop a knowledge of when and why to make a specific choice. (Huinker 2015) • (9-12 7H) Examine

materials to look for which representations are used most often and be aware of which are privileged and why. For example, all five modes of representation should be equally validated and part of mathematical discussions. • (K-2 7I) Plan experiences so that students can practice moving flexibly between representations once they have conceptual understanding. For example, explicitly ask for more than one representation from students as they engage in a task. • (K-2 7J) Understand the progression of learning that comes before and after grade level content being taught in order to connect representations over	materials to look for which representations are used most often and be aware of which are privileged and why. For example, all five modes of representation should be equally validated and part of mathematical discussions. • (3-5 7I) Plan experiences so that students can practice moving flexibly between representations once they have conceptual understanding. For example, explicitly ask for more than one representation from students as they engage in a task. • (3-5 7J) Understand the progression of learning that comes before and after grade level content being taught in order to connect representations over	materials to look for which representations are used most often and be aware of which are privileged and why. For example, all five modes of representation should be equally validated and part of mathematical discussions. • (6-8 7I) Plan experiences so that students can practice moving flexibly between representations once they have conceptual understanding. For example, explicitly ask for more than one representation from students as they engage in a task. • (6-8 7J) Understand the progression of learning that comes before and after grade level content being taught in order to connect representations over	materials to look for which representations are used most often and be aware of which are privileged and why. For example, all five modes of representation should be equally validated and part of mathematical discussions. • (9-12 7I) Plan experiences so that students can practice moving flexibly between representations once they have conceptual understanding. For example, explicitly ask for more than one representation from students as they engage in a task. • (9-12 7J) Understand the progression of learning that comes before and after grade level content being taught in order to connect

time. For example, students begin by anchoring numeric thinking to five as a landmark. One of the ways they show their thinking about this landmark is through the use of a five frame. Next, a ten frame supports students as they anchor their thinking to ten as a landmark. As larger numbers and place value become a focus, double ten frames and other representations build on this progression. • (K-2 7K) Connect the representation or tool to the mathematical strategy so that students can see where the mathematics used in different solutions may be the same, but it's the representation that is different. For example, an open number line is a visual	time. For example, students can think about multiplication through an array. Later, the array representation can be connected to an area model. Then, if the dimensions of the area model are decomposed by place value, connections can be made to the U.S. Standards Algorithm for Multiplication. • (3-5 7K) Connect the representation or tool to the mathematical strategy so that students can see where the mathematics used in different solutions may be the same, but it's the representation that is different. For example, an open number line is a visual	time. For example, students think about multiplication through arrays, numerical expressions, and area models. Later, these representations are connected to the solving of algebraic equations through the use of the distributive property.	representations over time. For example, students think about multiplication through arrays, numerical expressions, and area models. Later, these representations are connected to the solving of algebraic equations through the use of factoring.

representation that can be used to illustrate a strategy in which an initial addend is kept whole and the second added is decomposed into efficient “jumps” using landmark numbers. A series of equations could also be used as a different representation to illustrate this same strategy. • (K-2 7L) Use developmentally appropriate math manipulatives and tools to provide opportunities for students to visualize mathematics. For example, primary students begin using a number path to make sense of number before transitioning to a number line.[23]	representation that can be used to illustrate a strategy in which an initial addend is kept whole and the second added is decomposed into efficient “jumps” using landmark numbers. A series of equations could also be used as a different representation to illustrate this same strategy. • (3-5 7L) Use developmentally appropriate math manipulatives and tools to provide opportunities for students to visualize mathematics. For example, students make sense of volume by using cubes to build three dimensional figures.	• (6-8 7K) Use developmentally appropriate math manipulatives and tools to provide opportunities for students to visualize mathematics. For example, students make sense of volume by using cubes to build three dimensional figures.	• (9-12 7K) Use developmentally appropriate math manipulatives and tools to provide opportunities for students to visualize mathematics. For example, students use algebra tiles as they factor a polynomial based on an area model.

Access and Engagement: Every student must have access to and engage in meaningful, challenging, and rigorous mathematics.

Students are successful when they are engaged in doing mathematics. As mathematicians "students are active learners, constructing their knowledge through exploration, discussion, and reflection. The tasks in which students engage are both challenging and interesting and cannot be answered quickly by applying a known rule or procedure. Students must reason about and make sense of a situation and persevere when a pathway is not immediately evident." (NCTM 2014)

8 (K-2) Elicit and use evidence of student thinking.	8 (3-5) Elicit and use evidence of student thinking.	8 (6-8) Elicit and use evidence of student thinking.	8 (9-12) Elicit and use evidence of student thinking.
"Effective mathematics teaching elicits evidence of students' current mathematical understanding and uses it as a basis for making instructional decisions" (NCTM, 2014, p. 53). Effective teaching includes the skills of surfacing and identifying student thinking, interpreting student thinking during instruction, and then deciding how to respond on the basis of student thinking in order to advance learning. Teachers may gather information regarding what students know and understand about the mathematics they are learning by listening to what students say, observing	"Effective mathematics teaching elicits evidence of students' current mathematical understanding and uses it as a basis for making instructional decisions" (NCTM, 2014, p. 53). Effective teaching includes the skills of surfacing and identifying student thinking, interpreting student thinking during instruction, and then deciding how to respond on the basis of student thinking in order to advance learning. Teachers may gather information regarding what students know and understand about the mathematics they are learning by listening to what students say, observing	"Effective mathematics teaching elicits evidence of students' current mathematical understanding and uses it as a basis for making instructional decisions" (NCTM, 2014, p. 53). Effective teaching includes the skills of surfacing and identifying student thinking, interpreting student thinking during instruction, and then deciding how to respond on the basis of student thinking in order to advance learning. Teachers may gather information regarding what students know and understand about the mathematics they are learning by listening to what students say, observing	"Effective mathematics teaching elicits evidence of students' current mathematical understanding and uses it as a basis for making instructional decisions" (NCTM, 2014, p. 53). Effective teaching includes the skills of surfacing and identifying student thinking, interpreting student thinking during instruction, and then deciding how to respond on the basis of student thinking in order to advance learning. Teachers may gather information regarding what students know and understand about the mathematics they are learning by listening to what students say, observing

their actions, and analyzing their written work. Intentionally planning for how to elicit students' thinking during mathematics lessons is a key aspect of effective lesson planning (NCTM, 2017). In responsive mathematics classrooms, eliciting and using evidence of student thinking is central to informing the daily, in-the-moment instructional decisions of teachers. By carefully listening to and thoughtfully responding to student thinking, teachers position students' contributions as mathematically valuable and contributing to a broader collective understanding of the mathematical ideas at hand. Whose work gets selected and discussed during a lesson sends important messages to students about the solution paths that are valued and valid in the mathematics classroom. Listening to what students say, observing their actions, and analyzing their written work are all ways to gather	their actions, and analyzing their written work. Intentionally planning for how to elicit students' thinking during mathematics lessons is a key aspect of effective lesson planning (NCTM, 2017). In responsive mathematics classrooms, eliciting and using evidence of student thinking is central to informing the daily, in-the-moment instructional decisions of teachers. By carefully listening to and thoughtfully responding to student thinking, teachers position students' contributions as mathematically valuable and contributing to a broader collective understanding of the mathematical ideas at hand. Whose work gets selected and discussed during a lesson sends important messages to students about the solution paths that are valued and valid in the mathematics classroom. Listening to what students say, observing their actions, and analyzing their written work are all ways to gather	their actions, and analyzing their written work. Intentionally planning for how to elicit students' thinking during mathematics lessons is a key aspect of effective lesson planning (NCTM, 2017). In responsive mathematics classrooms, eliciting and using evidence of student thinking is central to informing the daily, in-the-moment instructional decisions of teachers. By carefully listening to and thoughtfully responding to student thinking, teachers position students' contributions as mathematically valuable and contributing to a broader collective understanding of the mathematical ideas at hand. Whose work gets selected and discussed during a lesson sends important messages to students about the solution paths that are valued and valid in the mathematics classroom. Listening to what students say, observing their actions, and analyzing their written work are all ways to gather	their actions, and analyzing their written work. Intentionally planning for how to elicit students' thinking during mathematics lessons is a key aspect of effective lesson planning (NCTM, 2017). In responsive mathematics classrooms, eliciting and using evidence of student thinking is central to informing the daily, in-the-moment instructional decisions of teachers. By carefully listening to and thoughtfully responding to student thinking, teachers position students' contributions as mathematically valuable and contributing to a broader collective understanding of the mathematical ideas at hand. Whose work gets selected and discussed during a lesson sends important messages to students about the solution paths that are valued and valid in the mathematics classroom. Listening to what students say, observing their actions, and analyzing their written work are all ways to gather

information on what students currently know and understand about key mathematical ideas. Therefore, eliciting and using evidence of student thinking is an aspect of formative assessment. Establishing a classroom culture in which mistakes are viewed as important reasoning opportunities promotes a wider range of students to engage in mathematical discussions with their peers and the teacher. Correct answers matter but not as a sole indicator of who is able to do mathematics. Eliciting student thinking and "engaging in mathematical discourse is essential for developing mathematical identity and should be recognized as an indicator of mathematical competence" (Barry, 2018). • (K-2 8A) Build a safe community of learners so that students can share successes, questions, mistakes, and challenges.	information on what students currently know and understand about key mathematical ideas. Therefore, eliciting and using evidence of student thinking is an aspect of formative assessment. Establishing a classroom culture in which mistakes are viewed as important reasoning opportunities promotes a wider range of students to engage in mathematical discussions with their peers and the teacher. Correct answers matter but not as a sole indicator of who is able to do mathematics. Eliciting student thinking and "engaging in mathematical discourse is essential for developing mathematical identity and should be recognized as an indicator of mathematical competence" (Barry, 2018). • (3-5 8A) Build a safe community of learners so that students can share successes, questions, mistakes, and challenges.	information on what students currently know and understand about key mathematical ideas. Therefore, eliciting and using evidence of student thinking is an aspect of formative assessment. Establishing a classroom culture in which mistakes are viewed as important reasoning opportunities promotes a wider range of students to engage in mathematical discussions with their peers and the teacher. Correct answers matter but not as a sole indicator of who is able to do mathematics. Eliciting student thinking and "engaging in mathematical discourse is essential for developing mathematical identity and should be recognized as an indicator of mathematical competence" (Barry, 2018). • (6-8 8A) Build a safe community of learners so that students can share successes, questions, mistakes, and challenges.	information on what students currently know and understand about key mathematical ideas. Therefore, eliciting and using evidence of student thinking is an aspect of formative assessment. Establishing a classroom culture in which mistakes are viewed as important reasoning opportunities promotes a wider range of students to engage in mathematical discussions with their peers and the teacher. Correct answers matter but not as a sole indicator of who is able to do mathematics. Eliciting student thinking and "engaging in mathematical discourse is essential for developing mathematical identity and should be recognized as an indicator of mathematical competence" (Barry, 2018). • (9-12 8A) Build a safe community of learners so that students can share successes, questions, mistakes, and challenges.

• (K-2 8B) Encourage, make public, and value student observations, conjectures, and initial reasoning. • (K-2 8C) Consider how and why to elicit mathematical thinking prior to a lesson. • (K-2 8D) Use tasks that have the potential to elicit student thinking. These include high-level tasks that may have multiple solutions or paths to the solution. • (K-2 8E) Hone the skills of noticing students' mathematical thinking, interpreting student understandings, and deciding how to respond to what is found. Professional learning communities can provide an important place for teachers to look at and	• (3-5 8B) Encourage, make public, and value student observations, conjectures, and initial reasonings. • (3-5 8C) Consider how and why to elicit mathematical thinking prior to a lesson. • (3-5 8D) Use tasks that have the potential to elicit student thinking. These include high-level tasks that may have multiple solutions or paths to the solution. • (3-5 8E) Hone the skills of noticing students' mathematical thinking, interpreting student understandings, and deciding how to respond to what is found. Professional learning communities can provide an important place for teachers to look at and	• (6-8 8B) Encourage, make public, and value student observations, conjectures, and initial reasonings. • (6-8 8C) Consider how and why to elicit mathematical thinking prior to a lesson. • (6-8 8D) Use tasks that have the potential to elicit student thinking. These include high-level tasks that may have multiple solutions or paths to the solution. • (6-8 8E) Hone the skills of noticing students' mathematical thinking, interpreting student understandings, and deciding how to respond to what is found. Professional learning communities can provide an important place for teachers to look at and analyze student	• (9-12 8B) Encourage, make public, and value student observations, conjectures, and initial reasonings. • (9-12 8C) Consider how and why to elicit mathematical thinking prior to a lesson. • (9-12 8D) Use tasks that have the potential to elicit student thinking. These include high-level tasks that may have multiple solutions or paths to the solution. • (9-12 8E) Hone the skills of noticing students' mathematical thinking, interpreting student understandings, and deciding how to respond to what is found. Professional learning communities can provide an important place for teachers to look at and

analyze student evidence alongside each other. • (K-2 8F) Ask permission before sharing student work with the class as a whole in order to honor the student's authorship and ownership of the work. After talking about a student's work, ask if what was shared accurately portrays the student's thinking. • (K-2 8G) Sequence student work for discussion in a variety of purposeful ways to shine light on the important thinking that each student brings to the mathematics. For example, student work can be shared by most common strategy to less common strategy, as a series of misconceptions, or as a	analyze student evidence alongside each other. • (3-5 8F) Ask permission before sharing student work with the class as a whole in order to honor the student's authorship and ownership of the work. After talking about a student's work, ask if what was shared accurately portrays the student's thinking. • (3-5 8G) Sequence student work for discussion in a variety of purposeful ways to shine light on the important thinking that each student brings to the mathematics. For example, student work can be shared by most common strategy to less common strategy, as a series of misconceptions, or as a	evidence alongside each other. • (6-8 8F) Ask permission before sharing student work with the class as a whole in order to honor the student's authorship and ownership of the work. After talking about a student's work, ask if what was shared accurately portrays the student's thinking. • (6-8 8G) Sequence student work for discussion in a variety of purposeful ways to shine light on the important thinking that each student brings to the mathematics. For example, student work can be shared by most common strategy to less common strategy, as a series of misconceptions, or as a	analyze student evidence alongside each other. • (9-12 8F) Ask permission before sharing student work with the class as a whole in order to honor the student's authorship and ownership of the work. After talking about a student's work, ask if what was shared accurately portrays the student's thinking. • (9-12 8G) Sequence student work for discussion in a variety of purposeful ways to shine light on the important thinking that each student brings to the mathematics. For example, student work can be shared by most common strategy to less common strategy, as a series of misconceptions, or as a

comparison and contrast of two work samples. ● (K-2 8H) Validate student mathematical thinking even when they do not get to a final answer. Position students' ideas as worthy of exploring, adjusting instruction to incorporate student thinking that supports learning. ● (K-2 8I) Use writing as a powerful way to elicit student thinking. Through writing, students clarify and organize their mathematical ideas. Writing can help students self-identify misunderstandings or questions that they have. The writing process continues the learning as students merge, connect, and consolidate their ideas. (Huinker and Bill	comparison and contrast of two work samples. ● (3-5 8H) Validate student mathematical thinking even when they do not get to a final answer. Position students' ideas as worthy of exploring, adjusting instruction to incorporate student thinking that supports learning. ● (3-5 8I) Use writing as a powerful way to elicit student thinking. Through writing, students clarify and organize their mathematical ideas. Writing can help students self-identify misunderstandings or questions that they have. The writing process continues the learning as students merge, connect, and consolidate their ideas. (Huinker and Bill 2017,	comparison and contrast of two work samples. ● (6-8 8H) Validate student mathematical thinking even when they do not get to a final answer. Position students' ideas as worthy of exploring, adjusting instruction to incorporate student thinking that supports learning. ● (6-8 8I) Use writing as a powerful way to elicit student thinking. Through writing, students clarify and organize their mathematical ideas. Writing can help students self-identify misunderstandings or questions that they have. The writing process continues the learning as students merge, connect, and consolidate their ideas (Huinker and Bill 2017,	comparison and contrast of two work samples. ● (9-12 8H) Validate student mathematical thinking even when they do not get to a final answer. Position students' ideas as worthy of exploring, adjusting instruction to incorporate student thinking that supports learning. ● (9-12 8I) Use writing as a powerful way to elicit student thinking. Through writing, students clarify and organize their mathematical ideas. Writing can help students self-identify misunderstandings or questions that they have. The writing process continues the learning as students merge, connect, and consolidate their ideas (Huinker and Bill 2017,

2017, p. 196)	p. 196)	p. 196)	p. 196)
9 (K-2) Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships (NCTM, 2014, p. 10). In responsive mathematics classrooms, teachers engage students in cognitively demanding tasks with the intent of giving students positive math experiences that encourage students to see themselves as competent problem solvers (Aguirre et al., 2014). Strengthening mathematics learning and cultivating positive mathematical identities begins with teachers knowing and believing in their students' abilities as competent and	**9 (3-5) Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships (NCTM, 2014, p. 10). In responsive mathematics classrooms, teachers engage students in cognitively demanding tasks with the intent of giving students positive math experiences that encourage students to see themselves as competent problem solvers (Aguirre et al., 2014). Strengthening mathematics learning and cultivating positive mathematical identities begins with teachers knowing and believing in their students' abilities as competent and	**9 (6-8) Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships (NCTM, 2014, p. 10). In responsive mathematics classrooms, teachers engage students in cognitively demanding tasks with the intent of giving students positive math experiences that encourage students to see themselves as competent problem solvers (Aguirre et al., 2014). Strengthening mathematics learning and cultivating positive mathematical identities begins with teachers knowing and believing in their students' abilities as competent and	**9 (9-12) Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships (NCTM, 2014, p. 10). In responsive mathematics classrooms, teachers engage students in cognitively demanding tasks with the intent of giving students positive math experiences that encourage students to see themselves as competent problem solvers (Aguirre et al., 2014). Strengthening mathematics learning and cultivating positive mathematical identities begins with teachers knowing and believing in their students' abilities as competent and

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anticipated needs, know when to use questions to focus student thinking and when to provide scaffolds while maintaining the integrity of the task. • (K-2 9E) Continually examine unconscious prejudices as to student abilities based on cultural grouping. A Hidden Bias Test is one way to measure unconscious, or automatic, biases. • (K-2 9F) Focus on the mathematical process rather than answers when providing feedback to students, including the risks students take during the process. For example, highlight the process, not the answer. Say, "You tried another strategy when you noticed this one didn't work!"	anticipated needs, know when to use questions to focus student thinking and when to provide scaffolds while maintaining the integrity of the task. • (3-5 9E) Continually examine unconscious prejudices as to student abilities based on cultural grouping. A Hidden Bias Test is one way to measure unconscious, or automatic, biases. • (3-5 9F) Focus on the mathematical process rather than answers when providing feedback to students, including the risks students take during the process. For example, highlight the process, not the answer. Say, "You tried another strategy when you noticed this one didn't work!"	anticipated needs, know when to use questions to focus student thinking and when to provide scaffolds while maintaining the integrity of the task. • (6-8 9E) Continually examine unconscious prejudices as to student abilities based on cultural grouping. A Hidden Bias Test is one way to measure unconscious, or automatic, biases. • (6-8 9F) Focus on the mathematical process rather than answers when providing feedback to students, including the risks students take during the process. For example, highlight the process, not the answer. Say, "You tried another strategy when you noticed this one didn't work!"	anticipated needs, know when to use questions to focus student thinking and when to provide scaffolds while maintaining the integrity of the task. • (9-12 9E) Continually examine unconscious prejudices as to student abilities based on cultural grouping. A Hidden Bias Test is one way to measure unconscious, or automatic, biases. • (9-12 9F) Focus on the mathematical process rather than answers when providing feedback to students, including the risks students take during the process. For example, highlight the process, not the answer. Say, "You tried another strategy when you noticed this one didn't work!"

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The Mathematics Teaching Framework (p. 7 of this Guide) is a reminder of the relationships among the NCTM Mathematical Practices. Depending on the goal for instruction, a teacher might select tasks that promote reasoning and problem solving or engage students in developing procedural fluency from conceptual understanding. Tasks that promote reasoning and problem solving provide the conceptual base on which fluency can be developed. Thus these two teaching practices strongly support one another.</td>
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• (K-2 10C) Plan for opportunities for students to engage in all levels of tasks, keeping in mind that experience with high-demand tasks in particular is critical in developing thinking, reasoning, and problem solving skills (Huinker and Bill 2017, p. 62.) • (K-2 10D) Connect relevant tasks to student interests. Connections can be made to home, school and community contexts. If tasks from current instructional materials are not relevant, adjustments are made. For example, while participating in a structured counting experience like Counting Collections, students take inventory of classroom supplies and fill their classroom's need to know "how many" through their use of mathematics.	• (3-5 10C) Plan for opportunities for students to engage in all levels of tasks, keeping in mind that experience with high-demand tasks in particular is critical in developing thinking, reasoning, and problem solving skill (Huinker and Bill 2017, p. 62). • (3-5 10D) Connect relevant tasks to student interests. Connections can be made to home, school and community contexts. If tasks from current instructional materials are not relevant, adjustments are made. For example, while preparing for a bake sale to raise money for a community cause, students solve real world problems using the four operations to organize the baking, packaging and selling of the baked goods.	• (6-8 10C) Plan for opportunities for students to engage in all levels of tasks, keeping in mind that experience with high-demand tasks in particular is critical in developing thinking, reasoning, and problem solving skills (Huinker and Bill 2017, p. 62). • (6-8 10D) Connect relevant tasks to student interests. Connections can be made to home, school and community contexts. If tasks from current instructional materials are not relevant, adjustments are made. For example, students use relevant data to interpret differences in shape, center and spread in the context of the data sets.	• (9-12 10C) Plan for opportunities for students to engage in all levels of tasks, keeping in mind that experience with high-demand tasks in particular is critical in developing thinking, reasoning, and problem solving skills (Huinker and Bill 2017, p. 62). • (9-12 10 D) Connect relevant tasks to student interests. Connections can be made to home, school and community contexts. If tasks from current instructional materials are not relevant, adjustments are made. For example, students use relevant data to interpret differences in shape, center and spread in the context of the data sets.

• (K-2 10E) Provide tasks that have multiple solutions or solution paths so that students have choice in how they engage in the task. Putting the decision of a pathway in students' hands empowers them to be mathematical problem solvers. For example, given an open number line with 0 and 42 already placed on the line, students are asked to approximately place 85, 21 and 31. They are also asked to explain their reasoning.	• (3-5 10E) Provide tasks that have multiple solutions or solution paths so that students have choice in how they engage in the task. Putting the decision of a pathway in students' hands empowers them to be mathematical problem solvers. For example, students are asked, "Name three other fractions that are close in size to ⅝ and explain your reasoning for each fraction."	• (6-8 10E) Provide tasks that have multiple solutions or solution paths so that students have choice in how they engage in the task.. Putting the decision of a pathway in students' hands empowers them to be mathematical problem solvers. For example, students are given a set of data about how many ducklings are in each family of ducks in the local lake. Students are asked to find the number of ducklings in a typical duck family (Smith, Steele, and Raith 2017, p. 34).	• (9-12 10E) Provide tasks that have multiple solutions or solution paths so that students have choice in how they engage in the task. Putting the decision of a pathway in students' hands empowers them to be mathematical problem solvers. For example, students are asked to determine the temperature of a cooling cup of coffee at a specific time. Students might create an algebraic model, use technology to create a regression, or use an accepted formula like Newton's Law of Cooling.
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lesson, and plan for ways to maximize accessibility and elicit mathematical discourse for all students. ● (K-2 10G) Plan for student use of math tools during engagement with the task. Tools are made available to students in a variety of ways. This may look like shelves with organized tools available to all students, personal bins of supplies, or table caddies. 2. *Guide student learning during the implementation of the math task(s).* ● (K-2 10H) Choose problems that students will feel a need to solve, or build a student need at the beginning of the lesson. ● (K-2 10I) Make explicit	lesson, and plan for ways to maximize accessibility and elicit mathematical discourse for all students. ● (3-5 10G) Plan for student use of math tools during engagement with the task. Tools are made available to students in a variety of ways. This may look like shelves with organized tools available to all students, personal bins of supplies, or table caddies. 2. *Guide student learning during the implementation of the math task(s).* ● (3-5 10H) Choose problems that students will feel a need to solve, or build a student need at the beginning of the lesson. ● (3-5 10I) Make explicit	lesson, and plan for ways to maximize accessibility and elicit mathematical discourse for all students. ● (6-8 10G) Plan for student use of math tools during engagement with the task. Tools are made available to students in a variety of ways. This may look like shelves with organized tools available to all students, personal bins of supplies, or table caddies. 2. *Guide student learning during the implementation of the math task(s).* ● (6-8 10H) Choose problems that students will feel a need to solve, or build a student need at the beginning of the lesson. ● (6-8 10I) Make explicit	lesson, and plan for ways to maximize accessibility and elicit mathematical discourse for all students. ● (9-12 10G) Plan for student use of math tools during engagement with the task. Tools are made available to students in a variety of ways. This may look like shelves with organized tools available to all students, personal bins of supplies, or table caddies. 2. *Guide student learning during the implementation of the math task(s).* ● (9-12 10H) Choose problems that students will feel a need to solve, or build a student need at the beginning of the lesson. ● (9-12 10I) Make explicit

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procedure they are using. They are able to demonstrate a flexible use of strategies and methods and can justify their choice of procedure for specific types of problems. Over time, this strong conceptual understanding leads to the meaningful use of general methods and algorithms. Building procedural fluency takes time and in no way identifies or implies memorization as a way to reach fluency. In fact, procedural fluency extends from a deep understanding of how numbers and operation work together and is grounded in an understanding of the meanings and properties of the operations (e.g., commutative and associative). If fluency is achieved through an emphasis on memorization, this may convey the message that mathematics is not about knowing and doing, but about memorizing. In responsive mathematics classrooms, teachers build procedural fluency from	procedure they are using. They are able to demonstrate a flexible use of strategies and methods and can justify their choice of procedure for specific types of problems. Over time, this strong conceptual understanding leads to the meaningful use of general methods and algorithms. Building procedural fluency takes time and in no way identifies or implies memorization as a way to reach fluency. In fact, procedural fluency extends from a deep understanding of how numbers and operation work together and is grounded in an understanding of the meanings and properties of the operations (e.g., commutative and associative). If fluency is achieved through an emphasis on memorization, this may convey the message that mathematics is not about knowing and doing, but about memorizing. In responsive mathematics classrooms, teachers build procedural fluency from	procedure they are using. They are able to demonstrate a flexible use of strategies and methods and can justify their choice of procedure for specific types of problems. Over time, this strong conceptual understanding leads to the meaningful use of general methods and algorithms. Building procedural fluency takes time and in no way identifies or implies memorization as a way to reach fluency. In fact, procedural fluency extends from a deep understanding of how numbers and operation work together and is grounded in an understanding of the meanings and properties of the operations (e.g., commutative and associative). If fluency is achieved through an emphasis on memorization, this may convey the message that mathematics is not about knowing and doing, but about memorizing. In responsive mathematics classrooms, teachers build procedural fluency from	procedure they are using. They are able to demonstrate a flexible use of strategies and methods and can justify their choice of procedure for specific types of problems. Over time, this strong conceptual understanding leads to the meaningful use of general methods and algorithms. Building procedural fluency takes time and in no way identifies or implies memorization as a way to reach fluency. In fact, procedural fluency extends from a deep understanding of how numbers and operation work together and is grounded in an understanding of the meanings and properties of the operations (e.g., commutative and associative). If fluency is achieved through an emphasis on memorization, this may convey the message that mathematics is not about knowing and doing, but about memorizing. In responsive mathematics classrooms, teachers build procedural fluency from

conceptual understanding helping students make sense of the mathematics, and in turn supporting the development of a positive disposition toward mathematics (NCTM, 2018). Learning mathematical procedures and facts with understanding provides students with a wider range of options for entering a task and building understanding supports the development of students' agency and ownership of mathematical knowledge. Ensuring students understand the mathematics behind the procedures they are employing helps them see mathematics as coherent and connected and not solely grounded in rote memorization and speed. Moreover, mathematics instruction that "focuses solely on remembering and applying procedures advantages students who are strong in memorization skills and disadvantages students who are not" (NCTM, 2018, p. 31). When teachers intentionally build procedural fluency from	conceptual understanding helping students make sense of the mathematics, and in turn supporting the development of a positive disposition toward mathematics (NCTM, 2018). Learning mathematical procedures and facts with understanding provides students with a wider range of options for entering a task and building understanding supports the development of students' agency and ownership of mathematical knowledge. Ensuring students understand the mathematics behind the procedures they are employing helps them see mathematics as coherent and connected and not solely grounded in rote memorization and speed. Moreover, mathematics instruction that "focuses solely on remembering and applying procedures advantages students who are strong in memorization skills and disadvantages students who are not" (NCTM, 2018, p. 31). When teachers intentionally build procedural fluency from	conceptual understanding helping students make sense of the mathematics, and in turn supporting the development of a positive disposition toward mathematics (NCTM, 2018). Learning mathematical procedures and facts with understanding provides students with a wider range of options for entering a task and building understanding supports the development of students' agency and ownership of mathematical knowledge. Ensuring students understand the mathematics behind the procedures they are employing helps them see mathematics as coherent and connected and not solely grounded in rote memorization and speed. Moreover, mathematics instruction that "focuses solely on remembering and applying procedures advantages students who are strong in memorization skills and disadvantages students who are not" (NCTM, 2018, p. 31). When teachers intentionally build procedural fluency from	conceptual understanding helping students make sense of the mathematics, and in turn supporting the development of a positive disposition toward mathematics (NCTM, 2018). Learning mathematical procedures and facts with understanding provides students with a wider range of options for entering a task and building understanding supports the development of students' agency and ownership of mathematical knowledge. Ensuring students understand the mathematics behind the procedures they are employing helps them see mathematics as coherent and connected and not solely grounded in rote memorization and speed. Moreover, mathematics instruction that "focuses solely on remembering and applying procedures advantages students who are strong in memorization skills and disadvantages students who are not" (NCTM, 2018, p. 31). When teachers intentionally build procedural fluency from

conceptual understanding they reduce mathematical anxiety in their students and help position them as confident knowers and doers of mathematics (Ashcraft, 2002; Ramirez, Gunderson, Levine, & Beilock, 2013). 1. *Support the development of deep conceptual understanding of mathematical ideas, relationships, and operations as a foundation for all mathematics.* • (K-2 11A) Encourage students to use a variety of strategies when solving problems. • (K-2 11B) Use intentionally-focused problem strings to promote critical strategies through finding patterns and reasoning. For example, the following string of expressions can focus a discussion around a making ten strategy:	conceptual understanding they reduce mathematical anxiety in their students and help position them as confident knowers and doers of mathematics (Ashcraft, 2002; Ramirez, Gunderson, Levine, & Beilock, 2013). 1. *Support the development of deep conceptual understanding of mathematical ideas, relationships, and operations as a foundation for all mathematics.* • (3-5 11A) Encourage students to use a variety of strategies when solving problems. • (3-5 11B) Use intentionally-focused problem strings to promote critical strategies through finding patterns and reasoning. For example, the following expressions can focus a discussion around doubling and the	conceptual understanding they reduce mathematical anxiety in their students and help position them as confident knowers and doers of mathematics (Ashcraft, 2002; Ramirez, Gunderson, Levine, & Beilock, 2013). 1. *Support the development of deep conceptual understanding of mathematical ideas, relationships, and operations as a foundation for all mathematics.* • (6-8 11A) Encourage students to use a variety of strategies when solving problems. • (6-8 11B) Use intentionally-focused problem strings to promote critical strategies through finding patterns and reasoning. For example, the following expressions highlight the idea of a constant	conceptual understanding they reduce mathematical anxiety in their students and help position them as confident knowers and doers of mathematics (Ashcraft, 2002; Ramirez, Gunderson, Levine, & Beilock, 2013). 1. *Support the development of deep conceptual understanding of mathematical ideas, relationships, and operations as a foundation for all mathematics.* • (9-12 11A) Encourage students to use a variety of strategies when solving problems. • (9-12 11B) Use intentionally-focused problem strings to promote critical strategies through finding patterns and reasoning. For example, the following problem string of equations highlights the different

4+4, 5+5, 6+4, 6+5, 7+3, 7+4, and 9+5. The progression of expressions begins with low doubles that establish making ten.. The sequence then progresses through expressions that can be solved using the previous problems and promote a making ten strategy. • (K-2 11C) Expect students to explain and justify their thinking and allow time and space for those explanations to be formed and communicated. Doing math is not simply about getting an answer. • (K-2 11D) Support students in connecting new ideas to previous learning. For example, 5 + 4 can help students think about 50 + 40.	distributive property: 2 x 23 4 x 23 8 x 23 10 x 23 18 x 23 • (3-5 11C) Expect students to explain and justify their thinking and allow time and space for those explanations to be formed and communicated. Doing math is not simply about getting an answer. • (3-5 11D) Support students in connecting new ideas to previous learning. For example, 5 + 4 can help students think about 500 + 400.	ratio to support the division of rational numbers: 32 ÷ 4 320 ÷ 40 3200 ÷ 400 3.2 ÷ .4 5.6 ÷ .8 • (6-8 11C) Expect students to explain and justify their thinking and allow time and space for those explanations to be formed and communicated. Doing math is not simply about getting an answer. • (6-8 11D) Support students in connecting new ideas to previous learning. For example, explore how repeated multiplication (2x2x2x2) can be	structural types of quadratic equations: $X^2 - 4 = 0$ $(x-4)^2 - 6 = 0$ $X^2 - 4x = 0$ $X^2 - 4x - 5 = 0$ $X^2 - 4x - 6 = 0$ • (9-12 11C) Expect students to explain and justify their thinking and allow time and space for those explanations to be formed and communicated. Doing math is not simply about getting an answer. • (9-12 11D) Support students in connecting new ideas to previous learning. For example, connect rigid transformations to triangle congruence.

2. *Teachers guide students in developing conceptual understanding with an eye toward the eventual connections to procedures.* (Huinker and Bill 2017, p. 68) • (K-2 11E) Promote fluency through meaningful instruction based on student work with quantities, number relationships, and number sense. A meaningful approach toward fluency ensures this work recognizes and validates students' thinking and strategies without compromising or negating their conceptual understanding. • (K-2 11F) Recognize that it takes time to develop fluency and that students move at	2. *Teachers guide students in developing conceptual understanding with an eye toward the eventual connections to procedures.* (Huinker and Bill 2017, p. 68) • (3-5 11E) Promote fluency through meaningful instruction based on student work with quantities, number relationships, and number sense. A meaningful approach toward fluency ensures this work recognizes and validates students' thinking and strategies without compromising or negating their conceptual understanding. • (3-5 11F) Recognize that it takes time to develop fluency and that students move at	written in exponential form (2^4). 2. *Teachers guide students in developing conceptual understanding with an eye toward the eventual connections to procedures.* (Huinker and Bill 2017, p. 68) • (6-8 11E) Promote fluency through meaningful instruction based on student work with quantities, number relationships, and number sense. A meaningful approach toward fluency ensures this work recognizes and validates students' thinking and strategies without compromising or negating their conceptual understanding. • (6-8 11F) Recognize that it takes time to develop fluency and that students move at	2. *Teachers guide students in developing conceptual understanding with an eye toward the eventual connections to procedures.* (Huinker and Bill 2017, p. 68) • (9-12 11E) Consider continuing conversations around fluency and number sense when the situation arises. In a geometric sequence with a common ratio of 1.5 finding the next term provides an opportunity for students to engage in flexible strategies, in this case, using the distributive property.

their own pace. As students move toward procedural fluency, teachers support students with intentional practice and recognize that fluency means that students can solve problems flexibly, accurately, and efficiently. • (K-2 11G) Help students move among concrete models, representational drawings, and abstract ways of engaging with mathematical concepts. • (K-2 11H) Promote the writing of symbolic equations to connect with contextual situations and ask learners to write contextual situations that stem from symbolic equations. • (K-2 11I) Connect student-generated strategies to more	their own pace. As students move toward procedural fluency, teachers support students with intentional practice and recognize that fluency means that students can solve problems flexibly, accurately, and efficiently. • (3-5 11G) Help students move among concrete models, representational drawings, and abstract ways of engaging with mathematical concepts. • (3-5 11H) Promote the writing of symbolic equations to connect with contextual situations and ask learners to write contextual situations that stem from symbolic equations. • (3-5 11I) Connect student-generated strategies to more	their own pace. As students move toward procedural fluency, teachers support students with intentional practice and recognize that fluency means that students can solve problems flexibly, accurately, and efficiently. • (6-8 11G) Help students move among concrete models, representational drawings, and abstract ways of engaging with mathematical concepts. • (6-8 11H) Promote the writing of symbolic equations to connect with contextual situations and ask learners to write contextual situations that stem from symbolic equations. • (6-8 11I) Connect student-generated strategies to more	• (9-12 11F) Help students move among concrete models, representational drawings, and abstract ways of engaging with mathematical concepts. • (9-12 11G) Promote the writing of symbolic equations to connect with contextual situations and ask learners to write contextual situations that stem from symbolic equations. • (9-12 11H) Connect student-generated strategies to more

efficient procedures during instruction to help students identify the underlying mathematics that is many times shrouded in generalized strategies. • (K-2 11J) Pair procedures with visual models offering students another critical way to see the mathematics in the procedures they are learning. For example, layered place value cards are used to solve a two-digit addition problem along with a written representation using tens and ones. • (K-2 11K) Intentionally use mathematical games and activities to promote conceptual understanding or procedural fluency. For example, a game can be played slowly, with time for thoughtful explanations of the	efficient procedures during instruction to help students identify the underlying mathematics that is many times shrouded in generalized strategies. • (3-5 11J) Pair procedures with visual models offering students another critical way to see the mathematics in the procedures they are learning. For example, area models are used when solving multi-digit multiplication problems. • (3-5 11K) Intentionally use mathematical games and activities to promote conceptual understanding or procedural fluency. For example, a game can be played slowly, with time for thoughtful explanations of the	efficient procedures during instruction to help students identify the underlying mathematics that is many times shrouded in generalized strategies. • (6-8 11J) Pair procedures with visual models offering students another critical way to see the mathematics in the procedures they are learning. For example, tape diagrams are used when dividing with fractions. • (6-8 11K) Intentionally use mathematical games and activities to promote conceptual understanding or procedural fluency. For example, a game can be played slowly, with time for thoughtful explanations of the	efficient procedures during instruction to help students identify the underlying mathematics that is many times shrouded in generalized strategies. • (9-12 11I)Pair procedures with visual models offering students another critical way to see the mathematics in the procedures they are learning. For example, using a right triangle to pair the visual model of the Pythagorean Theorem to the distance formula.

mathematical thinking that accompanies each turn in order to promote conceptual understanding and reasoning. Only a few turns may be taken due to the deep thinking associated with the game at that time. Later, that same game can be played from start to finish. Less discourse is necessary during the turns as conceptual understanding moves toward procedural fluency.	mathematical thinking that accompanies each turn in order to promote conceptual understanding and reasoning. Only a few turns may be taken due to the deep thinking associated with the game at that time. Later, that same game can be played from start to finish. Less discourse is necessary during the turns as conceptual understanding moves toward procedural fluency.	mathematical thinking that accompanies each turn in order to promote conceptual understanding and reasoning. Only a few turns may be taken due to the deep thinking associated with the game at that time. Later, that same game can be played from start to finish. Less discourse is necessary during the turns as conceptual understanding moves toward procedural fluency.	
• (K-2 11L) Ensure that all students engage in challenging mathematics while developing procedural fluency. For example, addition and subtraction fact fluency is not a prerequisite for challenging mathematics involving place value thinking.	• (3-5 11L) Ensure that all students engage in challenging mathematics while developing procedural fluency. For example, addition and subtraction fact fluency is not a prerequisite for challenging mathematics involving place value thinking.	• (6-8 11L) Ensure that all students engage in challenging mathematics while developing procedural fluency. For example, mastery of fraction operations is not a prerequisite for challenging mathematics involving solving equations.	• (9-12 11J) Ensure that all students engage in challenging mathematics while developing procedural fluency. For example, using abstract symbolic manipulation to find key features of a graph is not a prerequisite for modeling applications that involve analyzing graphs.

(K-2 11M) Measure progress for both conceptual understanding and procedural fluency. (K-2 11N) Intentionally use the analysis of solutions as the learning task.	(3-5 11M) Measure progress for both conceptual understanding and procedural fluency. (3-5 11N) Intentionally use the analysis of solutions as the learning task.	(6-8 11M) Measure progress for both conceptual understanding and procedural fluency. (6-8 11N) Intentionally use the analysis of solutions as the learning task.	(9-12 11K) Measure progress for both conceptual understanding and procedural fluency. (9-12 11L) Intentionally use the analysis of solutions as the learning task.
Collaboration, Discourse and Reflection: Effective mathematics learning leverages collaboration, discourse and reflection to engage students. The heart of any lesson is mathematical discourse. This is represented by the large rectangle in The Mathematics Teaching Framework (p. 7 of this Guide). As students are provided with opportunities to communicate their mathematical understanding to others, either orally, visually, or in writing, they are engaged in discourse. Discourse is mediated by the use of the four mathematics teaching practices situated within the discourse rectangle in the Framework, that is, pose purposeful questions, use and connect mathematical representations, elicit and use evidence of students' thinking, and support productive struggle. Together, these teaching practices interact (in service of the goals and reliant on the tasks) to engage students in meaningful discourse (NCTM, 2017). To ensure these practices are both effective and equitable, classroom environments must be intentionally structured to ensure each and every student is supported to actively contribute their mathematical ideas and reasoning and that all student voices are heard and valued.			
12 (K-2) Facilitate meaningful mathematical discourse. "Effective mathematics teaching engages students in discourse to advance the	**12 (3-5) Facilitate meaningful mathematical discourse.** "Effective mathematics teaching engages students in discourse to advance the	**12 (6-8) Facilitate meaningful mathematical discourse.** "Effective mathematics teaching engages students in discourse to advance the	**12 (9-12) Facilitate meaningful mathematical discourse.** "Effective mathematics teaching engages students in discourse to advance the

mathematical learning of the whole class” (NCTM, 2014, p. 29). Mathematical discourse is viewed as central to meaningful learning of mathematics and includes all forms of communication students use to share their ideas, including verbal, visual (e.g., drawings and gestures), and written formats. Discourse provides a platform for students to share ideas and clarify understandings, construct mathematical arguments and justifications, develop mathematical vocabulary in meaningful and contextually rich settings, and learn to see ideas from other perspectives (NCTM, 2000). Teachers support and encourage meaningful discourse as they create and sustain a classroom environment and establish discourse norms that prioritize student to student collaboration and justification of solution strategies. In responsive mathematics classrooms, students’ approaches and reasoning are	mathematical learning of the whole class” (NCTM, 2014, p. 29). Mathematical discourse is viewed as central to meaningful learning of mathematics and includes all forms of communication students use to share their ideas, including verbal, visual (e.g., drawings and gestures), and written formats. Discourse provides a platform for students to share ideas and clarify understandings, construct mathematical arguments and justifications, develop mathematical vocabulary in meaningful and contextually rich settings, and learn to see ideas from other perspectives (NCTM, 2000). Teachers support and encourage meaningful discourse as they create and sustain a classroom environment and establish discourse norms that prioritize student to student collaboration and justification of solution strategies. In responsive mathematics classrooms, students’ approaches and reasoning are	mathematical learning of the whole class” (NCTM, 2014, p. 29). Mathematical discourse is viewed as central to meaningful learning of mathematics and includes all forms of communication students use to share their ideas, including verbal, visual (e.g., drawings and gestures), and written formats. Discourse provides a platform for students to share ideas and clarify understandings, construct mathematical arguments and justifications, develop mathematical vocabulary in meaningful and contextually rich settings, and learn to see ideas from other perspectives (NCTM, 2000). Teachers support and encourage meaningful discourse as they create and sustain a classroom environment and establish discourse norms that prioritize student to student collaboration and justification of solution strategies. In responsive mathematics classrooms, students’ approaches and reasoning are	mathematical learning of the whole class” (NCTM, 2014, p. 29). Mathematical discourse is viewed as central to meaningful learning of mathematics and includes all forms of communication students use to share their ideas, including verbal, visual (e.g., drawings and gestures), and written formats. Discourse provides a platform for students to share ideas and clarify understandings, construct mathematical arguments and justifications, develop mathematical vocabulary in meaningful and contextually rich settings, and learn to see ideas from other perspectives (NCTM, 2000). Teachers support and encourage meaningful discourse as they create and sustain a classroom environment and establish discourse norms that prioritize student to student collaboration and justification of solution strategies. In responsive mathematics classrooms, students’ approaches and reasoning are

the starting point for mathematical discussions. Responsive teachers utilize mathematical discourse as a way to elicit students' ideas and strategies providing stronger access to the mathematics for each and every student, including those who have immediate ideas or solution strategies, those who need more time to grapple with ideas and develop a reasonable approach, as well as those who have faulty reasoning or misconceptions (Huinker & Bill, 2017). Effective classroom discourse engages teachers in providing opportunity and time for students to analyze, compare, justify, and prove their own solution strategies and representations, and to engage with the thinking of others. When discourse is facilitated in collaborative and supportive environments each and every student knows and believes that their explanations, ideas, and solution strategies are valued and worthy of being heard (Berry, 2018). Therefore, in	the starting point for mathematical discussions. Responsive teachers utilize mathematical discourse as a way to elicit students' ideas and strategies providing stronger access to the mathematics for each and every student, including those who have immediate ideas or solution strategies, those who need more time to grapple with ideas and develop a reasonable approach, as well as those who have faulty reasoning or misconceptions (Huinker & Bill, 2017). Effective classroom discourse engages teachers in providing opportunity and time for students to analyze, compare, justify, and prove their own solution strategies and representations, and to engage with the thinking of others. When discourse is facilitated in collaborative and supportive environments each and every student knows and believes that their explanations, ideas, and solution strategies are valued and worthy of being heard (Berry, 2018). Therefore, in	the starting point for mathematical discussions. Responsive teachers utilize mathematical discourse as a way to elicit students' ideas and strategies providing stronger access to the mathematics for each and every student, including those who have immediate ideas or solution strategies, those who need more time to grapple with ideas and develop a reasonable approach, as well as those who have faulty reasoning or misconceptions (Huinker & Bill, 2017). Effective classroom discourse engages teachers in providing opportunity and time for students to analyze, compare, justify, and prove their own solution strategies and representations, and to engage with the thinking of others. When discourse is facilitated in collaborative and supportive environments each and every student knows and believes that their explanations, ideas, and solution strategies are valued and worthy of being heard (Berry, 2018). Therefore, in	the starting point for mathematical discussions. Responsive teachers utilize mathematical discourse as a way to elicit students' ideas and strategies providing stronger access to the mathematics for each and every student, including those who have immediate ideas or solution strategies, those who need more time to grapple with ideas and develop a reasonable approach, as well as those who have faulty reasoning or misconceptions (Huinker & Bill, 2017). Effective classroom discourse engages teachers in providing opportunity and time for students to analyze, compare, justify, and prove their own solution strategies and representations, and to engage with the thinking of others. When discourse is facilitated in collaborative and supportive environments each and every student knows and believes that their explanations, ideas, and solution strategies are valued and worthy of being heard (Berry, 2018). Therefore, in

responsive mathematics classrooms teachers purposefully ensure diversity and equity in student voice and ideas. • (K-2 12A) Identify a learning goal to anchor the mathematical discussion. • (K-2 12B) Plan for accountability during classroom discourse. Routines are established for techniques like Notice and Wonder, number talks, gallery walks and other forms of student discourse that position all students as valuable resources. • (K-2 12C) Anticipate accurate and inaccurate solution strategies that students might use in order to prepare for opportunities for discourse. (Huinker and Bill 2017, p. 160)	responsive mathematics classrooms teachers purposefully ensure diversity and equity in student voice and ideas. • (3-5 12A) Identify a learning goal to anchor the mathematical discussion. • (3-5 12B) Plan for accountability during classroom discourse. Routines are established for techniques like Notice and Wonder, number talks, gallery walks and other forms of student discourse that position all students as valuable resources. • (3-5 12C) Anticipate accurate and inaccurate solution strategies that students might use in order to prepare for opportunities for discourse. (Huinker and Bill 2017, p. 160)	responsive mathematics classrooms teachers purposefully ensure diversity and equity in student voice and ideas. • (6-8 12A) Identify a learning goal to anchor the mathematical discussion. • (6-8 12B) Plan for accountability during classroom discourse. Routines are established for techniques like Notice and Wonder, number talks, gallery walks and other forms of student discourse that position all students as valuable resources. • (6-8 12C) Anticipate accurate and inaccurate solution strategies that students might use in order to prepare for opportunities for discourse. (Smith, Steele, and Raith 2017, p. 142)	responsive mathematics classrooms teachers purposefully ensure diversity and equity in student voice and ideas. • (9-12 12A) Identify a learning goal to anchor the mathematical discussion. • (9-12 12B) Plan for accountability during classroom discourse. Routines are established for techniques like Notice and Wonder, number talks, gallery walks and other forms of student discourse that position all students as valuable resources. • (9-12 12C) Anticipate accurate and inaccurate solution strategies that students might use in order to prepare for opportunities for discourse. (Smith, Steele, and Raith 2017, p. 142)

• (K-2 12D) Use mathematically accurate language while being careful to nurture discourse by beginning with and scaffolding from the language generated by students. • (K-2 12E) Provide opportunities for students to talk with, respond to, and question one another in ways that support and advance learning for the whole class. Students come to mathematical understandings without relying on the teacher as the authority. • (K-2 12F) Provide language support to allow all students to engage in mathematical discourse. Possible supports include the use of object visuals, sentence frames, sentence stems, word	• (3-5 12D) Use mathematically accurate language while being careful to nurture discourse by beginning with and scaffolding from the language generated by students. • (3-5 12E) Provide opportunities for students to talk with, respond to, and question one another in ways that support and advance learning for the whole class. Students come to mathematical understandings without relying on the teacher as the authority. • (3-5 12F) Provide language support to allow all students to engage in mathematical discourse. Possible supports include the use of object visuals, sentence frames, sentence stems, word	• (6-8 12D) Use mathematically accurate language while being careful to nurture discourse by beginning with and scaffolding from the language generated by students. • (6-8 12E) Provide opportunities for students to talk with, respond to, and question one another in ways that support and advance learning for the whole class. Students come to mathematical understandings without relying on the teacher as the authority. • (6-8 12F) Provide language support to allow all students to engage in mathematical discourse. Possible supports include the use of object visuals, sentence frames, sentence stems, word	• (9-12 12D) Use mathematically accurate language while being careful to nurture discourse by beginning with and scaffolding from the language generated by students. • (9-12 12E) Provide opportunities for students to talk with, respond to, and question one another in ways that support and advance learning for the whole class. Students come to mathematical understandings without relying on the teacher as the authority. • (9-12 12F) Provide language support to allow all students to engage in mathematical discourse. Possible supports include the use of object visuals, sentence frames, sentence stems, word

banks, modeling of language, familiar contexts, and the use of cognates. • (K-2 12G) Ensure that all students have access to linguistic resources, considering their English mastery. Refer to WIDA's English Language Development standards, specifically standard 3 Language of Mathematics, to make connections between language development and math content. • (K-2 12H) Use classroom discourse to position all students as competent and capable. Be intentional about ensuring there is diversity in voice, and that every student has a voice. • (K-2 12I) Connect visual models to the	banks, modeling of language, familiar contexts, and the use of cognates. • (3-5 12G) Ensure that all students have access to linguistic resources, considering their English mastery. Refer to WIDA's English Language Development standards, specifically standard 3 Language of Mathematics, to make connections between language development and math content. • (3-5 12H) Use classroom discourse to position all students as competent and capable. Be intentional about ensuring there is diversity in voice, and that every student has a voice. • (3-5 12I) Connect visual models to the conversations that	banks, modeling of language, familiar contexts, and the use of cognates. • (6-8 12G) Ensure that all students have access to linguistic resources, considering their English mastery. Refer to WIDA's English Language Development standards, specifically standard 3 Language of Mathematics, to make connections between language development and math content. • (6-8 12H) Use classroom discourse to position all students as competent and capable. Be intentional about ensuring there is diversity in voice, and that every student has a voice. • (6-8 12I) Connect visual models to the conversations that	banks, modeling of language, familiar contexts, and the use of cognates. • (9-12 12G) Ensure that all students have access to linguistic resources, considering their English mastery. Refer to WIDA's English Language Development standards, specifically standard 3 Language of Mathematics, to make connections between language development and math content. • (9-12 12H) Use classroom discourse to position all students as competent and capable. Be intentional about ensuring there is diversity in voice, and that every student has a voice. • (9-12 I) Connect visual models to the conversations that

conversations that support the mathematics. For example, while choral counting forward by ten, numerals are written in rows and columns that highlight the patterns and place value found in the number sequence. • (K-2 12J) Allow multiple and varied opportunities for discourse which appreciate different cultural speaking norms. For example: ◦ Talk moves like wait-time, revoicing, repeating, rephrasing, adding on, etc. ◦ Turn and talk ◦ Written response ◦ Think-Pair-Share	support the mathematics. For example, while having conversations about the comparison of fractions, bar models and number lines are used to illustrate the magnitude of the fractions and their relationship to benchmark fractions and each other. • (3-5 12J) Allow multiple and varied opportunities for discourse which appreciate different cultural speaking norms. For example: ◦ Talk moves like wait-time, revoicing, repeating, rephrasing, and adding on, etc. ◦ Turn and talk ◦ Written response ◦ Think-Pair-Share ◦ Gallery Walk to	support the mathematics. Students can use algebra tiles to illustrate the distributive property, for example, $3(2x+2) = 6x+6$. • (6-8 12J) Allow multiple and varied opportunities for discourse which appreciate different cultural speaking norms. For example: ◦ Talk moves like wait-time, revoicing, repeating, rephrasing, and adding on, etc. ◦ Turn and talk ◦ Written response ◦ Think-Pair-Share ◦ Gallery Walk to	support the mathematics. Students can use algebra tiles to illustrate the distributive property, for example, $3(2x+2) = 6x+6$. • (9-12 12J) Allow multiple and varied opportunities for discourse which appreciate different cultural speaking norms. For example: ◦ Talk moves like wait-time, revoicing, repeating, rephrasing, and adding on, etc. ◦ Turn and talk ◦ Written response ◦ Think-Pair-Share ◦ Gallery Walk to

• Gallery Walk to analyze the work of others • Notice and Wonder • Strategic student groupings: whole group, small group, partner, individual, or interview • (K-2 12K) Structure conversations to help students make mathematical connections and apply precise mathematical language. For example, the teacher monitors students while working and purposefully chooses three students to share their group's thinking during the class reflection. These three students are selected and ordered specifically to explain and justify the underlying	analyze the work of others • Notice and Wonder • Strategic student groupings: whole group, small group, partner, individual, or interview • (3-5 12K) Structure conversations to help students make mathematical connections and apply precise mathematical language. For example, the teacher monitors students while working and purposefully chooses three students to share their group's thinking during the class reflection. These three students are selected and ordered specifically to explain and justify the underlying mathematical ideas.	analyze the work of others • Notice and Wonder • Strategic student groupings: whole group, small group, partner, individual, or interview • (6-8 12K) Structure conversations to help students make mathematical connections and apply precise mathematical language. For example, the teacher monitors students while working and purposefully chooses three students to share their group's thinking during the class reflection. These three students are selected and ordered specifically to explain and justify the underlying mathematical ideas.	analyze the work of others • Notice and Wonder • Strategic student groupings: whole group, small group, partner, individual, or interview • (9-12 12K) Structure conversations to help students make mathematical connections and apply precise mathematical language. For example, the teacher monitors students while working and purposefully chooses three students to share their group's thinking during the class reflection. These three students are selected and ordered specifically to explain and justify the underlying mathematical ideas.

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but also develops a strong mathematical identity (Aguirre et al., 2013). Purposeful questions reveal insight into students' understanding and strategies and orient students to each other's reasoning. Teacher questioning and positioning of students and student responses influence how students view themselves as members of the mathematics learning community in the classroom. When teachers pose purposeful questions and then listen in order to understand students' thinking they signal to their students that their thinking is valued. Teachers carefully consider what questions to ask their students and they are also conscious of which students are given voice and authority in those mathematical conversations. Teachers follow up on the students' responses in such a way that supports the students' development of a positive mathematical identity and sense of agency as a thinker and doer of mathematics.	but also develops a strong mathematical identity (Aguirre et al., 2013). Purposeful questions reveal insight into students' understanding and strategies and orient students to each other's reasoning. Teacher questioning and positioning of students and student responses influence how students view themselves as members of the mathematics learning community in the classroom. When teachers pose purposeful questions and then listen in order to understand students' thinking they signal to their students that their thinking is valued. Teachers carefully consider what questions to ask their students and they are also conscious of which students are given voice and authority in those mathematical conversations. Teachers follow up on the students' responses in such a way that supports the students' development of a positive mathematical identity and sense of agency as a thinker and doer of mathematics.	but also develops a strong mathematical identity (Aguirre et al., 2013). Purposeful questions reveal insight into students' understanding and strategies and orient students to each other's reasoning. Teacher questioning and positioning of students and student responses influence how students view themselves as members of the mathematics learning community in the classroom. When teachers pose purposeful questions and then listen in order to understand students' thinking they signal to their students that their thinking is valued. Teachers carefully consider what questions to ask their students and they are also conscious of which students are given voice and authority in those mathematical conversations. Teachers follow up on the students' responses in such a way that supports the students' development of a positive mathematical identity and sense of agency as a thinker and doer of mathematics.	but also develops a strong mathematical identity (Aguirre et al., 2013). Purposeful questions reveal insight into students' understanding and strategies and orient students to each other's reasoning. Teacher questioning and positioning of students and student responses influence how students view themselves as members of the mathematics learning community in the classroom. When teachers pose purposeful questions and then listen in order to understand students' thinking they signal to their students that their thinking is valued. Teachers carefully consider what questions to ask their students and they are also conscious of which students are given voice and authority in those mathematical conversations. Teachers follow up on the students' responses in such a way that supports the students' development of a positive mathematical identity and sense of agency as a thinker and doer of mathematics.

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