WISCONSIN ESSENTIAL ELEMENTS FOR

Mathematics

Wisconsin Department of Public Instruction

Jill K. Underly, PhD, State Superintendent

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Wisconsin Department of Public Instruction
125 South Webster Street

Madison, WI 53703

(608) 266-8960

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Representation matters. In finding a cover photo for these standards, DPI valued the following:

- The cover should include a photograph of children rather than a photograph of objects or an illustration.
- Children who use Essential Elements have significant cognitive disabilities. While not all significant cognitive disabilities are visible, it was important to us to show a child with significant cognitive disabilities engaged in learning.
- Children with significant cognitive disabilities can and should (to the extent determined by their IEP team) learn with their grade-level peers. For this reason, we looked for a photograph that showed students learning together.
- Multiple identities (such as race, gender, and disability) should be represented in the photo.
- Children in the photo should be engaged in the major thinking of the discipline. This eliminated photographs of children engaged in rote tasks (such as flashcards or copying equations).
- Emphasis on the representation of a wide variety of disabilities, not limited to learners with down syndrome and in wheelchairs, as well as diverse races and cultures.

In searching photos available open source and with payment, we were not able to find a photo that incorporated all of the values described. This led DPI to question how students with significant cognitive disabilities – especially those who are from under- served populations – are represented in images commonly used in publications.   What you see on the cover is the best option DPI could find.

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Foreword

On March 23, 2022, I formally adopted the Wisconsin Essential Elements for Mathematics. This revised set of alternate academic achievement standards provides a foundational framework that identifies what students with the most significant cognitive disabilities should know and be able to do in Mathematics.

Mathematics is an essential part of a comprehensive PK-12 education for all students. Wisconsin students learn to use mathematics to understand and empower themselves and their worlds. The knowledge, skills, and habits of mind gained through mathematics education in Wisconsin schools support the Wisconsin Department of Public Instruction's vision of helping all students graduate college, career, and community ready. The knowledge and skills described provide a framework for parents, educators, school personnel, and community members to support their work in teaching students with most significant cognitive disabilities the mathematical skills necessary to succeed in life after graduation.

The Wisconsin Essential Elements for Mathematics were reviewed and revised by a group of Wisconsin educators and stakeholders who shared their expertise in mathematics and special education, as well as their expertise in teaching students in kindergarten through grade 12. As part of Wisconsin's Academic Standards review and revision process, the public and legislature provided feedback for the writing committee to consider.

The Wisconsin Department of Public Instruction will continue to build on this work to support implementation of the standards with resources for the field. I am excited to share the Wisconsin Essential Elements for Mathematics, which aim to build mathematics skills, knowledge, and engagement opportunities for all Wisconsin students.

Dr. Jill Underly

State Superintendent

Acknowledgements

The Wisconsin Department of Public Instruction (DPI) wishes to acknowledge the ongoing work, commitment, and various contributions of individuals to revise our state's alternate academic achievement standards for mathematics. Thank you to the State Superintendent's Standards Review Council for their work and guidance through the standards process. A special thanks to the Essential Elements for Mathematics Writing Committee for taking on this important project that will shape the classrooms of today and tomorrow. Thanks to the many staff members across the division and other teams at DPI who have contributed their time and talent to this project. Finally, a special thanks to Wisconsin educators, businesspeople, parents, and citizens who provided comment and feedback to drafts of these standards.

Wisconsin Essential Elements for Mathematics Writing Team

Co-Chairs: **Annie Sims,** Program Support Teacher, Marathon County Special Education
Lori Williams, K-12 Math Specialist, Manitowoc Public School District

DPI Liaisons: **Julie Bormett,** Education Consultant, Literacy and Mathematics
Iris Jacobson, Education Consultant, Special Education

Shawn Bowie
Cedarburg School District

Valencia Caldwell
Woodlands School Inc.

Jessica Cappeart
Pulaski Community School District

Chaitali Dutta
School District of Brown Deer

Sarah Harke
Oconto Falls School District

Melissa Hedges
Milwaukee Public Schools

Jodi Lee
School District of Holmen

Cori Moran
School District of South Milwaukee

Lauren Richards
Milwaukee Academy of Science

Cynthia Cuellar Rodriguez
Milwaukee Public Schools

Holly Smith
Walworth County CDEB-Lakeland School

Mallory Smith
Hartford Union High School District

Teresa Stockinger
Manitowoc Public School District

Bernice Sweo
School District of Crandon

Sara Vold
School District of Janesville

Karen Weidner
Mishicot School District

Department of Public Instruction, Academic Standards

Nicole Horsley, Director, Literacy and Mathematics, and Director for Academic Standards

Meri Annin, Lead Visual Communications Designer

David McHugh, Strategic Planning and Professional Learning Consultant

Department of Public Instruction Leaders

Duy Nguyen, Assistant State Superintendent, Division of Academic Excellence

Paul Manriquez, Assistant State Superintendent, Division for Learning Support

Julia Hartwig, Director, Special Education Team

Daniel Parker, Assistant Director, Special Education Team

Dr. John Johnson, Deputy State Superintendent, Office of the State Superintendent

Section I

Wisconsin's Approach to Academic Standards

Purpose of the Document

The purpose of this guide is to improve mathematics education for students and for communities. The Wisconsin Department of Public Instruction (DPI) has developed standards to assist Wisconsin educators and stakeholders in understanding, developing, and implementing course offerings and curriculum in school districts across Wisconsin.

This publication provides a vision for student success and follows *The Guiding Principles for Teaching and Learning (DPI 2011)*. In brief, the principles are:

1. Every student has the right to learn.
2. Instruction must be rigorous and relevant.
3. Purposeful assessment drives instruction and affects learning.
4. Learning is a collaborative responsibility.
5. Students bring strengths and experiences to learning.
6. Responsive environments engage learners.

Program leaders will find the guide valuable for making decisions about:

- Program structure and integration
- Curriculum redesign
- Staffing and staff development
- Scheduling and student grouping
- Facility organization
- Learning spaces and materials development
- Resource allocation and accountability
- Collaborative work with other units of the school, district, and community

What Are the Academic Standards?

Wisconsin Academic Standards specify what students should know and be able to do in the classroom. They serve as goals for teaching and learning. Setting high standards enables students, parents, educators, and citizens to know what students should have learned at a given point in time. In Wisconsin, all state standards serve as a model. Locally elected school boards adopt academic standards in each subject area to best serve their local communities. We must ensure that all children have equal access to high-quality education programs. Clear statements about what students must know and be able to do are essential in making sure our schools offer opportunities to get the knowledge and skills necessary for success beyond the classroom.

Adopting these standards is voluntary. Districts may use the academic standards as guides for developing local grade-by-grade level curriculum. Implementing standards may require some school districts to upgrade school and district curriculums. This may result in changes in instructional methods and materials, local assessments, and professional development opportunities for the teaching and administrative staff.

What is the Difference between Academic Standards and Curriculum?

Standards are statements about what students should know and be able to do, what they might be asked to do to give evidence of learning, and how well they should be expected to know or do it. Curriculum is the program devised by local school districts used to prepare students to meet standards. It consists of activities and lessons at each grade level, instructional materials, and various instructional techniques. In short, standards define what is to be learned at certain points in time, and from a broad perspective, what performances will be accepted as evidence that the learning has occurred. Curriculum specifies the details of the day-to-day schooling at the local level.

Developing the Academic Standards

DPI has a transparent and comprehensive process for reviewing and revising academic standards. The process begins with a notice of intent to review an academic area with a public comment period. The State Superintendent's Standards Review Council examines those comments and may recommend revision or development of standards in that academic area. The state superintendent authorizes whether or not to pursue a revision or development process. Following this, a state writing committee is formed to work on those standards for all grade levels. That draft is then made available for open review to get feedback from the public, key stakeholders, educators, and the Legislature with further review by the State Superintendent's Standards Review Council. The state superintendent then determines adoption of the standards.

Aligning for Student Success

To build and sustain schools that support every student in achieving success, educators must work together with families, community members, and business partners to connect the most promising practices in the most meaningful contexts. The release of the *Wisconsin Essential Elements for Mathematics* provides a set of important academic standards for school districts to implement. This is connected to a larger vision of every child graduating college and career ready. Academic standards work together with other critical principles and efforts to educate every child to graduate college and career ready. Here, the vision and set of Guiding Principles form the foundation for building a supportive process for teaching and learning rigorous and relevant content. The following sections articulate this integrated approach to increasing student success in Wisconsin schools and communities.

Relating the Academic Standards to All Students

Grade-level standards serve as goals for teaching and learning. Relating the standards to students requires all educators to center the learner as they plan for instruction and assessment and create systems that prioritize student understanding. This means that ALL students need to have access to grade-level high quality instruction and assessment in ways that fit their strengths, needs, and interests. This applies to students with Individualized Education Plans (IEPs), multilingual learners, and gifted and talented pupils, consistent with all other students.

Academic standards serve as a valuable basis for establishing concrete, meaningful goals as part of each student's developmental progress and demonstration of proficiency. Students with IEPs must be provided specially designed instruction that meets their individual needs. It is expected that each individual student with an IEP will require unique services and supports matched to their strengths and needs to close achievement gaps in grade-level standards. Alternate academic achievement standards are only available for students with the most significant cognitive disabilities. Multilingual learners deserve high expectations that emphasize the vital role of language and communication in solving mathematical problems, in developing mathematical thinking, and demonstrating knowledge in classroom interactions (Chval, Pinnow, Smith, and Trigos-Carrillo 2021, xv). As with all students, gifted and talented students should experience daily engagement with the Standards for Mathematical Practice. Students need ongoing opportunities to experience the joy of investigating rich concepts in depth and applying mathematical reasoning and justification to a variety of scientific, engineering, and other problems. Pacing for gifted and talented students means that they have the time and opportunity to delve deeply and creatively into topics, projects, and problems of interest (Johnsen and Sheffield 2013, 15-16, 18-19).

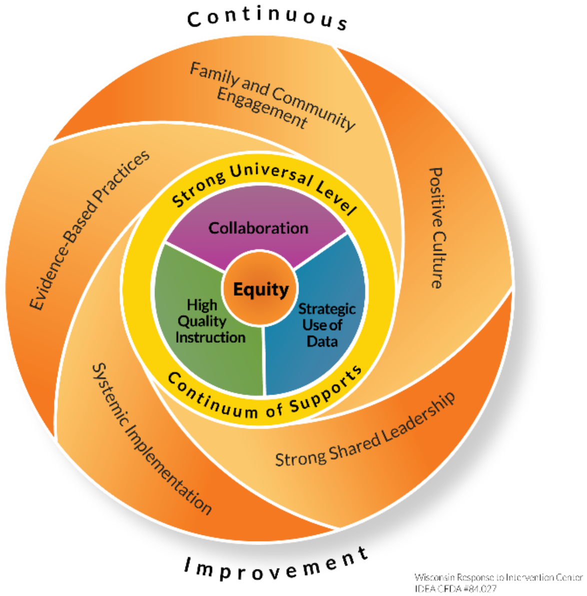
Our Vision: Every Child a Graduate, College and Career Ready

We are committed to ensuring every child graduates from high school academically prepared and socially and emotionally competent. A successful Wisconsin student is proficient in academic content and can apply their knowledge through skills such as critical thinking, communication, collaboration, and creativity. The successful student will also possess critical habits such as perseverance, responsibility, adaptability, and leadership. This vision for every child as a college and career ready graduate guides our beliefs and approaches to education in Wisconsin.

Guided by Principles

All educational initiatives are guided and impacted by important and often unstated attitudes or principles for teaching and learning. *The Guiding Principles for Teaching and Learning (DPI 2011)* emerge from research and provide the touchstone for practices that truly affect the vision of Every Child a Graduate Prepared for College and Career (DPI, n.d.). When made transparent, these principles inform what happens in the classroom, direct the implementation and evaluation of programs, and most importantly, remind us of our own beliefs and expectations for students.

Ensuring a Process for Student Success



For Wisconsin schools and districts, implementing the *Framework for Equitable Multi-Level Systems of Supports (DPI 2017)* means providing equitable services, practices, and resources to every learner based upon responsiveness to effective instruction and intervention. In this system, high-quality instruction, strategic use of data, and collaboration interact within a continuum of supports to facilitate learner success. Schools provide varying types of supports with differing levels of intensity to proactively and responsibly adjust to the needs of the whole child.

These include the knowledge, skills and habits learners need for success beyond high school, including developmental, academic, behavioral, social, and emotional skills.

Connecting to Content: Wisconsin Academic Standards

Within this vision for increased student success, rigorous, internationally benchmarked academic standards provide the content for high-quality curriculum and instruction and for a strategic assessment system aligned to those standards. With the adoption of the standards, Wisconsin has the tools to design curriculum, instruction, and assessments to maximize student learning. The standards articulate what we teach so that educators can focus on how instruction can best meet the needs of each student. When implemented within an equitable multi-level system of support, the standards can help to ensure that every child will graduate college and career ready.

Section II

Wisconsin Essential Elements for Mathematics

Wisconsin Essential Elements for Mathematics

All students, including students with the most significant cognitive disabilities, deserve and have a right to a quality educational experience, including engagement in meaningful mathematics. This right includes, to the maximum extent possible, the opportunity to be involved in and meet the same challenging expectations that have been established for all students. *Wisconsin Essential Elements for Mathematics* are aligned with college, career, and community ready expectations and include rigorous content and application for students with the most significant cognitive disabilities. These alternate academic achievement standards are clearly linked to *Wisconsin Standards for Mathematics*, promote access to the general education curriculum, and reflect professional judgement of the highest expectation possible.

What is Mathematics Education in Wisconsin?

Wisconsin Vision for Mathematics

The Wisconsin Vision for Mathematics is shaped by Wisconsin practitioners and experts and is informed by work at the national level and in other states. The overarching goal of the Wisconsin Vision for Mathematics (DPI 2021) is that mathematics is understood to be a human activity of experiencing, interacting with, and relating to the world. Relationships between and among people as users of mathematics as well as between people and the mathematics content support strong mathematical identities and agency. The following four components of the Vision Statement help to illustrate the overarching goal:

1. Every student is a knower and doer of mathematics.

2. Wondering, reasoning, and understanding are at the heart of mathematics.

3. Developing positive dispositions towards mathematics cultivates self-efficacy and lifelong interactions with mathematics.

4. Mathematics provides a lens for reflecting upon and appreciating the beauty in everyday practices in families, communities, and the world.

Wisconsin's Guiding Principles for Teaching and Learning (DPI 2011) provide important guidance for approaching the vision of mathematics. Each of the six guiding principles has implications for equity, pedagogy, instruction, and assessment. Mathematics educators should consider how teaching and learning systems and structures are in service of students with respect to each of the principles.

Every student has the right to learn significant mathematics.

Mathematical proficiency is essential for every student in Wisconsin. Students need to be able to formulate, represent, and solve problems; explain and justify solutions and solution paths; and see mathematics as sensible, useful, and worthwhile. To achieve this vision, all students must have access to challenging, rigorous, and meaningful mathematics. Schools and classrooms need to be organized to convey the message that all students can learn mathematics and should be expected to achieve.

Mathematics instruction should be rigorous and relevant.

Teachers focus on engaging students in using mathematical reasoning, making mathematical connections, and modeling and representing mathematical ideas in a variety of ways. The mathematics curriculum needs to integrate and sequence important mathematical ideas so that mathematics makes sense. Teachers use rich tasks to engage students in the development of conceptual understanding and procedural skills. An emphasis on connections within mathematics helps students see mathematics as a coherent and integrated whole rather than as a set of isolated and disconnected skills and procedures. Through mathematical applications, students recognize the usefulness of mathematics and appreciate the need to study and understand mathematical skills and concepts.

Purposeful assessment drives mathematics instruction and affects learning.

Teachers measure mathematical proficiency by using a variety of purposeful assessments before, during, and after instruction. Rich assessment tasks ask students to demonstrate their understanding by representing mathematical situations, solving problems as developed in the classroom, and justifying their solutions. Valuable assessments provide both students and teachers with the opportunity to reflect on students' mathematical communication, precision, and reasoning. Teachers use resulting data to adapt their instruction and the learning environment so that all students will understand new mathematics concepts and content.

Learning mathematics is a collaborative responsibility.

Collaborative structures, within the mathematics classroom as well as in the school community, support the teaching and learning of mathematics. Students develop mathematical habits of mind through purposeful interactions in the classroom. Teachers co-create contexts, conditions, and assessment strategies for an interdependent learning environment. Opportunities for students to communicate the solutions, solution paths, and justifications are present in mathematics lessons.

Students bring strengths and experiences to mathematics learning.

Students bring informal experiences of mathematics from their home and community to the mathematics classroom. They may enter classrooms with varying levels of mathematical misconceptions and confidence in their ability to do mathematics. Schools and teachers must build upon students' prior knowledge and intuitive understanding of mathematical ideas to connect the formal study of mathematics to students' ongoing experiences. Teachers need to continually identify students' strengths and weaknesses as a basis to develop tasks and experiences that will capitalize on student strengths and address weaknesses and misconceptions.

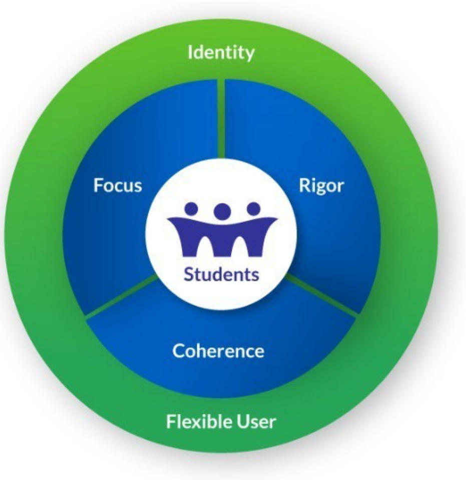
Responsive environments engage mathematics learners.

Teachers utilize strategies that create effective mathematics environments. These environments use high quality mathematics curriculum and instruction in response to the understanding that not all students learn at the same pace or in the same way. Student engagement, perseverance, and learning are increased when teachers respond to students' interests, learning profiles, and readiness. The Standards for Mathematical Practice are evident in a responsive environment.

Efforts to create and sustain a district or school mathematics program that effectively implements the *Wisconsin Essential Elements for Mathematics* should involve Wisconsin's Guiding Principles for Teaching and Learning (DPI 2011). This must be ongoing work in all Wisconsin schools and districts. It is critical that these Guiding Principles are used as a framework to continually inform the conversations around how to best create systems and structures that are designed for equitable outcomes. These conversations include, but are not limited to, determining a district's vision for mathematics, and considering how pedagogy impacts instruction and assessment.

Wisconsin's Approach to Alternate Academic Achievement Standards for Mathematics

The *Wisconsin Essential Elements for Mathematics (2022)* are built on the *Dynamic Learning Maps Essential Elements for Mathematics* (University of Kansas Center for Research 2014) as specific statements of knowledge and skills linked to expectations identified in *Wisconsin Standards for Mathematics* (2021). The recently revised, *Wisconsin Standards for Mathematics* (2021) are built on the foundation of existing standards (National Governors Association Center for Best Practices, Council of Chief State School Officers 2010) and incorporate shifts that reflect new research and broader expectations of mathematics. Three of the five shifts in *Wisconsin Standards for Mathematics* (2021) were expanded upon to emphasize advancing educational equity in mathematics. Two of the five shifts are new and unique to Wisconsin. In total, there are five important shifts to consider. These shifts, although initially written for *Wisconsin Standards for Mathematics* (2021), can be applied to the *Wisconsin Essential Elements for Mathematics* (2022) and instructional practices of educators of students with the most significant cognitive disabilities.



Shift #1: Learning mathematics emphasizes recognizing, valuing, and fostering mathematical identities and agency in all students.

The *Wisconsin Standards for Mathematics* (2021) and *Wisconsin Essential Elements for Mathematics* (2022) expect opportunities for inclusion of broader ways to think and do mathematics. This shift supports recognizing and valuing the mathematical ways of thinking students bring with them to school, mathematics from their culture, their families or previous grade level. By leveraging multiple mathematical competencies, drawing on multiple resources of knowledge and going deep into the mathematical concepts students develop stronger mathematical understanding (Aguirre, Martin, and Mayfield-Ingram 2013, 43).

Shift #2: All students are flexible users of mathematics who see how mathematics can be used to understand their world and the world around them.

The *Wisconsin Standards for Mathematics* (2021) and *Wisconsin Essential Elements for Mathematics* (2022) call for empowering students to be thinkers and doers of mathematics. Engaging students in mathematizing and modeling is a way to bring this shift to life in students' mathematical journeys. The standards are calling for an intentional pairing of the Standards for Mathematical Practice and the Essential Elements for Mathematics or Standards for Mathematical Content that allow for students to be mathematically curious and gain a lifelong appreciation of mathematics and how mathematics is used to understand, critique, and create solutions for the world (NCTM 2020, 15).

Shift #3: All students engage in mathematics that is focused on developing deep understanding of and connections among mathematical concepts in order to gain strong foundations to move their mathematics learning forward.

The *Wisconsin Essential Elements for Mathematics* (2022) continue to call for a focus in mathematics content across grade levels. This means focusing deeply based on the four claims and nine conceptual areas of the Dynamic Learning Maps (DLM) as follows:

Claim	Conceptual Area
Number Sense: Students demonstrate increasingly complex understanding of number sense.	Understand number structures.
	Compare, compose, and decompose numbers and sets.
	Calculate accurately and efficiently using simple arithmetic operations.
Geometry: Students demonstrate increasingly complex spatial reasoning and understanding of geometric principles.	Understand and use geometric properties of two- and three-dimensional shapes.
	Solve problems involving area, perimeter, and volume.
Measurement Data and Analysis: Students demonstrate increasingly complex understanding of measurement, data, and analytic procedures.	Understand and use measurement principles and units of measure.
	Represent and interpret data displays.
Algebraic and functional reasoning: Students solve increasingly complex mathematical problems, making productive use of algebra and functions.	Use operations and models to solve problems.
	Understand patterns and functional thinking.

Shift #4: All students engage in coherent mathematics that connects concepts and mathematical thinking within and across domains and grades.

Mathematics is not a list of disconnected topics, tricks, or mnemonics; it is a coherent body of knowledge made up of interconnected concepts. Therefore, the Essential Elements for Mathematics are designed around coherent progressions from grade to grade. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years.

Shift #5: All students engage in rigorous mathematics within a balanced approach developing conceptual understanding, procedural flexibility and efficiency and application to authentic contexts.

Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades. To help students meet the standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, flexible and efficient procedural skills, and application.

Conceptual understanding: The Essential Elements call for conceptual understanding of key concepts, such as place value and ratios. Students must be able to access concepts from a number of perspectives to see math as more than a set of mnemonics or discrete procedures.

Flexible and efficient procedural skills: The Essential Elements call for efficiency and accuracy in calculation. Flexible and efficient procedural skills build from an initial exploration and discussion of number concepts to using informal reasoning strategies and the properties of operations to develop general methods for solving problems (NCTM 2014).

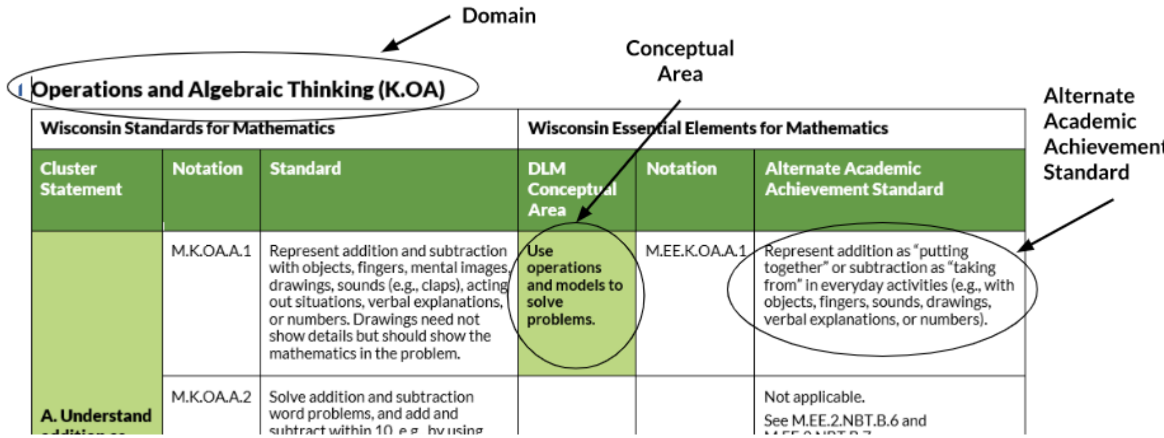
Application: The Essential Elements call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural flexibility and efficiency.

The *Wisconsin Essential Elements for Mathematics* (2022) are integrated through a variety of classes and experiences. Each district, school, and program area should determine the means by which students meet these standards. Through the collaboration of multiple stakeholders, these foundational alternate academic achievement standards will set the stage for high-quality, successful, contemporary mathematics courses and programs throughout Wisconsin's PK-12 systems.

Standards Structure

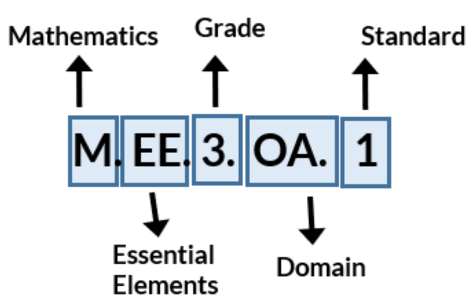
Wisconsin Standards for Mathematics (2021) side by side with *Wisconsin Essential Elements for Mathematics (2022)* have the following design features:

Wisconsin Standards for Mathematics (2021)	Wisconsin Essential Elements for Mathematics (2022)
Domains - Larger groups of related standards. Standards from different domains may sometimes be closely related. In high school standards there is an additional grouping of domains called **Conceptual Categories**.	
Clusters - Groups of related standards. Notice that the cluster statements appear at the far left of each standard table to visually highlight an emphasis on the cluster statement. Individual standards stem from cluster statements and provide the details of what students should understand and be able to do. Standards from different clusters may sometimes be closely related because mathematics is a connected subject.	**Conceptual Areas** – Define the knowledge and skills required to meet DLM broad claims. Each Essential Element is connected to one of nine conceptual areas.
Standards - Define what students should understand and be able to do.	
(M) Mathematical Modeling - Mathematical modeling is best interpreted, not as a collection of isolated topics, but rather in relation to other standards. To support these relationships, content standards that may be particularly valuable in middle school and high school have been indicated with an (M*)* symbol. The (M) symbol appears following those cluster statements throughout grades 6-12.	**(M) Mathematical Modeling -** Mathematical modeling is best interpreted, not as a collection of isolated topics, but rather in relation to other standards. To support these relationships, content standards that may be particularly valuable in middle school and high school have been indicated with an (M*)* symbol. The (M) symbol appears following specific conceptual areas throughout grades 6-12.
(F2Y) First Two Years - This denotes content standards that should be completed by all students in their first two years of high school mathematics, providing the shift of focus to the high school standards. The (F2Y) symbol appears following those standards throughout the high school section.	Rather than using the **(F2Y) First Two Years** notation, the shift of focus is brought to the *Essential Elements for Mathematics* high school alternate achievement standards through the DLM broad claims and conceptual areas described above in this chart.

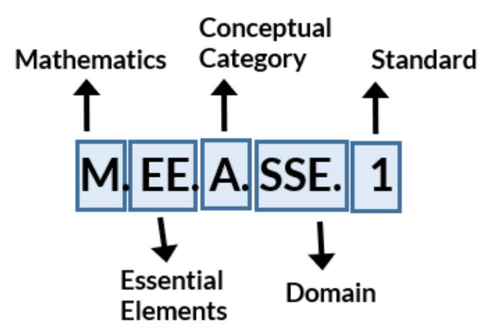


The *Wisconsin Essential Elements for Mathematics* use a specific coding structure. "EE" is added to the coding for *Wisconsin Standards for Mathematics* (2021) and the cluster letter is not included to notate each Essential Element. The notation aims to link the alternate academic achievement standards to the *Wisconsin Standards for Mathematics* with as similar of notation as possible.

K-8 Standards Coding



High School Standards Coding



Additional resources follow both the Standards for Mathematical Practice and the Essential Elements for grades K-12:

- **Tables** (Appendix 1) **-** Provides problem situation types and examples as well as the properties of operations, equality, and inequality. Notice that these are the same updated tables found in *Wisconsin Standards for Mathematics (2021).*

- **Glossary** (Appendix 2) - Supports the understanding of all grade level standards and is the same glossary found in *Wisconsin Standards for Mathematics (2021)* with a few additional terms.

- **Essential Elements by Conceptual Area** (Appendix 3) – Provides the Essential Elements in tables by DLM Conceptual Area in order support educators in understanding the Essential Elements that contribute to each mathematical area being developed across grade levels.

Section III

Essential Elements for Mathematics

Alternate Academic Achievement Standards

These revised alternate academic achievement state standards (2022), demonstrate the belief that every student can develop deep mathematical understanding as a confident and capable learner. To achieve this and develop strong mathematical identities in the process, students with the most significant cognitive disabilities need access to alternate academic achievement standards that in turn promote access to the general education curriculum. It is important that users of this document keep the five Wisconsin Key Shifts at the forefront of their minds while interacting with both the Standards for Mathematical Practice and the Essential Elements for Mathematics.

Essential Elements for Mathematics also provides schools/districts with opportunities to make local decisions about curriculum, materials, and assessments. To provide guidance for these decisions, efforts have been made to ensure the Essential Elements promote educational equity. Examples include:

- Broadening standard language to include specific examples, but not be limited to them.
- Intentionally using the aspects of fluency—flexibility and efficiency—to prioritize student understanding and strategic competence as a way to build toward mathematical proficiency.
- Using mathematical modeling throughout K-12 mathematical experiences that allow for exploration of authentic math problems that arise in everyday lives to support each student's identity as a problem solver.

Standards for Mathematical Practice

The Standards for Mathematical Practice are central to the teaching and learning of mathematics. These practices describe the behaviors and habits of mind that are exhibited by students who are mathematically proficient and are identical in both the *Wisconsin Standards for Mathematics* (2021) and the *Wisconsin Essential Elements for Mathematics* (2022). Mathematical understanding is the intersection of these practices and mathematics content. It is critical that the Standards for Mathematical Practice are embedded in daily mathematics instruction.

The Standards for Mathematical Practice include:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and appreciate and critique the reasoning of others (Gutiérrez 2017, 17-18).
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The *Essential Elements for Mathematics* (2022) includes K-12 descriptive versions of the Standards for Mathematical Practice, illustrating that these habits of mind are consistent at all levels of mathematics and develop over time. The descriptions aim to provide examples of how these practices might come to life within the mathematical content of the Essential Elements.

Essential Elements for Mathematics

The Essential Elements for Mathematics describe the sequence of important mathematics content that students learn. They are a combination of procedures and understandings. These Essential Elements are organized around Dynamic Learning Map (DLM) claims and conceptual areas, kindergarten through high school. "The learning maps are highly connected representations of how academic skills are acquired, based on current research. The claims help to organize the structure in the learning map for students with the most significant cognitive disabilities and the conceptual areas further define the knowledge and skills required to meet the broad claims (University of Kansas Center for Research 2014, 5-7).

Learning mathematics with understanding is a focus of the *Wisconsin Essential Elements for Mathematics* (2022). Several of the DLM conceptual areas begin with the verb "understand" and are crucial for mathematical proficiency. It is generally agreed that students understand a concept in mathematics if they can use mathematical reasoning with a variety of representations and connections to explain the concept to someone else or apply the concept to another situation. This is how 'understand' should be interpreted when implementing the *Wisconsin Essential Elements for Mathematics* (2022).

While the Standards for Mathematical Practice should be addressed with all of the Essential Elements, the Essential Elements connected to conceptual areas that begin with the verb "understand" are a natural intersection between the two.

Understanding Mathematics

Wisconsin Essential Elements for Mathematics (2022) define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can provide the product for a multiplication fact and a student who can share justification for that product. The student who can justify their reasoning may have a better chance to succeed at a less familiar task such as a more complex multiplication fact or the multiplication of a two-digit number times a one-digit number. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

K-12 Standards for Mathematical Practice

Math Practice 1: Make sense of problems and persevere in solving them.

Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem and plan and choose a solution pathway. For example, instead of hunting for “key words” in a word problem, students might use concrete objects or pictures to show the actions of a problem, such as counting out and joining two sets. If students are not at first making sense of a problem or seeing a way to begin, they ask questions about what is happening in the problem that will help them get started. As they work, they continually ask themselves, “Does this make sense?" When they find that their solution pathway does not make sense, they look for another pathway that does. They may consider simpler forms of the original problem; for example, they might replace multi-digit numbers in a word problem with single-digit numbers to better appreciate the quantities in the problem and how they relate.

Mathematically proficient students consider different solution pathways, both their own and those of other students, in order to identify and analyze connections among approaches. Mathematically proficient students can explain how alternate representations of problem conditions relate to each other. For example, they can identify connections between the solution to a word problem that uses only arithmetic and a solution that uses variables and algebra, and they can navigate among verbal descriptions, tables, graphs, and equations.

Math Practice 2: Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They can contextualize quantities and operations by using visual representations or stories. They interpret symbols as having meaning, not just as directions to carry out a procedure. Even as they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent.

Mathematically proficient students know and flexibly use different properties of operations, numerical relationships, numbers, and geometric objects. They can contextualize an abstract problem by placing it in a context that they can then use to make sense of the mathematical ideas. For example, if a student is evaluating the expression, 9 x 25, the student might think of a context to help produce a strategy—for example, by thinking “Eight groups of 25 is like having 8 quarters.” This prompts a strategy of thinking "I know that 4 quarters is $1.00 and another 4 quarters is another $1.00. $1.00 and $1.00 is $2.00." In this example the student uses a context to think through a strategy for solving the problem, using their knowledge of money and of decomposing one factor (8 = 4 + 4). The student then uses the context to identify the solution to the original problem.

Mathematically proficient students can also make sense of a contextual problem and express the actions or events that are described in the problem using numbers and symbols. If they work with the symbols to solve the problem, they can then interpret their solution in terms of the context. Consider the problem: A teacher wants to bring 10 pumpkins to school to decorate the classroom. Some are big pumpkins, and some are small pumpkins. How many of each size pumpkin might the teacher bring to school? When students create the number sentence 4+6=10, they have decontextualized the problem and expressed it with numbers and symbols. When they can explain that the number sentence means, "4 big pumpkins plus 6 small pumpkins equal 10 pumpkins," they demonstrate their ability to recontextualize the numbers and equation back to the word problem.

Math Practice 3: Construct viable arguments and appreciate and critique the reasoning of others.

Mathematically proficient students construct mathematical arguments that explain the reasoning underlying a strategy, solution, or conjecture. Arguments might use concrete referents such as objects, drawings, diagrams, and actions. Arguments might also rely on definitions, previously established results, properties, or structures. For example, a student might argue that 1/2 >1/10 on the basis that one of two equal parts of a whole is larger than one of 10 equal parts of that whole, because with more equal parts, the size of each part must be smaller. Another example is reasoning that two different shapes have equal area because it has already been demonstrated that they are each half of the same rectangle. Students might also use counterexamples to argue that a conjecture is not true— for example, a rectangle is an example that shows that not all quadrilaterals with four equal sides are squares; or, multiplying by one shows that a product of two whole numbers is not always greater than each factor.

Mathematically proficient students present their arguments in the form of representations, actions on those representations, explanations, or a combination of these three. Some of their arguments apply to individual problems, but others are about conjectures based on regularities they have noticed across multiple problems (Math Practice 8). As they articulate and justify generalizations, students consider to which mathematical objects (numbers or shapes, for example) their generalizations apply. For example, students may believe a generalization about the behavior of addition applies to positive whole numbers to 20 because those are the numbers with which they are currently familiar. As they expand their understanding of the number system, they may reexamine their conjecture for numbers within 100. Students also return to their conjectures and arguments about whole numbers to determine whether they apply to fractions and decimals.

While communicating their own mathematical ideas is important, students learn to be open to others' mathematical ideas as well. They appreciate a different perspective or approach to a problem and learn how to respond to those ideas, respecting the reasoning of others (Gutiérrez 2017, 17-18).

Math Practice 4: Model with mathematics.

"In the course of a student's mathematics education, the word 'model' is used in many ways. Several of these, such as manipulatives, demonstration, role modeling, and conceptual models of mathematics, are valuable tools for teaching and learning. However, they are different from the practice of mathematical modeling. Mathematical modeling … uses mathematics to answer big, messy, reality-based questions" (Bliss and Libertini 2016, 7).

“Mathematically proficient students formulate their own problems that emerge from natural circumstances as they mathematize the world around them. They can identify the mathematical elements of a situation and generate questions that can be addressed using mathematics (e.g., noticing and wondering). Students dig into the context and make assumptions as they decide “what matters”. Mathematically proficient students understand that there are multiple solutions to a modeling problem, so they are working to find a solution rather than the solution. Students make judgements about what matters and assess the quality of their solution (Bliss and Libertini 2016, 10). They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving their mathematical modeling approach if it has not served its purpose. As they decontextualize the situation and represent it mathematically, they are also reasoning abstractly (Math Practice 2).

Students encounter mathematical modeling opportunities each and every day at school and at home. Children might be presented with a bag of apples and simply asked “Is this enough for our class/family?” or consider the question, “Is the carpet in our classroom big enough for our bodies?” Students might also plan a fundraising event involving selling popcorn after school. In this example, sometimes students will be engaged in only a part of the modeling cycle such as making assumptions about how much to charge or how much popcorn to make (Godbold, Malkevitch, Teague, and van der Kooij 2016, 50).

Note: Although physical objects and visuals can be used to model a situation, using these tools, absent a contextual situation is not an example of Math Practice 4. For example, solving a word problem using counters or a tape diagram would not be modeling with mathematics, instead this is modeling the mathematics. Math Practice 4 is about engaging in solving authentic real-world problems.

Math Practice 5: Use appropriate tools strategically.

Mathematically proficient students strategically consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. These tools might include physical objects (e.g., manipulatives, pencil and paper, rulers); conceptual tools (e.g., properties of operations, algorithms); drawings or diagrams (e.g., number lines, tally marks, tape diagrams, arrays, tables, graphs) and available technologies (e.g., calculators, online apps).

Mathematically proficient students are sufficiently familiar with tools appropriate for their areas of content to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained from their use as well as their limitations. Students choose tools that are relevant and useful to the problem at hand. For example, when determining how to measure length, students may compare the benefits of using non-standard units of measure (e.g., their own hands, paperclips) versus standard units and tools (e.g., an inch or centimeter ruler). As another example, when presented with 1022-3 or 101-98, students subtract strategically, which may involve reasoning, counting, or decomposing rather than using a written algorithm.

When making mathematical models, students know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Math Practice 6: Attend to precision.

Mathematically proficient students use precise communication. They come to appreciate, understand, and use mathematical vocabulary not in isolation, but in the context of doing mathematical thinking and problem solving. They may start by using everyday language to express their mathematical ideas, and gradually select words with greater clarity and specificity. For example, they may initially use the word “triangle” to refer only to equilateral triangles resting on their bases but come to understand and use a more precise definition of a triangle as a closed figure with three straight sides. As another example, they may initially explain a solution by saying, “it works" without explaining what “it" means but later clarify their explanation with specific details.

In using mathematical representations, students provide appropriate labels to precisely communicate the meaning of their representations (e.g., charts, graphs, and drawings). As they progress in mathematics, students carefully formulate explanations, resulting in the ability to examine claims and make explicit use of definitions.

Students use mathematical symbols correctly and can describe the meaning of the symbols they use. For example, they use the equal sign consistently and appropriately. They state the meaning of the symbols they choose in relation to the problem at hand.

Students use tools and strategies (e.g., measuring tools, estimation) effectively, to maintain a level of precision that is appropriate to the situation. They specify units of measure where needed.

Perseverance and attention to detail are mathematical habits of mind; mathematically proficient students check for reasonableness and accuracy by solving a problem a second way, analyzing errors and learning from them.

Math Practice 7: Look for and make use of structure.

Mathematically proficient students use structures such as place value, the properties of operations, and attributes of shapes to solve problems. In many cases, they have identified and described these structures through repeated reasoning (Math Practice 8).

When students recognize that adding 1 results in the next counting number, they are identifying the basic structure of whole numbers. Later, when students calculate 6 x 9, they might apply the structure of the distributive property to find the product: 6 x 9 = (5 + 1) x 9 = (5 x 9) + (1 x

9). To determine the volume of a 3 x 4 x 5 rectangular prism, students might see the structure of the prism as five layers of 3 x 4 arrays of cubes.

Students look for and make use of structure when they view expressions as objects to observe and interpret. For example, students might observe that 120 – 41 must be one less than 120 – 40 because "if you subtract one more, the result will be one less" (Math Practice 8).

Students can interpret the expression 5 x 3 + 2 x 3 as "five groups of three and two more groups of three" or notice there are a total of 7 groups of 3.

Math Practice 8: Look for and express regularity in repeated reasoning.

Mathematically proficient students look for and identify regularities as they solve multiple related problems. Students make and test conjectures, reason about and express these regularities as generalizations about structures and relationships, and then use those generalizations to solve problems (Math Practice 7).

For example, students might notice that when tossing two-color counters to find combinations of a given number, over time students will notice a pattern (commutative property of addition). For example, when tossing six 2-sided counters, they may get 2 red, 4 yellow and 4 red, 2 yellow and when tossing 4 counters, they get 1 red, 3 yellow and 3 red, 1 yellow.

Students can recognize and use patterns to help them become flexible with addition. For example, given the number string below, students may recognize they can take one away from the 5 and add it to the first number to make a multiple of ten. They also may notice a pattern related to the first digit increasing by 10, therefore the answer increases by 10.

9+5   19+5   29+5   39+5

Students notice if calculations are repeated and look both for general methods and for shortcuts. For example, when examining the equation

5 + 5 + 5 + 5 = 20, students may notice repeated addition and it may lead them to the general operations of multiplication for 4 x 5 = 20.

As students practice sharing their observations, they learn to communicate with greater precision (Math Practice 6). As they explain why these generalizations must be true, they construct, critique, appreciate and compare arguments (Math Practice 3).

Kindergarten

Counting and Cardinality (K.CC)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Know number names and the count sequence.	M.K.CC.A.1	Count to 100 by ones and by tens.	**Understand number structures.**	M.EE.K.CC.1	Starting with one, count to 10 by ones.
	M.K.CC.A.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).			Not applicable. See M.EE.2.NBT.2c
	M.K.CC.A.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).			Not applicable. See M.EE.2.NBT.3

Counting and Cardinality (K.CC) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Tell the number of objects.	M.K.CC.B.4	Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object (one to one correspondence). Understand that the last number name said tells the number of objects counted (cardinality). The number of objects is the same regardless of their arrangement or the order in which they were counted (number conservation). c. Understand that each successive number name refers to a quantity that is one larger and the previous number is one smaller (hierarchical inclusion).	**Understand number structures.**	M.EE.K.CC.4	Connect counting to cardinality to 10. a. Demonstrate one-to-one correspondence, pairing each object with one and only one number and each number with one and only one object (one-to-one correspondence). Understand that the last number name said tells the number of objects counted (cardinality).
	M.K.CC.B.5	Quickly recognize and name the quantity of up to 5 objects briefly shown in structured or unstructured arrangements without counting (perceptual subitizing).			Not applicable.

Counting and Cardinality (K.CC) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Tell the number of objects. (cont'd)	M.K.CC.B.6 [WI.2010. K.CC.B.5]	Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.	**Understand number structures.**	M.EE.K.CC.6	Count out up to three objects from a larger set, pairing each object with one and only one number name to tell how many.
	M.K.CC.C.7 [WI.2010. K.CC.C.6]	Identify whether the number of objects (up to 10) in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.	**Compare, compose, and decompose numbers and sets.**	M.EE.K.CC.7	Identify whether the number of objects in one group is greater than (more) or less than (when the quantities are clearly different) or equal to the number objects in another group.
	M.K.CC.C.8 [WI.2010. K.CC.C.7]	Compare two numbers between 1 and 10 presented as written numerals using student generated ways to record the comparison.			Not applicable. See M.EE.1.NBT.4 and M.EE.2.NBT.4.

Operations and Algebraic Thinking (K.OA)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand addition as putting together and adding to and understand subtraction as taking apart and taking from.	M.K.OA.A.1	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, or numbers. Drawings need not show details but should show the mathematics in the problem.	**Use operations and models to solve problems.**	M.EE.K.OA.1	Represent addition as "putting together" or subtraction as "taking from" in everyday activities (e.g., with objects, fingers, sounds, drawings, verbal explanations, or numbers).
	M.K.OA.A.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. *See Appendix, Table 1 for specific problem situations and category information.*			Not applicable. See M.EE.2.NBT.6 and M.EE.2.NBT.7.

Operations and Algebraic Thinking (K.OA) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand addition as putting together and adding to and understand subtraction as taking apart and taking from. (cont'd)	M.K.OA.A.3	Compose and decompose quantities within 10. a. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition with drawings or numbers. Quickly name the quantity of objects briefly shown in structured arrangements anchored to 5 (e.g., fingers, ten frames, math rack/rekenrek) with totals up to 10 without counting by recognizing the arrangement or seeing the quantity in subgroups that are combined (conceptual subitizing).			Not applicable. See M.EE.1.NBT.6.
	M.K.OA.A.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or numbers.			Not applicable. See M.EE.1.NBT.2.
	M.K.OA.A.5	Flexibly and efficiently add and subtract within 5 using mental images and composing/decomposing numbers up to 5.			Not applicable. See M.EE.1.NBT.4 and M.EE.1.NBT.6.

Number and Operations in Base Ten (K.NBT)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Work with numbers 11-19 to gain foundations for place value.	M.K.NBT.A.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or numbers; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.			Not applicable. See M.EE.1.NBT.4 and M.EE.1.NBT.6.

Measurement and Data (K.MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Describe and compare measurable attributes.	M.K.MD.A.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.K.MD.1	Classify objects according to attributes (e.g., big/small, heavy/light).
	M.K.MD.A.2	Directly compare two objects with a measurable attribute in common, to see which object has "more of" / "less of" the attribute and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.K.MD.2	Classify objects according to attributes (e.g., big/small, heavy/light).
B. Classify objects and count the number of objects in each category.	M.K.MD.B.3	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. Limit category counts to be less than or equal to 10.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.K.MD.3	Classify objects according to attributes (e.g., big/small, heavy/light).

Geometry (K.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	M.K.G.A.1	Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.			Not applicable. See M.EE.1.G.1
	M.K.G.A.2	Correctly name shapes regardless of their orientations or overall size.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.K.G.2	Match shapes of the same size and orientation, including circles, squares, rectangles, and triangles.
	M.K.G.A.3	Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.K.G.3	Match shapes of the same size and orientation, including circles, squares, rectangles, and triangles.

Geometry (K.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Analyze, compare, create, and compose shapes.	M.K.G.B.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).			Not applicable. See M.EE.7.G.1
	M.K.G.B.5	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.			Not applicable.
	M.K.G.B.6	Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*			Not applicable. See M.EE.1.G.A.3.

Grade 1

Operations and Algebraic Thinking (1.OA)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Represent and solve problems involving addition and subtraction.	M.1.OA.A.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. *See Appendix, Table 1 for specific problem situations and category information.*	**Use operations and models to solve problems.**	M.EE.1.OA.1	Represent addition and subtraction e.g., by using objects, fingers, mental images, drawings, sounds (e.g., claps), or acting out situations.
	M.1.OA.A.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**Use operations and models to solve problems.**	M.EE.1.OA.2	Use "putting together" to solve problems with two sets.

Operations and Algebraic Thinking (1.OA) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Understand and apply properties of operations and the relationship between addition and subtraction.	M.1.OA.B.3	Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Informal use of the commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12.* *(Informal use of the associative property of addition.)*			Not applicable. See M.EE.6.EE.3 and M.EE.N.CN.2.
	M.1.OA.B.4	Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.*			Not applicable. See M.EE.1.NBT.4 and M.EE.1.NBT.6.

Operations and Algebraic Thinking (1.OA) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Add and subtract within 20.	M.1.OA.C.5	Use counting and subitizing strategies to explain addition and subtraction. a. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). b. Use conceptual subitizing in unstructured arrangements with totals up to 10 and structured arrangements anchored to 5 or 10 (e.g., 10 frames, double ten frames, math rack/rekenrek) with totals up to 20 to relate the compositions and decompositions to addition and subtraction.	**Use operations and models to solve problems.**	M.EE.1.OA.5	Relate counting to addition and subtraction. a. Use manipulatives or visual representations to indicate the number that results when adding one more. b. Apply knowledge of "one less" to subtract one from a number.

Operations and Algebraic Thinking (1.OA) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Add and subtract within 20. (cont'd)	M.1.OA.C.6	Use multiple strategies to add and subtract within 20. a. Flexibly and efficiently add and subtract within 10 using strategies that may include mental images and composing/decomposing up to 10. b. Add and subtract within 20 using objects, drawings or equations. Use multiple strategies that may include counting on; making a ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).			Not applicable. See M.EE.3.OA.6.
D. Work with addition and subtraction equations (cont'd)	M.1.OA.D.7	Understand the meaning of the equal sign as "has the same value/amount as" and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*	**Use operations and models to solve problems.**	M.EE.1.OA.7	Recognize two groups that have the same or equal quantity.

Number and Operations in Base Ten (1.NBT)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Extend the counting sequence.	M.1.NBT.A.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	**Understand number structures**	M.EE.1.NBT.1	Know the count sequence and understand numbers and quantities. a. Count by ones to 30. b. Count up to 10 objects and represent the quantity with the corresponding numeral.
B. Understand place value.	M.1.NBT.B.2	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a.10 can be thought of as a bundle of ten ones -- called a "ten". b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	**Compare, compose, and decompose numbers and sets.**	M.EE.1.NBT.2	Create sets of 10.
	M.1.NBT.B.3	Compare two two-digit numbers based on meanings of the tens and ones digits and describe the result of the comparison using words and symbols (>, =, and <).	**Compare, compose, and decompose numbers and sets.**	M.EE.1.NBT.3	Compare two groups of 10 or fewer items when the number of items in each group is similar.

Number and Operations in Base Ten (1.NBT) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Use place value understanding and properties of operations to add and subtract.	M.1.NBT.C.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	**Compare, compose, and decompose numbers and sets.**	M.EE.1.NBT.4	Compose quantities less than or equal to five in more than one way.
	M.1.NBT.C.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.			Not applicable. See M.EE.1.OA.5.
	M.1.NBT.C.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**Compare, compose, and decompose numbers and sets.**	M.EE.1.NBT.6	Decompose quantities less than or equal to five in more than one way.

Measurement and Data (1.MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Measure lengths indirectly and by iterating length units.	M.1.MD.A.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.	**Understand and use measurement principles and units of measure.**	M.EE.1.MD.1	Compare lengths to identify which is longer/shorter or taller/shorter.
	M.1.MD.A.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.	**Understand and use measurement principles and units of measure.**	M.EE.1.MD.2	Compare lengths to identify which is longer/shorter or taller/shorter.

Measurement and Data (1.MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Tell and write time.	M.1.MD.B.3	Tell and write time in hours and half-hours using analog and digital clocks.	**Understand and use measurement principles and units of measure.**	M.EE.1.MD.3	Understand concepts of time. a. Demonstrate an understanding of *tomorrow*, *yesterday*, and *today*. b. Demonstrate an understanding of *morning*, *afternoon*, *day*, and *night*. c. Identify activities that come before, next, and after. d. Demonstrate an understanding that telling time is the same every day.

Measurement and Data (1.MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Represent and interpret data.	M.1.MD.C.4	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**Represent and interpret data displays.**	M.EE.1.MD.4	Organize objects into categories by sorting (e.g., color, size, shape). Count the number of objects in each category and identify the category with the most objects.

Geometry (1.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Reason with shapes and their attributes.	M.1.G.A.1	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.1.G.1	Identify the relative position of objects that are on, off, in, and out.
	M.1.G.A.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Student use of formal names such as "right rectangular prism" is not expected.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.1.G.2	Sort shapes of the same size and orientation including circles, squares, rectangles, and triangles.
	M.1.G.A.3	Partition circles and rectangles into two and four equal shares, describe and count the shares using the words *halves and fourths*, and use the phrases *half of and fourth of the whole*. Describe the whole as being two of the shares, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	**Solve problems involving area, perimeter, and volume.**	M.EE.1.G.3	Put together two shapes to make a shape that relates to the whole (e.g., two semicircles to make a circle, two squares to make a rectangle).

Grade 2
Operations and Algebraic Thinking (2.OA)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Represent and solve problems involving addition and subtraction.	M.2.OA.A.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. *See Appendix, Table 1 for specific problem situations and category information.*			Not applicable. See M.EE.3.OA.6.
B. Add and subtract within 20.	M.2.OA.B.2	Flexibly and efficiently add and subtract within 20 using multiple mental strategies which may include counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).			Not applicable. See M.EE.2.NBT.7 and M.EE.3.OA.6.

Grade 2
Operations and Algebraic Thinking (2.OA)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Work with equal groups of objects to gain foundation for multiplications.	M.2.OA.C.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	**Use operations and models to solve problems.**	M.EE.2.OA.3	Equally distribute even numbers of objects between two groups.
	M.2.OA.C.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	**Use operations and models to solve problems.**	M.EE.2.OA.4	Use addition to find the total number of objects arranged within equal groups up to a total of 10.

Number and Operations in Base Ten (2.NBT)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand place value.	M.2.NBT.A.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens -- called a "hundred". b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	**Compare, compose, and decompose numbers and sets.**	M.EE.2.NBT.1	Represent numbers up to 30 with objects in sets of tens and ones (e.g., ten frames, hundreds chart, columns).
	M.2.NBT.A.2	Count within 1000; skip-count by 5s, 10s, and 100s.	**Understand number structures**	M.EE.2.NBT.2	Extend understanding of numbers and quantities. a. Count to answer "how many?" questions about as many as 30 things. b. Given a number from 1-30, count out that many objects. c. Name the next number in a sequence between 1 and 10.
	M.2.NBT.A.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	**Understand number structures**	M.EE.2.NBT.3	Identify numerals 1 to 30.

Number and Operations in Base Ten (2.NBT)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand place value. (cont'd)	M.2.NBT.A.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, and describe the result of the comparison using words and symbols (>, =, and <).	**Compare, compose, and decompose numbers and sets.**	M.EE.2.NBT.4	Compare sets of objects and numbers using accurate vocabulary (e.g., greater, more, less, equal).

Number and Operations in Base Ten (2.NBT) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Use place value understanding and properties of operations to add and subtract.	M.2.NBT.B.5	Flexibly and efficiently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In Grade 2, subtraction with decomposition is an exception and may include drawings/representations.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.2.NBT.5	Identify the meaning of the "+" sign (including combine, plus, add), "-" sign (including separate, subtract, take), and the "=" sign (including the same amount as, equal).
	M.2.NBT.B.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.	**Compare, compose, and decompose numbers and sets.**	M.EE.2.NBT.6	Using concrete models, compose and decompose numbers up to 10 in more than one way.
	M.2.NBT.B.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.2.NBT.7	Use objects, representations, and numbers (0-20) to add and subtract.

Number and Operations in Base Ten (2.NBT)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Use place value understanding and properties of operations to add and subtract. (cont'd)	M.2.NBT.B.8	Mentally add 10 or 100 to a given number 100 - 900, and mentally subtract 10 or 100 from a given number 100 - 900.			Not applicable.
	M.2.NBT.B.9	Explain why addition and subtraction strategies work, using place value and the properties of operations. These explanations may be supported by drawings or objects.			Not applicable.

Measurement and Data (2.MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Measure and estimate lengths in standard units.	M.2.MD.A.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	**Understand and use measurement principles and units of measure.**	M.EE.2.MD.1	Measure the length of objects using non-standard units.
	M.2.MD.A.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.			Not applicable.
	M.2.MD.A.3	Estimate lengths using units of inches, feet, centimeters, and meters.	**Understand and use measurement principles and units of measure.**	M.EE.2.MD.3	Order objects by length using non-standard units.
	M.2.MD.A.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard-length unit.	**Understand and use measurement principles and units of measure.**	M.EE.2.MD.4	Order objects by length using non-standard units.

Measurement and Data (2.MD)

B. Relate addition and subtraction to length.	M.2.MD.B.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as number lines) and equations with a symbol for the unknown number to represent the problem.	**Understand and use measurement principles and units of measure.**	M.EE.2.MD.5	Increase or decrease length by adding or subtracting unit(s).
	M.2.MD.B.6	Represent whole numbers as lengths from 0 on a number line with equally spaced points corresponding to the numbers 0, 1, 2 ... and represent whole-number sums and differences within 100 on a number line.	**Understand and use measurement principles and units of measure.**	M.EE.2.MD.6	Use a number line to add one more unit of length.

Measurement and Data (2.MD) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Work with time and money.	M.2.MD.C.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	**Understand and use measurement principles and units of measure.**	M.EE.2.MD.7	Identify, on a digital clock, the hour that matches a routine activity.
	M.2.MD.C.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*	**Understand and use measurement principles and units of measure.**	M.EE.2.MD.8	Recognize that money has value.
D. Represent and interpret data.	M.2.MD.D.9	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	**Represent and interpret data displays.**	M.EE.2.MD.9	Create physical bar graphs from sorted objects (e.g., line up different colored toy cars).
	M.2.MD.D.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems using information presented in a bar graph. *See Appendix, Table 1 for specific problem situations.*	**Represent and interpret data displays.**	M.EE.2.MD.10	Create physical bar graphs from sorted objects (e.g., line up different colored toy cars).

Geometry (2.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Reason with shapes and their attributes.	M.2.G.A.1	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. Sizes are compared directly or visually, not compared by measuring.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.2.G.1	Identify common two-dimensional shapes including squares, circles, triangles, and rectangles of various sizes and in various orientations.
	M.2.G.A.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.			Not applicable.
	M.2.G.A.3	Partition circles and rectangles into two, three, or four equal shares, describe and count the shares using the words *halves*, *thirds*, and *fourths*, and use phrases *half of*, *a third of*, and *a fourth of the whole*. Describe the whole as composed of two halves, three thirds, and four fourths. Recognize that equal shares of identical wholes need not have the same shape.			Not applicable. See M.EE.3.G.2 and M.EE.4.NF.1.

Grade 3

Operations and Algebraic Thinking (3.OA)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Represent and solve problems involving multiplica-tion and division.	M.3.OA.A.1	Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7.*	**Use operations and models to solve problems.**	M.EE.3.OA.1	Use repeated addition to find the total number of objects and determine the sum.
	M.3.OA.A.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	**Use operations and models to solve problems.**	M.EE.3.OA.2	Use repeated addition to find the total number of objects and determine the sum.
	M.3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. *See Appendix, Tables 2A and 2B for specific problem situations.*			Not applicable. See M.EE.3.OA.1 and M.EE.5.NBT.5.

Operations and Algebraic Thinking (3.OA) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Understand properties of multiplica-tion and the relationship between multiplica-tion and division.	M.3.OA.B.4 [WI.2010. 3.OA.B.5]	Apply properties of operations as strategies to multiply and divide. Student use of the formal terms for these properties is not necessary. *Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56. (Distributive property.)*			Not applicable. See M.EE. N.CN.2.
	M.3.OA.B.5 [WI.2010. 3.OA.B.6]	Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8*			Not applicable. See M.EE.5.NBT.6.
C. Multiply and divide within 100.	M.3.OA.C.6 [WI.2010. 3.OA.B.7]	Use multiplicative thinking to multiply and divide within 100. a. Use the meanings of multiplication and division, the relationship between the operations (e.g., knowing that 8 x 5 = 40, one could reason that 40 ÷ 5 = 8), and properties of operations (e.g., the distributive property) to develop and understand strategies to multiply and divide within 100. Flexibly and efficiently use strategies, the relationship between the operations, and properties of operations to find products and quotients with multiples of 0, 1, 2, 5, & 10 within 100.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.3.OA.6	Solve addition and subtraction problems within 20, when the result is unknown.

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Operations and Algebraic Thinking (3.OA) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
D. Solve problems involving the four operations and identify and explain patterns in arithmetic.	M.3.OA.D.7 [WI.2010. 3.OA.B.8]	Solve two-step word problems, posed with whole numbers and having whole number answers, using the four operations. Represent these problems using one or two equations with a letter standing for the unknown quantity. If one equation is used, grouping symbols (i.e., parentheses) may be needed. Assess the reasonableness of answers using mental computation and estimation strategies.	**Use operations and models to solve problems.**	M.EE.3.OA.7	Solve one-step real-world problems within 20, using addition or subtraction.
	M.3.OA.D.8 [WI.2010. 3.OA.B.9]	Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	**Understand patterns and functional thinking.**	M.EE.3.OA.8	Identify arithmetic patterns.

Number and Operations in Base Ten (3.NBT)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Use place value understand-ing and properties of operations to perform multi-digit arithmetic, using a variety of strategies.	M.3.NBT.A.1	Use place value understanding to generate estimates for problems in real-world situations, with whole numbers within 1,000, using strategies such as mental math, benchmark numbers, compatible numbers, and rounding. Assess the reasonableness of their estimates (e.g., Is my estimate too low or too high? What degree of precision do I need for this situation?).	**Understand number structures**	M.EE.3.NBT.1	Use decade numbers (10, 20, 30) as benchmarks to demonstrate understanding of place value for numbers 0-30.
	M.3.NBT.A.2	Flexibly and efficiently add and subtract within 1,000 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	**Understand number structures**	M.EE.3.NBT.2	Demonstrate understanding of place value to tens.
	M.3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.	**Understand number structures**	M.EE.3.NBT.3	Count by tens using models (e.g., objects, base ten blocks, money).

Number and Operations – Fractions (3.NF)

Grade 3 assessment expectations for the *Wisconsin Standards for Mathematics* in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8, but students should also have instructional experiences with other sized fractions.

Wisconsin Standards for Mathematics			**Wisconsin Essential Elements for Mathematics**		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Develop understand-ing of fractions as numbers.	M.3.NF.A.1	Understand a unit fraction as the quantity formed when a whole is partitioned into equal parts and explain that a unit fraction is one of those parts (e.g., 1/4). Understand fractions are composed of unit fractions. *For example, 7/4 is the quantity formed by 7 parts of the size 1/4.*	**Understand number structures**	M.EE.3.NF.1	Differentiate a fractional part from a whole.
	M.3.NF.A.2	Understand and represent a fraction as a number on the number line. a. Understand the whole on a number line is defined as the interval from 0 to 1 and the unit fraction is defined by partitioning the interval into equal parts (i.e., equal-sized lengths). b. Represent fractions on a number line by iterating lengths of the unit fraction from 0. Recognize that the resulting interval represents the size of the fraction and that its endpoint locates the fraction as a number on the number line. *For example, 5/3 indicates the length of a line segment from 0 by iterating the unit fraction 1/3 five times and its end point locates the fraction 5/3 on the number line.*	**Understand number structures**	M.EE.3.NF.2	Differentiate a fractional part from a whole.

Number and Operations – Fractions (3.NF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Develop understand-ing of fractions as numbers. (cont'd)	M.3.NF.A.3	Explain equivalence of fractions and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size or name the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., *1/2 = 2/4*, 4/6 = 2/3) and explain why the fractions are equivalent by using a visual fraction model (e.g., tape diagram or number line). c. Express whole numbers as fractions (3 = 3/1) and recognize fractions that are equivalent to whole numbers (4/4 = 1). d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Justify the conclusions by using a visual fraction model (e.g., tape diagram or number line) and describe the result of the comparison using words and symbols (>, =, and <).	**Understand number structures**	M.EE.3.NF.3	Differentiate a fractional part from a whole.

Measurement and Data (3.MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Solve problems involving measure-ment and estimation of intervals of time, liquid volumes, and masses of objects.	M.3.MD.A.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line.	**Understand and use measurement principles and units of measure.**	M.EE.3.MD.1	Tell time to the hour on a digital clock.
	M.3.MD.A.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l), excluding compound units such as cm^3 and finding the geometric volume of a container. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. *See Appendix, Table 2B for problem situations. Do not include multiplicative comparison problems.*	**Understand and use measurement principles and units of measure.**	M.EE.3.MD.2	Identify the appropriate measurement tool to solve one-step word problems involving mass and volume.

Measurement and Data (3.MD) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Represent and interpret data.	M.3.MD.B.3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.	**Represent and interpret data displays.**	M.EE.3.MD.3	Use picture or bar graphs to answer questions about the data.
	M.3.MD.B.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units -- whole numbers, halves, fourths.	**Understand and use measurement principles and units of measure.**	M.EE.3.MD.4	Measure the length of objects using standard tools including rulers, yardsticks, and meter sticks.

Measurement and Data (3.MD) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	M.3.MD.C.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square" is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.			Not applicable. See M.EE.4.MD.3.
	M.3.MD.C.6	Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units).			Not applicable. See M.EE.4.MD.3.

Measurement and Data (3.MD) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Geometric measure-ment: understand concepts of area and relate area to multiplica-tion and to addition. (cont'd)	M.3.MD.C.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b + c* is the sum of *a x b and a x c*. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.			Not applicable. See M.EE.4.MD.3.

Measurement and Data (3.MD) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
D. Geometric measure-ment: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	M.3.MD.D.8	Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.			Not applicable. See M.EE.3.MD.4.

Geometry (3.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Reason with shapes and their attributes.	M.3.G.A.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.3.G.1	Describe attributes of two-dimensional shapes.
	M.3.G.A.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	**Solve problems involving area, perimeter, and volume.**	M.EE.3.G.2	Recognize shapes that have been partitioned into equal areas.

Grade 4
Operations and Algebraic Thinking (4.OA)

Wisconsin Standards for Mathematics			**Wisconsin Essential Elements for Mathematics**		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Use the four operations with whole numbers to solve problems.	M.4.OA.A.1	Interpret a multiplication equation as a multiplicative comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	**Use operations and models to solve problems.**	M.EE.4.OA.1	Demonstrate the connection between repeated addition and multiplication.
	M.4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. *See Appendix, Tables 2A & 2B.*	**Use operations and models to solve problems.**	M.EE.4.OA.2	Demonstrate the connection between repeated addition and multiplication.
	M.4.OA.A.3	Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies.	**Use operations and models to solve problems.**	M.EE.4.OA.3	Solve one-step real-world problems using addition or subtraction within 100.

Grade 4

Operations and Algebraic Thinking (4.OA)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Gain familiarity with factors and multiples.	M.4.OA.B.4	Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	**Use operations and models to solve problems.**	M.EE.4.OA.4	Use an understanding of multiplication to show at least one way to determine a product.

Operations and Algebraic Thinking (4.OA) (cont'd)

Wisconsin Standards for Mathematics			**Wisconsin Essential Elements for Mathematics**		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Generate and analyze patterns.	M.4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	**Understand patterns and functional thinking.**	EE.4.OA.5	Use repeating patterns to make predictions.
D. Multiply and divide within 100.	M.4.OA.D.6	Flexibly and efficiently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations [e.g., knowing that 7 x 6 can be thought of as 7 groups of 6 so one could think 5 groups of 6 is 30 and 2 more groups of 6 is 12 and 30 + 12 = 42 (informal use of the distributive property)].			Not applicable.

Number and Operations in Base Ten (4.NBT)

Grade 4 expectations for the *Wisconsin Standards for Mathematics* in this domain are limited to whole numbers less than or equal to 1,000,000.

Wisconsin Standards for Mathematics			**Wisconsin Essential Elements for Mathematics**		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Generalize place value understand-ing for multi-digit whole numbers.	M.4.NBT.A.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*			Not applicable. See M.EE.5.NBT.1
	M.4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place and describe the result of the comparison using words and symbols (>, =, and <).	**Compare, compose, and decompose numbers and sets.**	M.EE.4.NBT.2	Compare whole numbers to 10 using symbols (<, >, =).
	M.4.NBT.A.3	Use place value understanding to generate estimates for real-world problem situations, with multi-digit whole numbers, using strategies such as mental math, benchmark numbers, compatible numbers, and rounding. Assess the reasonableness of their estimates. (e.g., Is my estimate too low or too high? What degree of precision do I need for this situation?)	**Compare, compose, and decompose numbers and sets.**	M.EE.4.NBT.3	Use place value understanding to generate estimates for real-world addition and subtraction problem situations within 30, using strategies such as mental math, benchmark numbers, compatible numbers, and rounding.

Number and Operations in Base Ten (4.NBT) (cont'd)

Grade 4 expectations for the *Wisconsin Standards for Mathematics* in this domain are limited to whole numbers less than or equal to 1,000,000.

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Use place value understanding and properties of operations to perform multi-digit arithmetic.	M.4.NBT.B.4	Flexibly and efficiently add and subtract multi-digit whole numbers using strategies or algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.4.NBT.4	Add and subtract two-digit whole numbers.
	M.4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.			Not applicable. See M.EE.4.OA.4 and M.EE.5.NBT.5.
	M.4.NBT.B.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.			Not applicable. See M.EE.5.NBT.6.

Number and Operations – Fractions (4.NF)

Grade 4 assessment expectations for the *Wisconsin Standards for Mathematics* in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100 but students should also have instructional experiences with other sized fractions.

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Extend understand-ing of fraction equivalence.	M.4.NF.A.1	Understand fraction equivalence. a. Explain why a fraction is equivalent to another fraction by using visual fraction models (e.g., tape diagrams and number lines), with attention to how the number and the size of the parts differ even though the two fractions themselves are the same size. b. Understand and use a general principle to recognize and generate equivalent fractions that name the same amount.	**Understand number structures**	M.EE.4.NF.1	Identify models of one half (½) and one fourth (¼).
	M.4.NF.A.2	Compare fractions with different numerators and different denominators while recognizing that comparisons are valid only when the fractions refer to the same whole. Justify the conclusions by using visual fraction models (e.g., tape diagrams and number lines) and by reasoning about the size of the fractions, using benchmark fractions (including whole numbers), or creating common denominators or numerators. Describe the result of the comparison using words and symbols (>, =, and <).	**Understand number structures**	M.EE.4.NF.2	Identify models of one half (½) and one fourth (¼).

Number and Operations – Fractions (4.NF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	M.4.NF.B.3	Understand composing and decomposing fractions. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of unit fractions and/or multiples of that unit fraction in more than one way, recording each decomposition by an equation. Justify decompositions with explanations, visual fraction models, or equations. *For example: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract fractions, including mixed numbers, with like denominators (e.g., 3/8 + 2/8) and related denominators (e.g., 1/2 +1/4, 1/3 + 1/6) by using visual fraction models (e.g., tape diagrams and number lines), properties of operations, and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions with like and related denominators, including mixed numbers, by using visual fraction models and equations to represent the problem. Students are not required to rename fractions in lowest terms nor use least common denominators.	**Understand number structures**	M.EE.4.NF.3	Differentiate between whole and half.

Number and Operations – Fractions (4.NF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Build fractions from unit fractions by applying and extending previous understand-ings of operations on whole numbers. (cont'd)	M.4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a whole number times a fraction. a. Understand a fraction as a group of unit fractions or as a multiple of a unit fraction. *For example, 5/4 can be represented visually as 5 groups of ¼, as a sum of unit fractions ¼ + ¼ + ¼ + ¼ + ¼, or as a multiple of a unit fraction 5 x ¼.* b. Represent a whole number times a non-unit fraction (e.g., 3 x 2/5) using visual fraction models and understand this as combining equal groups of the non-unit fraction (3 groups of 2/5) and as a collection of unit fractions (6 groups of 1/5), recognizing this product as 6/5. c. Solve word problems involving multiplication of a whole number times a fraction by using visual fraction models and equations to represent the problem. Understand a reasonable answer range when multiplying with fractions. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*			Not applicable. See M.EE.4.OA.4 and M.EE.5.NBT.5.

Number and Operations – Fractions (4.NF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Understand decimal notation for fractions and compare decimal fractions.	M.4.NF.C.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*			Not applicable. See M.EE.7.NS.2c-d.
	M.4.NF.C.6	Use decimal notation for fractions with denominators 10 or 100, connect decimals to real-world contexts, and represent with visual models (e.g., number line or area model). *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line.*			Not applicable. See M.EE.7.NS.2c-d.
	M.4.NF.C.7	Compare decimals to hundredths by reasoning about their size and using benchmarks. Recognize that comparisons are valid only when the decimals refer to the same whole. Justify the conclusions, by using explanations or visual models (e.g., number line or area model) and describe the result of the comparison using words and symbols (>, =, and <).			Not applicable. See M.EE.7.NS.2c-d.

Measurement and Data (4.MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	M.4.MD.A.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min., sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).*	**Understand and use measurement principles and units of measure.**	M.EE.4.MD.1	Identify the smaller measurement unit that comprises a larger unit within a measurement system including inches/foot, centimeter/meter, minutes/hour.
	M.4.MD.A.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as a number line that feature a measurement scale.	**Understand and use measurement principles and units of measure.**	M.EE.4.MD.2	Apply concepts of measurement. a. Tell time using a digital clock. Tell time to the nearest hour using an analog clock. b. Measure mass or volume using standard tools. c. Use standard measurement to compare lengths of objects. d. Identify coins including penny, nickel, dime, quarter and their values.

Measurement and Data (4.MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Solve problems involving measurement and conversion of measure-ments from a larger unit to a smaller unit. (cont'd)	M.4.MD.A.3	Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	**Solve problems involving area, perimeter, and volume.**	M.EE.4.MD.3	Determine the area of a square or rectangle by counting units of measure (unit squares).
B. Represent and interpret data.	M.4.MD.B.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	**Represent and interpret data displays.**	M.EE.4.MD.4	Represent and interpret data on a picture or bar graph, given a model and a graph to complete.

Measurement and Data (4.MD) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Geometric measure-ment: understand concepts of angle and measure angles.	M.4.MD.C.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle" and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.4.MD.5	Recognize angles in geometric shapes.
	M.4.MD.C.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.4.MD.6	Compare angles as larger than, smaller than, or the same size as another angle.

Measurement and Data (4.MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Geometric measure-ment: understand concepts of angle and measure angles. (cont'd)	M.4.MD.C.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the problems, e.g., by using an equation with a symbol for the unknown angle measure.			Not applicable. See M.EE.4.G.2.

Geometry (4.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Draw and identify lines and angles and classify shapes by properties of their lines and angles.	M.4.G.A.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.4.G.1	Recognize parallel lines and intersecting lines.
	M.4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.4.G.2	Describe the defining attributes of two-dimensional shapes.
	M.4.G.A.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	**Solve problems involving area, perimeter, and volume.**	M.EE.4.G.3	Recognize that lines of symmetry partition shapes into equal areas.

Grade 5

Operations and Algebraic Thinking (5.OA)

Wisconsin Standards for Mathematics			**Wisconsin Essential Elements for Mathematics**		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Write and interpret numerical expressions.	M.5.OA.A.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.			Not applicable.
	M.5.OA.A.2	Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*			Not applicable.
B. Analyze patterns and relation-ships.	M.5.OA.B.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	**Understand patterns and functional thinking.**	M.EE.5.OA.3	Identify and extend numerical patterns.

Number and Operations in Base Ten (5.NBT)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand the place value system.	M.5.NBT.A.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	**Compare, compose, and decompose numbers and sets.**	M.EE.5.NBT.1	Compare numbers up to 99 using place value models.
	M.5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	**Compare, compose, and decompose numbers and sets.**	M.EE.5.NBT.2	Use the number of zeros in numbers that are powers of 10 to determine which values are equal, greater than, or less than another number.
	M.5.NBT.A.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000). b. Compare decimals to thousandths based on meanings of the digits in each place and describe the result of the comparison using words and symbols (>, =, and <).	**Compare, compose, and decompose numbers and sets.**	M.EE.5.NBT.3	Compare whole numbers up to 100 using symbols (<, >, =).

Number and Operations in Base Ten (5.NBT)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand the place value system. (cont'd)	M.5.NBT.A.4	Use place value understanding to generate estimates for problems in real-world situations, with decimals, using strategies such as mental math, benchmark numbers, compatible numbers, and rounding. Assess the reasonableness of their estimates (e.g., Is my estimate too low or too high? What degree of precision do I need for this situation?	**Compare, compose, and decompose numbers and sets.**	M.EE.5.NBT.4	Use place value understanding to generate estimates for real-world addition and subtraction problem situations within 100, using strategies such as mental math, benchmark numbers, compatible numbers, and rounding.

Number and Operations in Base Ten (5.NBT) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Perform operations with multi-digit whole numbers and with decimals to hundredths.	M.5.NBT.B.5	Flexibly and efficiently multiply multi-digit whole numbers using strategies or algorithms based on place value, area models, and the properties of operations.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.5.NBT.5	Use the meaning of multiplication to develop and understand strategies to find products with multiples of 0, 1, 2, 5, and 10 within 100.
	M.5.NBT.B.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.5.NBT.6	Illustrate the concept of division using fair and equal shares.
	M.5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.5.NBT.7	Identify models of tenths (1/10, 2/10, 3/10, 4/10, 5/10, 6/10, 7/10, 8/10, 9/10, 10/10).

Number and Operations – Fractions (5.NF)

Students are not required to rename fractions in lowest terms nor use least common denominators.

Wisconsin Standards for Mathematics			**Wisconsin Essential Elements for Mathematics**		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Use equivalent fractions as a strategy to add and subtract fractions.	M.5.NF.A.1	Add and subtract fractions and mixed numbers using flexible and efficient strategies, including renaming fractions with equivalent fractions. Justify using visual models (e.g., tape diagrams or number lines) and equations. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12.*	**Understand number structures**	M.EE.5.NF.1	Identify models of halves (½, 2/2), thirds (⅓, ⅔, 3/3), and fourths (¼, 2/4, ¾, 4/4).
	M.5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 <1/2.*			Not applicable.

Number and Operations – Fractions (5.NF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Apply and extend previous understand-ings of multiplica-tion and division to multiply and divide fractions.	M.5.NF.B.3	Interpret a fraction as an equal sharing division situation, where a quantity (the numerator) is divided into equal parts (the denominator). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, by using visual fraction models (e.g., tape diagrams or area models) or equations to represent the problem. *For example, when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*			Not applicable. See M.EE.6.RP.1.

Number and Operations – Fractions (5.NF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Apply and extend previous understand-ings of multiplica-tion and division to multiply and divide fractions. (cont'd)	M.5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction times a whole number (e.g., 2/3 x 4) or a fraction times a fraction (e.g., 2/3 x 4/5), including mixed numbers. a. Represent word problems involving multiplication of fractions using visual models to develop flexible and efficient strategies. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15.* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.			Not applicable.

Number and Operations – Fractions (5.NF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Apply and extend previous understand-ings of multiplica-tion and division to multiply and divide fractions. (cont'd)	M.5.NF.B.5	Interpret multiplication as scaling (resizing) by estimating whether a product will be larger or smaller than a given factor on the basis of the size of the other factor, without performing the indicated multiplication. a. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number and explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number. b. Relate the principle of fraction equivalence to the effect of multiplying or dividing a fraction by 1 or an equivalent form of 1 (e.g., 3/3, 5/5).			Not applicable.
	M.5.NF.B.6	Solve real-world problems involving multiplication of fractions and mixed numbers by using visual fraction models (e.g., tape diagrams, area models, or number lines) and equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.			Not applicable. See M.EE.N.CN.2b.

Number and Operations – Fractions (5.NF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Apply and extend previous understand-ings of multiplica-tion and division to multiply and divide fractions. (cont'd)	M.5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers (e.g., *1/3* ÷ 4) and whole numbers by unit fractions (e.g., 4 ÷ 1/5). Students able to multiply fractions can develop strategies to divide fractions by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade. a. Interpret and represent division of a unit fraction by a non-zero whole number as an equal sharing division situation. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. Interpret and represent division of a whole number by a unit fraction as a measurement division situation. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*			Not applicable. See M.EE.7.NS.2b.

Measurement and Data (4.MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Apply and extend previous understand-ings of multiplica-tion and division to multiply and divide fractions. (cont'd)	M.5.NF.B.7 (cont'd)	c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 4 people share 1/3 lb. of chocolate equally? Each person gets 1/12 lb. of chocolate. How many 1/5-cup servings are in 4 cups of raisins? There are 20 servings of size 1/5-cup of raisins.*			Not applicable. See M.EE.7.NS.2b.

Measurement and Data (5.MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Convert like measure-ment units within a given measure-ment system.	M.5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.	**Understand and use measurement principles and units of measure.**	M.EE.5.MD.1	Extend understanding of measurement concepts. a. Tell time using an analog or digital clock to the half or quarter hour. b. Use standard units to measure weight and length of objects. c. Indicate relative value of collections of coins.
B. Represent and interpret data.	M.5.MD.B.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	**Represent and interpret data displays.**	M.EE.5.MD.2	Represent and interpret data on a picture graph, line plot, or bar graph.

Measurement and Data (5.MD) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Geometric measure-ment: understand concepts of volume and relate volume to multiplica-tion and to addition.	M.5.MD.C.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube", is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.5.MD.3	Identify common three-dimensional shapes.
	M.5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.	**Solve problems involving area, perimeter, and volume.**	M.EE5.MD.4	Determine the volume of a rectangular prism by counting units of measure (unit cubes).

Measurement and Data (5.MD) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Geometric measure-ment: understand concepts of volume and relate volume to multiplica-tion and to addition. (cont'd)	M.5.MD.C.5	Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.	**Solve problems involving area, perimeter, and volume.**	M.EE5.MD.5	Determine the volume of a rectangular prism by counting units of measure (unit cubes).

Geometry (5.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Graph points on the coordinate plane to solve real-world and mathe-matical problems.	M.5.G.A.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.5.G.1	Sort two-dimensional figures and identify the attributes, including number of angles, sides, or corners that they have in common.
	M.5.G.A.2	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.5.G.2	Sort two-dimensional figures and identify the attributes, including number of angles, sides, or corners that they have in common.

Geometry (5.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Classify two-dimensional figures into categories based on their properties.	M.5.G.B.3	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.5.G.3	Sort two-dimensional figures and identify the attributes, including number of angles, sides, or corners that they have in common.
	M.5.G.B.4	Classify two-dimensional figures in a hierarchy based on properties.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.5.G.4	Sort two-dimensional figures and identify the attributes, including number of angles, sides, or corners that they have in common.

Grade 6

Ratios and Proportional Relationships (6.RP)

Wisconsin Standards for Mathematics			**Wisconsin Essential Elements for Mathematics**		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand ratio concepts and use ratio reasoning to solve problems. (M)	M.6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	**Understand number structures**	M.EE.6.RP.1	Demonstrate a simple ratio relationship.
	M.6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0 and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* Expectations for unit rates in this grade are limited to non-complex fractions.			Not applicable. See M.EE.7.RP.1.

Ratios and Proportional Relationships (6.RP)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand ratio concepts and use ratio reasoning to solve problems. (M) (cont'd)	M.6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.			Not applicable. See M.EE.8.F.1.

The Number System (6.NS)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Apply and extend previous understand-ings of multiplica-tion and division to divide fractions by fractions.	M.6.NS.A.1	Interpret, represent and compute division of fractions by fractions; and solve word problems by using visual fraction models (e.g., tape diagrams, area models, or number lines), equations, and the relationship between multiplication and division. *For example, create a story context for (2/3) ÷ (3/4) such as "How many 3/4 -cup servings are in 2/3 of a cup of yogurt" or "How wide is a rectangular strip of land with length 3/4 mile and area 2/3 square mile?" Explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3.*	**Compare, compose, and decompose numbers and sets.**	M.EE.6.NS.1	Compare the relationships between two unit fractions.
B. Flexibly and efficiently compute with multi-digit numbers and find common factors and multiples.	M .6.NS.B.2	Flexibly and efficiently divide multi-digit whole numbers using strategies or algorithms based on place value, area models, and the properties of operations.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.6.NS.2	Apply the concept of fair share and equal shares to divide.
	M.6.NS.B.3	Flexibly and efficiently add, subtract, multiply, and divide multi-digit decimals using strategies or algorithms based on place value, visual models, the relationship between operations and the properties of operations.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.6.NS.3	Use the meaning of multiplication and the properties of operations (e.g., the distributive property) to develop and understand strategies to find products with multiples of 3, 4, and 9 within 100.

The Number System (6.NS)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Flexibly and efficiently compute with multi- digit numbers and find common factors and multiples. (cont'd)	M.6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*			Not applicable.
C. Apply and extend previous understand-ings of numbers to the system of rational numbers. (M)	M.6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**Compare, compose, and decompose numbers and sets.**	M.EE.6.NS.5	Describe quantities having opposite directions or values as positive and negative numbers (e.g., temperature above/below zero).

The Number System (6.NS) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Apply and extend previous understandings of numbers to the system of rational numbers. (M) (cont'd)	M.6.NS.C.6	Understand a rational number as a point on the number line. Extend number lines and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line; find and position pairs of integers and other rational numbers on a coordinate plane.	**Compare, compose, and decompose numbers and sets.**	M.EE.6.NS.6	Describe quantities having opposite directions or values as positive and negative numbers (e.g., temperature above/below zero).

The Number System (6.NS) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Apply and extend previous understand= ings of numbers to the system of rational numbers. (M) (cont'd)	M.6.NS.C.7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write -3 °C > -7 °C to express the fact that -3 °C is warmer than -7 °C.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*	**Compare, compose, and decompose numbers and sets.**	M.EE.6.NS.7	Describe quantities having opposite directions or values as positive and negative numbers (e.g., temperature above/below zero).

The Number System (6.NS)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Apply and extend previous understand=ings of numbers to the system of rational numbers. (M) (cont'd)	M.6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	**Compare, compose, and decompose numbers and sets.**	M.EE.6.NS.8	Describe quantities having opposite directions or values as positive and negative numbers (e.g., temperature above/below zero).

The Expressions and Equations (6.EE)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Apply and extend previous understand-ings of arithmetic to algebraic expressions.	M.6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.	**Use operations and models to solve problems. (M)**	M.EE.6.EE.1-2	Evaluate if an equation is true.
	M.6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*			

The Expressions and Equations (6.EE)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Apply and extend previous understand-ings of arithmetic to algebraic expressions. (cont'd)	M.6.EE.A.3	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*	**Use operations and models to solve problems. (M)**	M.EE.6.EE.3	Apply the properties of addition to identify equivalent numerical expressions.
	M.6.EE.A.4	Identify when two expressions are equivalent (e.g., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*			Not applicable.

The Expressions and Equations (6.EE) (cont'd)

Wisconsin Standards for Mathematics			**Wisconsin Essential Elements for Mathematics**		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Reason about and solve one-variable equations and inequalities.	M.6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	**Use operations and models to solve problems. (M)**	M.EE.6.EE.B.5-7	Match an equation to a real-world problem in which variables are used to represent numbers.
	M.6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	**Use operations and models to solve problems. (M)**	M.EE.6.EE.B.5-7	Match an equation to a real-world problem in which variables are used to represent numbers.
	M.6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	**Use operations and models to solve problems. (M)**	M.EE.6.EE.B.5-7	Match an equation to a real-world problem in which variables are used to represent numbers.
	M.6.EE.B.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.			Not applicable.

The Expressions and Equations (6.EE) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Represent and analyze quantitative relation-ships between dependent and independent variables. (M)	M.6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*			Not applicable.

Geometry (6.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Solve real-world and mathematical problems involving area, surface area, and volume. (M)	M.6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	**Solve problems involving area, perimeter, and volume. (M)**	M.EE.6.G.1	Solve real-world and mathematical problems about area using unit squares.
	M.6.G.A.2	Find volumes of right rectangular prisms with fractional edge lengths by using physical or virtual unit cubes. Develop (construct) and apply the formulas $V = l\ w\ h$ and $V = B\ h$ to find volumes of right rectangular prisms in the context of solving real-world and mathematical problems.	**Solve problems involving area, perimeter, and volume. (M)**	M.EE.6.G.2	Solve real-world and mathematical problems about volume using unit cubes.
	M.6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.			Not applicable.
	M.6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.			Not applicable.

Statistics and Probability (6.SP)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Develop understanding of statistical variability. (M)	M.6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	**Represent and interpret data displays. (M)**	M.EE.6.SP.1	Display data on a graph or table that shows variability in the data.
	M.6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	**Represent and interpret data displays. (M)**	M.EE.6.SP.2	Display data on a graph or table that shows variability in the data.
	M.6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.			Not applicable.

Statistics and Probability (6.SP) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Summarize and describe distribu-tions. (M)	M.6.SP.B.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	**Represent and interpret data displays. (M)**	M.EE.6.SP.4	Display data on a graph or table that shows variability in the data.
	M.6.SP.B.5	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered and the quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation) were given. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	**Represent and interpret data displays. (M)**	M.EE.6.SP.5	Summarize data distributions shown in graphs or tables.

Grade 7
Ratios and Proportional Relationships (7.RP)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Analyze proportional relation-ships and use them to solve real-world and mathema-tical problems. (M)	M.7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*	**Understand number structures**	M.EE.7.RP.1	Use a ratio to model or describe a relationship.

Ratios and Proportional Relationships (7.RP)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Analyze proportional relation- ships and use them to solve real-world and mathema-tical problems. (M) (cont'd)	M.7.RP.A.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate.	**Understand number structures**	M.EE.7.RP.2	Use a ratio to model or describe a relationship.
	M.7.RP.A.3	Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	**Understand number structures**	M.EE.7.RP.3	Use a ratio to model or describe a relationship.

The Number System (7.NS)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Apply and extend previous understand-ings of operations with fractions to add, subtract, multiply, and divide rational numbers.	M.7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line. a. Describe situations in which opposite quantities combine to make 0. *For example, if you earn $10 and then spend $10, you are left with $0.* b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.7.NS.1	Add fractions with like denominators (halves, thirds, fourths, and tenths) with sums less than or equal to one.

The Number System (7.NS)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Apply and extend previous understand-ings of operations with fractions to add, subtract, multiply, and divide rational numbers. (cont'd)	M.7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	**a-b.** **Calculate accurately and efficiently using simple arithmetic operations.** **c-d.** **Understand number structures**	M.EE.7.NS.2	Apply and extend previous understandings of multiplication, division, and fractions. a. Multiply within 100 using strategies such as the properties of operations [e.g., knowing that 7 x 6 can be thought of as 7 groups of 6 so one could think 5 groups of 6 is 30 and 2 more groups of 6 is 12 and 30 + 12 = 42 (informal use of the distributive property)]. b. Solve division problems within 100, including divisors of 1-5 and 10, without remainders. c-d. Express a fraction with a denominator of 10 as a decimal.

The Number System (7.NS) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Apply and extend previous understand-ings of operations with fractions to add, subtract, multiply, and divide rational numbers. (cont'd)	M.7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers. (Note: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)	**Compare, compose, and decompose numbers and sets.**	M.EE.7.NS.3	Using real world examples, compare quantities represented as decimals to tenths.

The Expressions and Equations (7.EE)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Use properties of operations to generate equivalent expressions.	M.7.EE.A.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	**Use operations and models to solve problems.**	M.EE.7.EE.1	Use the properties of operations as strategies to demonstrate that expressions are equivalent.
	M.7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*	**Understand patterns and functional thinking. (M)**	M.EE.7.EE.2	Identify an arithmetic sequence of whole numbers with a whole number common difference.
B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations. (M)	M.7.EE.B.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50.*			Not applicable.

The Expressions and Equations (7.EE)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Solve real-life and mathematic al problems using numerical and algebraic expressions and equations. (M) (cont'd)	M.7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Flexibly and efficiently apply the properties of operations and equality to solve equations of these forms. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make and describe the solutions.*	**Use operations and models to solve problems.**	M.EE.7.EE.4	Use the concept of equality to solve one-step addition and subtraction equations with models.

Geometry (7.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Draw, construct, and describe geometrical figures and describe the relation-ships between them.	M.7.G.A.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.7.G.1	Match two similar geometric shapes that are proportional in size and have the same orientation.
	M.7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.7.G.2	Recognize geometric shapes with given conditions.
	M.7.G.A.3	Describe the two-dimensional figures that result from slicing three dimensional figures parallel to the base, as in plane sections of right rectangular prisms and right rectangular pyramids.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.7.G.3	Match a two-dimensional shape with a three-dimensional shape that shares an attribute.

Geometry (7.G) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (M)	M.7.G.B.4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	**Solve problems involving area, perimeter, and volume. (M)**	M.EE.7.G.4	Determine the perimeter of a rectangle.
	M.7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.7.G.5	Recognize angles that are acute, obtuse, and right.
	M.7.G.B.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**Solve problems involving area, perimeter, and volume. (M)**	M.EE.7.G.6	Determine the area of a rectangle using the formula for length x width and confirm the result using tiling or partitioning into unit squares.

Statistics and Probability (7.SP)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Use random sampling to draw inferences about a population. (M)	M.7.SP.A.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	**Represent and interpret data displays. (M)**	M.EE.7.SP.1	Answer a data related question, given a model of the data from a student experiment or collection.
	M.7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	**Represent and interpret data displays. (M)**	M.EE.7.SP.2	Answer a data related question, given a model of the data from a student experiment or collection.

Statistics and Probability (7.SP) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Draw informal comparative inferences about two populations. (M)	M.7.SP.B.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	**Represent and interpret data displays. (M)**	M.EE.7.SP.3	Compare two sets of data found within a single data display such as a picture graph, line plot, or bar graph.
	M.7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*			Not applicable. See M.EE.SP.ID.4.

Statistics and Probability (7.SP) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Investigate chance processes and develop, use, and evaluate probability models. (M)	M.7.SP.C.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	**Represent and interpret data displays. (M)**	M.EE.7.SP.5	Describe the probability of events occurring as possible or impossible.
	M.7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	**Represent and interpret data displays. (M)**	M.EE.7.SP.6	Describe the probability of events occurring as possible or impossible.

Statistics and Probability (7.SP) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Investigate chance processes and develop, use, and evaluate probability models. (M) (cont'd)	M.7.SP.C.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	**Represent and interpret data displays. (M)**	M.EE.7.SP.7	Describe the probability of events occurring as possible or impossible.

Statistics and Probability (7.SP) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Investigate chance processes and develop, use, and evaluate probability models. (M) (cont'd)	M.7.SP.C.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*			Not applicable.

Grade 8

The Number System (8.NS)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Know that there are numbers that are not rational and approx-imate them by rational numbers.	M.8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and use patterns to rewrite a decimal expansion that repeats into a rational number.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.8.NS.1	Subtract fractions with like denominators, including halves, thirds, fourths, and tenths with minuends less than or equal to one.
	M.8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions (e.g., $\pi\pi 2$). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	**a. Understand number structures** **b. Compare, compose, and decompose numbers and sets.**	M.EE.8.NS.2	Represent and compare fractions and decimal fractions to hundredths. a. Express a fraction with a denominator of 100 as a decimal. b. Using real world examples, compare quantities represented as decimals to hundredths.

The Expressions and Equations (8.EE)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Work with radicals and integer exponents.	M.8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, 32 x 3-5 = 3-3 = 1/33 = 1/27.*	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.8.EE.1	Identify and understand the meaning of exponents.
	M.8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form x2 = p and x3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √ 2 is irrational.	**Understand patterns and functional thinking. (M)**	M.EE.8.EE.2	Identify a geometric sequence of whole numbers with a whole number common ratio.
	M.8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 x 108 and the population of the world as 7 x 109, and determine that the world population is more than 20 times larger.*	**Compare, compose, and decompose numbers and sets.**	M.EE.8.EE.3	Compose and decompose whole numbers up to 999.
	M.8.EE.A.4	Use technology to interpret and perform operations with numbers expressed in scientific notation. Choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading).	**Compare, compose, and decompose numbers and sets.**	M.EE.8.EE.4	Compose and decompose whole numbers up to 999.

The Expressions and Equations (8.EE)(cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Understand the connections between proportional relation-ships, lines, and linear equations.	M.8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	**Understand patterns and functional thinking. (M)**	M.EE.8.EE.5	Graph a simple ratio by connecting the origin to a point representing the ratio in the form of *y/x*. *For example, when given a ratio in standard form (2:1), convert to 2/1, and plot the point (1,2).*
	M.8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	**Understand patterns and functional thinking. (M)**	M.EE.8.EE.6	Graph a simple ratio by connecting the origin to a point representing the ratio in the form of *y/x*. *For example, when given a ratio in standard form (2:1), convert to 2/1, and plot the point (1,2).*

The Expressions and Equations (8.EE)(cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Analyze and solve linear equations and pairs of simulta-neous linear equations. (M)	M.8.EE.C.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into equivalent forms. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	**Use operations and models to solve problems. (M)**	M.EE.8.EE.7	Solve simple algebraic equations with one variable using addition and subtraction.

The Expressions and Equations (8.EE) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Analyze and solve linear equations and pairs of simulta- neous linear equations. (M) (cont'd)	M.8.EE.C.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables by graphing and analyzing tables. Solve simple cases represented in algebraic symbols by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*			Not applicable. See M.EE.8.EE.5.

Functions (8.F)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Define, evaluate, and compare functions.	M.8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a numerically valued function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.	**Understand patterns and functional thinking. (M)**	M.EE.8.F.1	Given a table of data for a linear function containing at least 2 complete ordered pairs, identify a missing number that completes another ordered pair.
	M.8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	**Understand patterns and functional thinking. (M)**	M.EE.8.F.2	Given a table of data for a linear function containing at least 2 complete ordered pairs, identify a missing number that completes another ordered pair.
	M.8.F.A.3	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	**Understand patterns and functional thinking. (M)**	M.EE.8.F.3	Given a table of data for a linear function containing at least 2 complete ordered pairs, identify a missing number that completes another ordered pair.

Functions (8.F) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Use functions to model relation-ships between quantities. (M)	M.8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	**Understand patterns and functional thinking. (M)**	M.EE.8.F.4	Determine the values or rule of a function using a graph or a table.
	M.8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear, continuous or discrete). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**Understand patterns and functional thinking. (M)**	M.EE.8.F.5	Describe how a graph represents a relationship between two quantities.

Geometry (8.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand congruence and similarity using physical models, transparen-cies, or geometry software.	M.8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.8.G.1	Recognize translations, rotations, and reflections of shapes.
	M.8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.8.G.2	Identify shapes that are congruent.
	M.8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.			Not applicable.
	M.8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.8.G.4	Identify similar shapes with and without rotation.

Geometry (8.G)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand congruence and similarity using physical models, transparen-cies, or geometry software. (cont'd)	M.8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.8.G.5	Compare any angle to a right angle, and describe the angle as greater than, less than, or congruent to the right angle.

Geometry (8.G) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Understand and apply the Pythago- rean Theorem. (M)	M.8.G.B.6	Justify the relationship between the lengths of the legs and the length of the hypotenuse of a right triangle, and the converse of the Pythagorean theorem.			Not applicable.
	M.8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.			Not applicable.
	M.8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.			Not applicable.
C. Solve real-world and mathe-matical problems involving volume of cylinders, cones, and spheres. (M)	M.8.G.C.9	Know the relationship among the formulas for the volumes of cones, cylinders, and spheres (given the same height and diameter) and use them to solve real-world and mathematical problems.	**Solve problems involving area, perimeter, and volume. (M)**	M.EE.8.G.9	Use the formulas for perimeter and area of rectangles, and volume of rectangular prisms to solve real-world and mathematical problems.

Statistics and Probability (8.SP)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Investigate patterns of association in bivariate data. (M)	M.8.SP.A.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.			Not applicable.
	M.8.SP.A.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.			Not applicable. See M.EE.SP.ID.1.
	M.8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*			Not applicable.

Statistics and Probability (8.SP)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Investigate patterns of association in bivariate data. (M) (cont'd)	M.8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	**Represent and interpret data displays. (M)**	M.EE.8.SP.4	Construct a graph or table from given categorical data and interpret the data.

High School

Number and Quantity

The Real Number System (N-RN)

Wisconsin Standards for Mathematics			**Wisconsin Essential Elements for Mathematics**		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Extend the properties of exponents to rational exponents.	M.N.RN.A.1 (F2Y)	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.N.RN.1	Determine the value of a quantity that is expressed with an exponent.
	M.N.RN.A.2 (F2Y)	Rewrite expressions involving radicals and rational exponents using the properties of exponents.			Not applicable.
B. Use properties of rational and irrational numbers.	M.N.RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.			Not applicable.

Quantities (N-Q)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Reason quan-titatively and use units to solve problems. (M)	M.N.Q.A.1 (F2Y)	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**Understand and use measurement principles and units of measure. (M)**	M.EE.N.Q.1	Express quantities as a precise measurement based on the context.
	M.N.Q.A.2 (F2Y)	Define appropriate quantities for the purpose of descriptive modeling.	**Understand and use measurement principles and units of measure. (M)**	M.EE.N.Q.2	Express quantities as a precise measurement based on the context.
	M.N.Q.A.3 (F2Y)	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	**Understand and use measurement principles and units of measure. (M)**	M.EE.N.Q.3	Express quantities as a precise measurement based on the context.

The Complex Number System (N-CN)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Perform arithmetic operations with complex numbers.	M.N.CN.A.1	Know there is a complex number i such that $ii^2 = -1$, and every complex number has the form a + bi with a and b real. Understand why complex numbers exist.			Not applicable.
	M.N.CN.A.2	(+) Use the relation $ii^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.N.CN.2	Use properties (commutative, associative, and distributive) and place value understandings to calculate and solve problems. a. Add, subtract, and multiply whole numbers. b. Add, subtract, and multiply decimals. c. Solve real-world problems involving addition, subtraction, and multiplication of whole numbers and decimals.
	M.N.CN.A.3	(+) Find the conjugate of a complex number; use conjugates to find moduli (absolute values) and quotients of complex numbers.			Not applicable.

The Complex Number System (N-CN) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Represent complex numbers and their operations on the complex plane.	M.N.CN.B.4	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number.			Not applicable.
	M.N.CN.B.5	(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1+\sqrt{3}i)^3 = 8$ *because* $(-1+\sqrt{3}i)$ *has modulus 2 and argument 120°.*			Not applicable.
	M.N.CN.B.6	(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.			Not applicable.

The Complex Number System (N-CN) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Use complex numbers in polynomial identities and equations.	M.N.CN.C.7	Solve quadratic equations with real coefficients that have complex solutions. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.			Not applicable.
	M.N.CN.C.8	(+) Extend polynomial identities to the complex numbers. For example, rewrite x^2+ 4 as $(x + 2i)(x - 2i)$.			Not applicable.
	M.N.CN.C.9	(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.			Not applicable.

Vector and Matrix Quantities (N-VM)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Represent and model with vector quantities. (M)	M.N.VM.A.1	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, ***v***).			Not applicable.
	M.N.VM.A.2	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.			Not applicable.
	M.N.VM.A.3	(+) Solve problems involving velocity and other quantities that can be represented by vectors.			Not applicable.

Vector and Matrix Quantities (N-VM) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Perform operations on vectors.	M.N.VM.B.4	(+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction **v** - **w** as **v** + (-**w**), where -**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise.			Not applicable.
	M.N.VM.B.5	(+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple *c***v** using $\|\|c\mathbf{v}\|\| = \|c\|\mathbf{v}$. Compute the direction of c**v** knowing that when $\|c\|\mathbf{v} \neq 0$, the direction of c**v** is either along **v** (for c > 0) or against **v** (for c < 0).			Not applicable.

Vector and Matrix Quantities (N-VM) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Perform operations on matrices and use matrices in applications. (M)	M.N.VM.C.6	(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.			Not applicable.
	M.N.VM.C.7	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.			Not applicable.
	M.N.VM.C.8	(+) Add, subtract, and multiply matrices of appropriate dimensions.			Not applicable.
	M.N.VM.C.9	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.			Not applicable.
	M.N.VM.C.10	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.			Not applicable.
	M.N.VM.C.11	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.			Not applicable.

Vector and Matrix Quantities (N-VM) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Perform operations on matrices and use matrices in applications. (M) (cont'd)	M.N.VM.C.12	(+) Work with 2 x 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area.			Not applicable.

Algebra
Seeing Structure in Expressions (A-SSE)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Interpret the structure of expressions. (M)	M.A.SSE.A.1 (F2Y)	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. *For example, in the expression representing height of a projective, -16t2+vt+c recognizing there are three terms in the expression, factors within some of the terms, and coefficients. Interpret within the context the meaning of the coefficient -16 as related to gravity, the factor of v as the initial velocity, and the c-term as initial height.* b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret the expression representing population growth P(1+r)n as the product of P and a factor not depending on P. Interpret the meaning of the P-factor as initial population, and the other factor as being related to growth rate and a period of time.*	**Use operations and models to solve problems. (M)**	M.EE.A.SSE.1	Identify an algebraic expression involving one arithmetic operation to represent a real-world problem.
	M.A.SSE.A.2 (F2Y)	Use the structure of an expression to identify ways to rewrite it. *For example, see x^4-y^4 as $(x^2)^2$-$(y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2$-$y^2)(x^2+y^2)$.*			Not applicable.

Seeing Structure in Expressions (A-SSE) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Write expressions in equivalent forms to solve problems. (M)	M.A.SSE.B.3 (F2Y)	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example, if the expression 1.15^t represents growth in an investment account at time t (measured in years), it can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly rate of return is 1.2% based on an annual growth rate of 15%.*	**Use operations and models to solve problems. (M)**	M.EE.A.SSE.3	Solve simple algebraic equations with one variable using multiplication and division.
	M.A.SSE.B.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. *For example, calculating mortgage payments or tracking the amount of an antibiotic in the human body when prescribed for an infection.*	**Understand patterns and functional thinking. (M)**	M.EE.A.SSE.4	Determine the successive term in a geometric sequence given the common ratio.

Arithmetic with Polynomials and Rational Expressions (A-APR)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Perform arithmetic operations on polynomials.	M.A.APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.			Not applicable.
B. Understand the relationship between zeros and factors of polynomials.	M.A.APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.			Not applicable.
	M.A.APR.B.3	Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.			Not applicable.

Arithmetic with Polynomials and Rational Expressions (A-APR) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Use polynomial identities to solve problems.	M.A.APR.C.4	Prove polynomial identities and use them to describe numerical relationships. *For example, use* $(a+20)^2=a^2+40a+400$ *to mentally or efficiently square numbers in the 20s. (e.g.,* $22^2=2^2+2*40+400=484$*). Generalize to other double-digit numbers. Use* $a^2=(A+b)(a-b)+b^2$ *and multiples of a*10 to square, e.g.,* $22^2=(22+12)(22-12)+12^2=340+144=484$*. Recognize the visual representation of* $(a+2b)^2-a^2=4ab$ *as the area of a frame and find equivalent expressions.*			Not applicable.
	M.A.APR.C.5	(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.			Not applicable.

Arithmetic with Polynomials and Rational Expressions (A-APR) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
D. Rewrite rational expressions.	M.A.APR.D.6	Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x)*, where *a(x)*, *b(x)*, *q(x)*, and *r(x)* are polynomials with the degree of *r*(*x*) less than the degree of *b*(*x*), using inspection, long division, or, for the more complicated examples, a computer algebra system.			Not applicable.
	M.A.APR.D.7	(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.			Not applicable.

Creating Equations (A-CED)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Create equations that describe numbers or relation-ships. (M)	M.A.CED.A.1 (F2Y)	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	**Use operations and models to solve problems. (M)**	M.EE.A.CED.1	Determine an equation involving at least one variable that can be used to solve a real-world problem.
	M.A.CED.A.2 (F2Y)	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	**Use operations and models to solve problems. (M)**	M.EE.A.CED.2	Solve one-step inequalities.
	M.A.CED.A.3 (F2Y)	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	**Use operations and models to solve problems. (M)**	M.EE.A.CED.3	Solve one-step inequalities.
	M.A.CED.A.4 (F2Y)	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange the formula C = 5/9 (F-32) so you solve for F.*	**Use operations and models to solve problems. (M)**	M.EE.A.CED.4	Solve one-step inequalities.

Reasoning with Equations and Inequalities (A-REI)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand solving equations as a process of reasoning and explain the reasoning.	M.A.REI.A.1 (F2Y)	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.			Not applicable.
	M.A.REI.A.2	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.			Not applicable. See M.EE.A.CED.1.
B. Solve equations and inequalities in one variable.	M.A.REI.B.3 (F2Y)	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.			Not applicable. See M.EE.A.CED.1.
	M.A.REI.B.4 (F2Y)	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, factoring, and graphing as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.			Not applicable.

Reasoning with Equations and Inequalities (A-REI) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Solve systems of equations.	M.A.REI.C.5 (F2Y)	Justify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.			Not applicable.
	M.A.REI.C.6 (F2Y)	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.			Not applicable. See M.EE.A.REI.10.
	M.A.REI.C.7 (F2Y)	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*			Not applicable. See M.EE.A.REI.10.
	M.A.REI.C.8	(+) Represent a system of linear equations as a single matrix equation in a vector variable.			Not applicable.
	M.A.REI.C.9	(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).			Not applicable.

Reasoning with Equations and Inequalities (A-REI) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
D. Represent and solve equations and inequalities graphically.	M.A.REI.D.10 (F2Y)	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	**Understand patterns and functional thinking. (M)**	M.EE.A.REI.10	Interpret the meaning of a point on the graph of a line. *For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.*
	M.A.REI.D.11 (F2Y)	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.	**Understand patterns and functional thinking. (M)**	M.EE.A.REI.11	Interpret the meaning of a point on the graph of a line. *For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.*
	M.A.REI.D.12 (F2Y)	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**Understand patterns and functional thinking. (M)**	M.EE.A.REI.12	Interpret the meaning of a point on the graph of a line. *For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.*

Functions

Interpreting Functions (F-IF)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand the concept of a function and use function notation.	M.F.IF.A.1 (F2Y)	Understand that a function from one set, discrete or continuous, (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.	**Understand patterns and functional thinking. (M)**	M.EE.F.IF.1	Use the concept of function to solve problems.
	M.F.IF.A.2 (F2Y)	Use function notation, evaluate functions. and interpret statements that use function notation in terms of a context.	**Understand patterns and functional thinking. (M)**	M.EE.F.IF.2	Use the concept of function to solve problems.
	M.F.IF.A.3 (F2Y)	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, in an arithmetic sequence, f(x) = f(x-1) + C or in a geometric sequence, f(x) = f(x-1) * C, where C is a constant.*	**Understand patterns and functional thinking. (M)**	M.EE.F.IF.3	Use the concept of function to solve problems.

Interpreting Functions (F-IF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Interpret functions that arise in applications in terms of context. (M)	M.F.IF.B.4 (F2Y)	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	**Understand patterns and functional thinking. (M)**	M.EE.F.IF.4	Construct graphs that represent linear functions with different rates of change and interpret the graphs. *For example, which rate is faster/slower or higher/lower.*
	M.F.IF.B.5	Relate the domain of a function to its graph and find an appropriate domain (discrete or continuous) in the context of the given problem.	**Understand patterns and functional thinking. (M)**	M.EE.F.IF.5	Construct graphs that represent linear functions with different rates of change and interpret the graphs. *For example, which rate is faster/slower or higher/lower.*
	M.F.IF.B.6	Calculate and interpret the average rate of change of a linear or nonlinear function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	**Understand patterns and functional thinking. (M)**	M.EE.F.IF.6	Construct graphs that represent linear functions with different rates of change and interpret the graphs. *For example, which rate is faster/slower or higher/lower.*

Interpreting Functions (F-IF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Analyze functions using different representa-tions. (M)	M.F.IF.C.7 M.F.IF.C.7a (F2Y)	Graph functions expressed symbolically and show key features of the graph using an efficient method. a. Graph linear and quadratic functions and show intercepts, maxima, and minima; and exponential functions, showing intercepts and end behavior. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.			Not applicable. See M.EE.F.IF.1.

Interpreting Functions (F-IF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Analyze functions using different represent-tations. (M) (cont'd)	M.F.IF.C.8 (F2Y)	Write a function defined by an expression in equivalent forms to reveal and explain different properties of the function. a. Use an efficient process to rewrite $f(x) = ax^2+bx+c$ as $f(x) = a(x-h)^2 +k$ or $f(x)=a(x-p)(x-q)$ to determine the characteristics of the function and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions, where t is in years, such as* $y = (1.01)^{12t}$ *is approximately* $y = (1.127)^t$*, where t is in years, meaning it is a 1% growth rate each month and a 12.7% growth rate each year. Identify percent rate of change in functions, where t is in years, such as* $y = (1.2)^{(t/10)}$ *is approximately* $y = (1.018)^t$*, meaning it is a 20% growth rate each decade and a 1.8% growth rate each year.*			Not applicable.
	M.F.IF.C.9 (F2Y)	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*			Not applicable.

Building Functions (F-BF)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Build a function that models a relationship between two quantities. (M)	M.F.BF.A1	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example: The temperature of a cup of coffee can be modeled by combining together a function representing difference in temperature and the actual room temperature, which results in an exponential model. An average cost function can be created by dividing the cost of purchasing n items by the number of n items purchased, which results in a rational function. c. Work with composition of functions using tables, graphs and symbols. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*	**Understand patterns and functional thinking. (M)**	M.EE.F.BF.1	Select a graph from the first quadrant of the coordinate plane that represents a situation involving constant rate of change. c. Not applicable.
	M.F.BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	**Understand patterns and functional thinking. (M)**	M.EE.F.BF.2	Determine an arithmetic sequence with whole numbers when provided a recursive rule.

Building Functions (F-BF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Build new functions from existing functions.	M.F.BF.B.3	Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* using transformations for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.			Not applicable.
	M.F.BF.B.4	Identify and create inverse functions, using tables, graphs, and symbolic methods to solve for the other variable. *For example: Each car in a state is assigned a unique license plate number and each license plate number is assigned to a unique car; thus, there is an inverse relationship. Rearrange the formula C=59(F-32) so you solve for F. You examine a table of values and realize the inputs and outputs are invertible. Two graphs are symmetrical about the line y = x.*			Not applicable.
	M.F.BF.B5	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.			Not applicable.

Linear, Quadratic, and Exponential Models (F-LE)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Construct and compare linear, quadratic, and exponential models and solve problems. (M)	M.F.LE.A.1 (F2Y)	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	**Understand patterns and functional thinking. (M)**	M.EE.F.LE.1	Model a simple linear function such as $y=mx$ to show that these functions increase by equal amounts over equal intervals.
	M.F.LE.A.2 (F2Y)	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	**Understand patterns and functional thinking. (M)**	M.EE.F.LE.2	Model a simple linear function such as $y=mx$ to show that these functions increase by equal amounts over equal intervals.
	M.F.LE.A.3 (F2Y)	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	**Understand patterns and functional thinking. (M)**	M.EE.F.LE.3	Model a simple linear function such as $y=mx$ to show that these functions increase by equal amounts over equal intervals.

Linear, Quadratic, and Exponential Models (F-LE)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Construct and compare linear, quadratic, and exponential models and solve problems. (M) (cont'd)	M.F.LE.A.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2,10, or *e*; evaluate the logarithm using technology.			Not applicable.
B. Interpret expressions for functions in terms of the situation they model.	M.F.LE.B.5 (F2Y)	Interpret the parameters in a linear or exponential function in terms of a context.			Not applicable. See M.EE.F.IF.1.

Trigonometric Functions (F-TF)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Extend the domain of the trigono-metric functions of the unit circle.	M.F.TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.			Not applicable.
	M.F.TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.			Not applicable.
	M.F.TF.A.3	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.			Not applicable.
	M.F.TF.A.4	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.			Not applicable.

Trigonometric Functions (F-TF) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Model periodic phenomena with trigono-metric functions. (M)	M.F.TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.			Not applicable.
	M.F.TF.B.6	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.			Not applicable.
	M.F.TF.B.7	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context.			Not applicable.
C. Prove and apply trigono-metric identities.	M.F.TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.			Not applicable.
	M.F.TF.C.9	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.			Not applicable.

Geometry

Congruence (G-CO)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Experiment with transfor-mations in the plane.	M.G.CO.A.1 (F2Y)	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.G.CO.1	Know the attributes of geometric elements and two-dimensional shapes. a. Know the attributes of points, rays, lines, line segments, perpendicular lines, parallel lines, and angles. b. Know the attributes of two-dimensional shapes.
	M.G.CO.A.2 (F2Y)	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).			Not applicable.
	M.G.CO.A.3 (F2Y)	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.			Not applicable.

Congruence (G-CO)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Experiment with transfor-mations in the plane. (cont'd)	M.G.CO.A.4 (F2Y)	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.G.CO.4	Given a geometric figure and a rotation, reflection, or translation of that figure, identify the components of the two figures that are congruent.
	M.G.CO.A.5 (F2Y)	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.G.CO.5	Given a geometric figure and a rotation, reflection, or translation of that figure, identify the components of the two figures that are congruent.

Congruence (G-CO) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Understand congruence in terms of rigid motion.	M.G.CO.B.6 (F2Y)	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.G.CO.6	Identify corresponding congruent and similar parts of shapes.
	M.G.CO.B.7 (F2Y)	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.G.CO.7	Identify corresponding congruent and similar parts of shapes.
	M.G.CO.B.8 (F2Y)	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.G.CO.8	Identify corresponding congruent and similar parts of shapes.

Congruence (G-CO) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Prove geometric theorems.	M.G.CO.C.9 (F2Y)	Prove theorems about lines and angles. Theorems should include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.			Not applicable.
	M.G.CO.C.10 (F2Y)	Prove theorems about triangles. Theorems should include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.			Not applicable.
	M.G.CO.C.11 (F2Y)	Prove theorems about parallelograms. Theorems should include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.			Not applicable.

Congruence (G-CO) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
D. Make geometric con-structions.	M.G.CO.D.12 (F2Y)	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.			Not applicable.
	M.G.CO.D.13 (F2Y)	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.			Not applicable.

Similarity, Right Triangles, and Trigonometry (G-SRT)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand similarity in terms of similarity trans-formations.	M.G.SRT.A.1 (F2Y)	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.			Not applicable. See M.EE.G.CO.6.
	M.G.SRT.A.2 (F2Y)	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.			Not applicable. See M.EE.G.CO.6.
	M.G.SRT.A.3 (F2Y)	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.			Not applicable. See M.EE.G.CO.6.

Similarity, Right Triangles, and Trigonometry (G-SRT) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Prove theorems involving similarity.	M.G.SRT.B.4 (F2Y)	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.			Not applicable.
	M.G.SRT.B.5 (F2Y)	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.			Not applicable. See M.EE.G.CO.6.

Similarity, Right Triangles, and Trigonometry (G-SRT) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Define trigo-nometric ratios and solve problems involving right triangles. (M)	M.G.SRT.C.6 (F2Y)	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.			Not applicable.
	M.G.SRT.C.7 (F2Y)	Explain and use the relationship between the sine and cosine of complementary angles.			Not applicable.
	M.G.SRT.C.8 (F2Y)	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.			Not applicable.
D. Apply trigonometry to general triangles.	M.G.SRT.D.9	(+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.			Not applicable.
	M.G.SRT.D.10	(+) Prove the Laws of Sines and Cosines and use them to solve problems.			Not applicable.
	M.G.SRT.D.11	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).			Not applicable.

Circles (G-C)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand and apply theorems about circles.	M.G.C.A.1 (F2Y) [WI.2010. G.C.A.2 and G.C.A.3]	Identify and describe relationships among inscribed angles, radii, and chords. Prove properties of angles for a quadrilateral inscribed in a circle. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*			Not applicable.
B. Find arc lengths and areas of sectors of circles.	M.G.C.B.2 (F2Y) [WI.2010. G.C.B.5]	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.			Not applicable.

Expressing Geometric Properties (G-GPE)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Translate between the geometric description and the equation for a conic section.	M.G.GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.			Not applicable.
	M.G.GPE.A.2	(+) Derive the equation of a parabola given a focus and directrix.			Not applicable.
	M.G.GPE.A.3	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.			Not applicable.

Expressing Geometric Properties (G-GPE) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Use coordinates to prove simple geometric theorems alge-braically.	M.G.GPE.B.4 (F2Y)	Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*			Not applicable.
	M.G.GPE.B.5 (F2Y)	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).			Not applicable. See M.EE.G.CO.1.
	M.G.GPE.B.6 (F2Y)	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.			Not applicable. See M.EE.G.CO.1.
	M.G.GPE.B.7 (F2Y)	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).	**Solve problems involving area, perimeter, and volume. (M)**	M.EE.G.GPE.7	Find perimeters and areas of squares and rectangles to solve real-world problems.

Geometric Measurement and Dimension (G-GMD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Explain volume formulas and use them to solve problems. (M)	M.G.GMD.A.1 (F2Y)	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	**Solve problems involving area, perimeter, and volume. (M)**	M.EE.G.GMD.1	Estimate the volumes of containers, the areas of figures, and the perimeters of figures. Check the estimates using formulas or models.
	M.G.GMD.A.2 (F2Y) [WI.2010. G.GMC.A.3]	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	**Solve problems involving area, perimeter, and volume. (M)**	M.EE.G.GMD.2	Estimate the volumes of containers, the areas of figures, and the perimeters of figures. Check the estimates using formulas or models.,
B. Visualize relation-ships between two-dimensional and three-dimensional objects.	M.G.GMD.B.3 (F2Y) [WI.2010. G.GMD.4]	Identify three-dimensional objects generated by rotations of two-dimensional objects.			Not applicable.

Geometric Measurement and Dimension (G-GMD) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Apply geometric concepts in modeling situations. (M)	M.G.GMD.C.4 (F2Y) [WI.2010. M.G.MG.A.1]	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.G.GMD.4	Use properties of geometric shapes to describe real life objects.
	M.G.GMD.C.5 (F2Y) [WI.2010. M.G.MG.A.2]	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.G.GMD.5	Use properties of geometric shapes to describe real life objects.
	M.G.GMD.C.6 (F2Y) [WI.2010. M.G.MG.A.3]	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	**Understand and use geometric properties of two- and three-dimensional shapes.**	M.EE.G.GMD.6	Use properties of geometric shapes to describe real life objects.

Statistics and Probability (SP)

Interpreting Categorical and Quantitative Data (S-ID)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Summarize, represent, and interpret data on a single count or measure-ment variable. (M)	M.SP.ID.A.1 (F2Y)	Represent data with plots on the real number line (dot plots, histograms, and box plots).	**Represent and interpret data displays. (M)**	M.EE.SP.ID.1	Given data, construct a graph (line, pie, bar, or picture) or table and interpret the data.
	M.SP.ID.A.2 (F2Y)	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	**Represent and interpret data displays. (M)**	M.EE.SP.ID.2	Given data, construct a graph (line, pie, bar, or picture) or table and interpret the data.
	M.SP.ID.A.3 (F2Y)	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**Represent and interpret data displays. (M)**	M.EE.SP.ID.3	Interpret trends on a graph or chart.
	M.SP.ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use statistical packages calculators, spreadsheets, and tables to estimate areas under the normal curve.	**Represent and interpret data displays. (M)**	M.EE.SP.ID.4	Calculate the mean of a given data set.

Interpreting Categorical and Quantitative Data (S-ID) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Summarize, represent, and interpret data on two categorical and quantitative variables. (M)	M.SP.ID.B.5 (F2Y)	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies as examples of proportionality and disproportionality). Recognize possible associations and trends in the data.			Not applicable. See M.EE.F.IF.1.
	M.SP.ID.B.6 (F2Y)	Represent data on two quantitative variables on a scatter plot and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize appropriate families of functions to model. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.			Not applicable.

Interpreting Categorical and Quantitative Data (S-ID) (cont'd)

Wisconsin Standards for Mathematics			**Wisconsin Essential Elements for Mathematics**		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
C. Interpret linear models (M)	M.SP.ID.C.7 (F2Y)	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.			Not applicable. See M.EE.F.IF.4.
	M.SP.ID.C.8 (F2Y)	Use technology to create a correlation coefficient for a linear fit and then interpret its meaning for the model.			Not applicable.
	M.SP.ID.C.9 (F2Y)	Distinguish between correlation and causation. *For example, cities with a higher number of fast food restaurants tend to have more hospitals. While there is a clear correlation (likely driven by population), we cannot conclude that fast food restaurants are causing more hospitals to open. There is a relationship between height and reading level in elementary school children. This is not because changes in height cause better reading ability. Rather as children get older, they get both taller and improve in their reading skills.*			Not applicable.

Making Inferences and Justifying Conclusions (S-IC)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand and evaluate random processes underlying statistical experi-ments. (M)	M.SP.IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.SP.IC.1	Determine the probability of an event occurring when the outcomes are equally likely to occur.
	M.SP.IC.A.2	Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation). *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	**Calculate accurately and efficiently using simple arithmetic operations. .**	M.EE.SP.IC.2	Determine the probability of an event occurring when the outcomes are equally likely to occur.
B. Make inferences and justify conclusions from sample surveys, experi-ments, and observa-tional studies. (M)	M.SP.IC.B.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.			Not applicable. See M.EE.SP.ID.1.
	M.SP.IC.B.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.			Not applicable. See M.EE.SP.ID.1.
	M.SP.IC.B.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.			Not applicable. See M.EE.SP.ID.1.

Making Inferences and Justifying Conclusions (S-IC)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Make inferences and justify conclusions from sample surveys, experi- ments, and observa- tional studies. (M)	M.SP.IC.B.6	Evaluate reports based on data.			Not applicable. See M.EE.SP.ID.1.

Conditional Probability and the Rules of Probability (S-CP)

Wisconsin Standards for Mathematics			**Wisconsin Essential Elements for Mathematics**		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand indepen-dence and conditional probability and use them to interpret data. (M)	M.SP.CP.A.1 (F2Y)	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.S.CP.1	Identify when events are independent or dependent.
	M.SP.CP.A.2 (F2Y)	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.S.CP.2	Identify when events are independent or dependent.
	M.SP.CP.A.3 (F2Y)	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.S.CP.3	Identify when events are independent or dependent.

Conditional Probability and the Rules of Probability (S-CP)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Understand indepen-dence and conditional probability and use them to interpret data. (M) (cont'd)	M.SP.CP.A.4 (F2Y)	Represent data from two categorical variables using two-way frequency tables and/or Venn diagrams. Interpret the representation when two categories are associated with each object being classified. Use the representation as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.S.CP.4	Identify when events are independent or dependent.
	M.SP.CP.A.5 (F2Y)	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*	**Calculate accurately and efficiently using simple arithmetic operations.**	M.EE.S.CP.5	Identify when events are independent or dependent.

Conditional Probability and the Rules of Probability (S-CP) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	M.SP.CP.B.6 (F2Y)	Use a representation such as a two-way table or Venn diagram to find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.			Not applicable. See M.EE. SP.IC.1.
	M.SP.CP.B.7 (F2Y)	Use a representation such as a two-way table or Venn diagram to apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model.			Not applicable. See M.EE. SP.IC.1.
	M.SP.CP.B.8	(+) Use a representation such as a tree diagram to apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B) and interpret the answer in terms of the model.			Not applicable.
	M.SP.CP.B.9	(+) Use permutations and combinations to compute probabilities of compound events and solve problems.			Not applicable.

Using Probability to Make Decisions (S-MD)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
A. Calculate expected values and use them to solve problems. (M)	M.SP.MD.A.1	(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.			Not applicable.
	M.SP.MD.A.2	(+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.			Not applicable.
	M.SP.MD.A.3	(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*			Not applicable.
	M.SP.MD.A.4	(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*			Not applicable.

Using Probability to Make Decisions (S-MD) (cont'd)

Wisconsin Standards for Mathematics			Wisconsin Essential Elements for Mathematics		
Cluster Statement	**Notation**	**Standard**	**DLM Conceptual Area**	**Notation**	**Alternate Academic Achievement Standard**
B. Use probability to evaluate outcomes of decisions. (M)	M.SP.MD.B.5	(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*			Not applicable.
	M.SP.MD.B.6	Use probabilities to make fair decisions (e.g., drawing for a party door prize where attendees earn one entry to the drawing for each activity they complete, using an electronic spinner to pick a team spokesperson at random from a group, flip a coin to decide which of two friends gets to choose the movie, using a random number generator to select people to include in a sample for an experiment).			Not applicable.
	M.SP.MD.B.7	Analyze decisions and strategies using probability concepts (e.g., balancing expected gains and risk, medical product testing, choosing an investment option, deciding when to kick an extra point vs. two-point conversion after a touchdown in football).			Not applicable.

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Appendix I
Tables

“There’s a moment in problem solving...when you...classify a problem and make connections between this problem and the catalog of problems you’ve tackled in the past” (Su 2020, 36).

Table 1. Addition and Subtraction Situations

(Adapted from Progressions for the Common Core State Standards 2019, 16-18)

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? *A* + *B* = ?	*A* bunnies were sitting on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first *A* bunnies? *A* + ? = *C*	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? ? + *B* = *C*
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? *C* - *B* = ?	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? C - ? = A	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? ? - *B* = *A*
	Total Unknown	**Both Addends Unknown**	**Addend Unknown**
Put Together/ Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? *A* + *B* = ?	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? *C* = ? + ?	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? *A* + ? = *C* *C* - *A* = ?

NOTE: This table continued on next page.

	Difference Unknown	Bigger Unknown	Smaller Unknown
Additive Compare	"How many more?" version. Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy? "*How many fewer*?" version. Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? A + ? = C C - A = ?	"*More*" version suggests operation. Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have? "*Fewer*" version suggests wrong operation. Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? *A* + *B* = ?	"*Fewer*" version suggests operation. Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? "*More*" suggests wrong operation. Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have? *C* - *B* = ? ? + *B* = *C*

- In each type of problem, shown as a row, any one of the three quantities in the situation can be unknown, leading to the specific problem situations shown in each cell of the row. The table also shows some important language variations which, while mathematically the same, require separate attention.
- Put Together/Take Apart Both Addends Unknown problem situations can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean "makes" or "results in" but always means "is the same amount as." Such problems are not a problem situation with one unknown, as is the Addend Unknown problem situation to the right. These problems are a productive variation with two unknowns that give experience with finding all the decompositions of a number and reflecting on the patterns involved.
- In Put Together/Take Apart Start Unknown problem situations either addend can be unknown. Both variations should be included.
- For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2A. Multiplication and Division Problem Situations

(Adapted from Progressions for the Common Core State Standards 2019, 32)

	A x B = ?	**A x ? = C and C ÷ A = ?**	**? x B = C and C ÷ B = ?**
Equal Groups of Objects	Unknown Product There are *A* bags with *B* plums in each bag. How many plums are there in all?	Group Size Unknown If *C* plums are shared equally into *A* bags, then how many plums will be in each bag?	Number of Groups Unknown If *C* plums are to be packed *B* to a bag, then how many bags are needed?
Arrays of Objects	**Equal Groups Language**		
	Unknown Product The apples in the grocery window are in *A* rows and *B* columns. How many apples are there?	Unknown Factor If *C* apples are arranged into *A* equal rows, how many apples will be in each row?	Unknown Factor If *C* apples are arranged into equal rows of *B* apples, how many rows will there be?
	Rows and Columns Language		
	Unknown Product The apples in the grocery window are in *A* rows and *B* columns. How many apples are there?	Unknown Factor If *C* apples are arranged into an array with *A* rows, how many columns of apples are there?	Unknown Factor If *C* apples are arranged into an array with *B* columns, how many rows are there?

NOTE: This table continued on next page.

Multiplicative Compare	**A > 1**		
	Larger Unknown A blue hat costs *$B*. A red hat costs *A* times as much as the blue hat. How much does the red hat cost?	Smaller Unknown A red hat costs *$C* and that is *A* times as much as a blue hat costs. How much does a blue hat cost?	Multiplier Unknown A red hat costs *$C* and a blue hat costs *$B*. How many times as much does the red hat cost as the blue hat?
	***A* < 1**		
	Smaller Unknown A blue hat costs *$B*. A red hat costs *A* as much as the blue hat. How much does the red hat cost?	Larger Unknown A red hat costs *$C* and that is *A* of the cost of a blue hat. How much does a blue hat cost?	Multiplier Unknown A red hat costs *$C* and a blue hat costs *$B*. What fraction of the cost of the blue hat is the cost of the red hat?

- Equal Groups problems can also be stated in terms of columns, exchanging the order of A and B, so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?
- In the row and column situations, the number of groups and group size are not distinguished.
- Division problems of the form A x ? = C are about finding an unknown multiplicand. For Equal Groups and Compare situations, these involve what is called the sharing, partitive, how-many-in-each-group, or what-is-the-unit interpretation of division. Array situations can be seen as Equal Groups situations, thus, also as examples of the sharing interpretation of division for problems about finding an unknown multiplicand.
- Division problems of the form ? x B = C are about finding an unknown multiplier. For Equal Groups and Compare situations, these involve what is called the measurement, quotitive, how-many-groups, or how-many-units interpretation of division. Array situations can be seen as Equal Groups situations, thus, also as examples of the measurement interpretation of division for problems about finding an unknown multiplier.

Table 2B. Multiplication and Division - Measurement Examples

(Adapted from Progressions for the Common Core State Standards 2019, 103)

	A x B = ?	**A x ? = C and C ÷ A = ?**	**? x B = C and C ÷ B = ?**
Grouped Objects (Units of Units)	You need *A* lengths of string, each *B* inches long. How much string will you need altogether?	You have *C* inches of string, which you will cut into *A* equal pieces. How long will each piece of string be?	You have *C* inches of string, which you will cut into pieces that are *B* inches long. How many pieces of string will you have?
Arrays of Objects (Spatial Structuring)	What is the area of a *A* cm by *B* cm rectangle?	A rectangle has area *C* square centimeters. If one side is *A* cm long, how long is a side next to it?	A rectangle has area *C* square centimeters. If one side is *B* cm long, how long is a side next to it?
Multiplicative Compare	A rubber band is *B* cm long. How long will the rubber band be when it is stretched to be *A* times as long?	A rubber band is stretched to be *C* cm long and that is *A* times as long as it was at first. How long was the rubber band at first?	A rubber band was *B* cm long at first. Now it is stretched to be *C* cm long. How many times as long is the rubber band now as it was at first?

- In the second column, division problems of the form *A* x *? = C* are about finding an unknown multiplicand. For Grouped and Compare situations, these involve what is called the sharing, partitive, how-many-in-each-group, or what-is-the-unit interpretation of division. Array situations can be seen as Equal Groups situations, thus, the Array situations in this column can also be seen as examples of the sharing interpretation of division.

- In the third column, division problems of the form *?* x *B = C* are about finding an unknown multiplier. For Equal Groups and Compare situations, these involve what is called the measurement, quotitive, how-many-groups, or how-many-units interpretation of division. Array situations can be seen as Equal Groups situations, thus, the Array situations in this column can also be seen as examples of the measurement interpretation of division.

Table 3. The Properties of Operations.

Here a, b, and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	*(a + b) + c = a + (b + c)*
Commutative property of addition	*a + b = b + a*
Additive identity property of 0	*a + 0 = 0 + a = a*
Existence of additive inverses	For every *a* there exists -*a* so that *a + (-a) = (-a) + a = 0*
Associative property of multiplication	*(a x b) x c = a x (b x c)*
Commutative property of multiplication	*a x b = b x a*
Multiplicative identity property of 1	*a x 1 = 1 x a = a*
Existence of multiplicative inverses	For every *a ≠ 0* there exists *1/a* so that *a x 1/a = 1/a x a = 1*
Distributive property of multiplication over addition	*a x (b + c) = a x b + a x c*

Table 4. The Properties of Equality.

Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a - c = b - c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then *b* may be substituted for *a* in any expression containing *a*.

Table 5. The Properties of Inequality.

Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$
If $a > b$, then $b < a$
If $a > b$, then $-a < b$
If $a > b$, then $a \pm c > b \pm c$
If $a > b$ and $c > 0$, then $a \times c > b \times c$
If $a > b$ and $c < 0$, then $a \times c < b \times c$
If $a > b$ and $c > 0$, then $a \div c > b \div c$
If $a > b$ and $c < 0$, then $a \div c < b \div c$

Appendix 2

Glossary

“Children build their own “working definitions” based on their initial experiences … the concepts will become more precise, and the vocabulary with which we name the concepts will, accordingly, carry more precise meanings. Formal definitions generally come last” (Education Development Center 2020).

Appendix 2. Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Area model. A rectangular shaped tool for understanding problems that involve multiplicative reasoning. An area model may be used to develop conceptual understanding, promote sense making, and build connections between topics.

Arithmetic sequence. A sequence of numbers in which the difference between consecutive terms is always the same.

Associative property of addition. See Table 3 in Appendix 1.

Associative property of multiplication. See Table 3 in Appendix 1.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.

Cardinality. The last number word said when counting tells how many objects have been counted.

Commutative property. See Table 3 in the Appendix.

Complex fraction. A fraction *A/B* where *A* and/or *B* are fractions (*B* nonzero).

Compose. To put together (e.g., numbers and shapes).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Conceptual subitizing. Recognizing that a collection of objects is composed of subcollections and quickly combining their cardinalities to find the cardinality of the collection (e.g., seeing a set as two subsets of cardinality 2 and saying "four").

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven". There are eleven books now."

Decompose. To take apart (e.g., numbers and shapes).

Dot plot. *See*: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Efficiently. In a way that uses strategic thinking to carry out a computation or apply procedures.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6. *See also:* median, third quartile, interquartile range.

Flexibly. In a way that can use, explain, and justify multiple strategies and be adept at choosing a meaningful strategy for the computation or procedure in a problem.

Fraction. A number expressible in the form *a/b* where *a* is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Function. Describes a situation in which one quantity is determined by another.

Function notation. Formal notation for a function that builds in the correspondence between the domain or set of input values and the range or set of output values. For example, *f* (5) is shorthand for "the output value of *f* when the input value is 5".

Geometric sequence. A sequence of numbers in which each term after the first is found by multiplying the previous one by the same value such that the ratio between consecutive terms is constant.

Hierarchical inclusion. Each whole number represents one more than the previous number in the counting sequence and includes all previous numbers within it.

Identity property of 0. See Table 3 in Appendix 1.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or -*a* for some whole number *a*.

Interquartile range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.

Linear function. Functions in the form $y = mx + b$, where m and b are constants.

Mathematical modeling. A process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.

Mathematize. To treat or regard mathematically.

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[4] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Measurement division. Division when finding the number of groups (an unknown multiplier) and of the form *? x B = C* . This type of division can also be referred to as quotitive, how-many-groups, or how-many-units interpretation of division. Also called quotitive division.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 x 4/3 = 4/3 x 3/4 = 1.

Number conservation. Understanding that the quantity of a set doesn't change if the set is rearranged.

Number line. Used to represent numbers and support reasoning about them.

One to one correspondence. Saying number words in correspondence with the objects counted.

Partitive division. Division when finding the size of each group (an unknown multiplicand) and of the form *A* x ? = *C*. This type of division can also be referred to as the sharing, how-many-in-each-group, or what-is-the-unit interpretation of division. *See also* sharing division.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Perceptual subitizing. Quickly recognizing the cardinalities of small groups without having to count the objects.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of equality. See Table 4 in Appendix 1.

Properties of inequality. See Table 5 in Appendix 1.

Properties of operations. See Table 3 in Appendix 1.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Quotitive division. Division when finding the number of groups (an unknown multiplier) and of the form *? x B = C*. This type of division can also be referred to as the measurement, how-many-groups, or how-many-units interpretation of division. Also called measurement division.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction *a/b*. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Recursive rule. A rule that continually takes a previous number and changes it to get the next number.

Rekenrek. Developed by mathematics education researchers at the Freudenthal Institute in the Netherlands. A visual model comprised of two strings of ten beads each, strategically broken into two groups: five red beads, and five white beads. Also known as an arithmetic rack or math rack.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.

Sharing division. Division when finding the size of each group (an unknown multiplicand) and of the form *A* x ? = *C*. This type of division can also be referred to as partitive, how-many-in-each-group, or what-is-the-unit interpretation of division. Also called partitive division.

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object *B*, and the length of object *B* is greater than the length of object C, then the length of object *A* is greater than the length of object *C*. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line, or area model.

Whole numbers. The numbers 0, 1, 2, 3, …

Appendix 3

Essential Elements by Conceptual Area

"Developing deep understanding of mathematics is a major goal of equity-based mathematics teaching practices" (Aguirre, Martin, and Mayfield-Ingram 2013, 43).

Essential Elements by Conceptual Area: Understand number structures.

M.EE.K.CC.1	Starting with one, count to 10 by ones.
M.EE.K.CC.4	Connect counting to cardinality to 10. a. Demonstrate one-to-one correspondence, pairing each object with one and only one number and each number with one and only one object (one-to-one correspondence). b. Understand that the last number name said tells the number of objects counted (cardinality).
M.EE.K.CC.6	Count out up to three objects from a larger set, pairing each object with one and only one number name to tell how many.
M.EE.1.NBT.1	Know the count sequence and understand numbers and quantities. a. Count by ones to 30. b. Count up to 10 objects and represent the quantity with the corresponding numeral.
M.EE.2.NBT.2	Extend understanding of numbers and quantities. a. Count to answer "how many?" questions about as many as 30 things. b. Given a number from 1-30, count out that many objects. c. Name the next number in a sequence between 1 and 10.
M.EE.2.NBT.3	Identify numerals 1 to 30.
M.EE.3.NBT.1	Use decade numbers (10, 20, 30) as benchmarks to demonstrate understanding of place value for numbers 0-30.
M.EE.3.NBT.2	Demonstrate understanding of place value to tens
M.EE.3.NBT.3	Count by tens using models (e.g., objects, base ten blocks, money).
M.EE.3.NF.1-3	Differentiate a fractional part from a whole.
M.EE.4.NF.1-2	Identify models of one half (½) and one fourth (¼).
M.EE.4.NF.3	Differentiate between whole and half.
M.EE.5.NF.1	Identify models of halves (½, 2/2), thirds (⅓, ⅔, 3/3). and fourths (¼, 2/4, ¾, 4/4).
M.EE.5.NF.2	Identify models of thirds (⅓, ⅔, 3/3) and tenths (1/10, 2/10, 3/10, 4/10, 5/10, 6/10, 7/10, 8/10, 9/10, 10/10).
M.EE.6.RP.1	Demonstrate a ratio relationship expressed in its simplest form.
M.EE.7.RP.1-3	Use a ratio to model or describe a relationship.
M.EE.7.NS.2c-d	Apply and extend previous understandings of multiplication, division, and fractions. c-d. Express a fraction with a denominator of 10 as a decimal.
M.EE.8.NS.2	Represent and compare fractions and decimal fractions to hundredths. a. Express a fraction with a denominator of 100 as a decimal.

Essential Elements by Conceptual Area: Compare, compose, and decompose numbers and sets.

M.EE.K.CC.7	Identify whether the number of objects in one group is greater than (more) or less than (when the quantities are clearly different) or equal to the number objects in another group.
M.EE.1.NBT.2	Create sets of 10.
M.EE.1.NBT.3	Compare two groups of 10 or fewer items when the number of items in each group is similar.
M.EE.1.NBT.4	Compose quantities less than or equal to five in more than one way.
M.EE.1.NBT.6	Decompose quantities less than or equal to five in more than one way.
M.EE.2.NBT.1	Represent numbers up to 30 with objects in sets of tens and ones (e.g., ten frames, hundreds chart, columns).
M.EE.2.NBT.4	Compare sets of objects and numbers using accurate vocabulary (e.g., greater, more, less, equal).
M.EE.2.NBT.6	Using concrete models, compose and decompose numbers up to 10 in more than one way.
M.EE.4.NBT.2	Compare whole numbers to 10 using symbols (<,>,=).
M.EE.4.NBT.3	Use place value understanding to generate estimates for real-world addition and subtraction problem situations within 30, using strategies such as mental math, benchmark numbers, compatible numbers, and rounding.
M.EE.5.NBT.1	Compare numbers up to 99 using place value models.
M.EE.5.NBT.2	Use the number of zeros in numbers that are powers of 10 to determine which values are equal, greater than, or less than another number.
M.EE.5.NBT.3	Compare whole numbers up to 100 using symbols (<,>,=).
M.EE.5.NBT.4	Use place value understanding to generate estimates for real-world addition and subtraction problem situations within 100, using strategies such as mental math, benchmark numbers, compatible numbers, and rounding.
M.EE.6.NS.1	Compare the relationships between two unit fractions.
M.EE.6.NS.5-8	Describe quantities having opposite directions or values as positive and negative numbers (e.g., temperature above/below zero).
M.EE.7.NS.3	Using real world examples, compare quantities represented as decimals to tenths.
M.EE.8.NS.2	Represent and compare fractions and decimal fractions to hundredths. b. Using real world examples, compare quantities represented as decimals to hundredths.
M.EE.8.EE.3-4	Compose and decompose whole numbers up to 999.

Essential Elements by Conceptual Area: Calculate accurately and efficiently using simple arithmetic operations.

M.EE.2.NBT.5	Identify the meaning of the "+" sign (including combine, plus, add), "-" sign (including separate, subtract, take), and the "=" sign (including the same amount as, equal).
M.EE.2.NBT.7	Use objects, representations, and numbers (0-20) to add and subtract.
M.EE.3.OA.6	Solve addition and subtraction problems within 20, when the result is unknown.
M.EE.4.NBT.4	Add and subtract two-digit whole numbers.
M.EE.5.NBT.5	Use the meaning of multiplication to develop and understand strategies to find products with multiples of 0, 1, 2, 5, and 10 within 100.
M.EE.5.NBT.6	Illustrate the concept of division using fair and equal shares.
M.EE.5.NBT.7	Identify models of tenths (1/10, 2/10, 3/10, 4/10, 5/10, 6/10, 7/10, 8/10, 9/10, 10/10).
M.EE.6.NS.2	Apply the concept of fair share and equal shares to divide.
M.EE.6.NS.3	Use the meaning of multiplication and the properties of operations (e.g., the distributive property) to develop and understand strategies to find products with multiples of 3, 4, and 9 within 100.
M.EE.7.NS.1	Add fractions with like denominators (halves, thirds, fourths, and tenths) with sums less than or equal to one.
M.EE.7.NS.2 a-b	Apply and extend previous understandings of multiplication, division, and fractions. a. Multiply within 100, using strategies such as the properties of operations [e.g., knowing that 7 x 6 can be thought of as 7 groups of 6 so one could think 5 groups of 6 is 30 and 2 more groups of 6 is 12 and 30 + 12 = 42 (informal use of the distributive property)]. b. Solve division problems within 100, including divisors of 1-5 and 10, without remainders.
M.EE.8.NS.1	Subtract fractions with like denominators, including halves, thirds, fourths, and tenths with minuends less than or equal to one.
M.EE.8.EE.1	Identify and understand the meaning of exponents.
M.EE.N.RN.1	Determine the value of a quantity that is expressed with an exponent.
M.EE.N.CN.2	Use properties (commutative, associative, and distributive) and place value understandings to calculate and solve problems. a. Add, subtract, and multiply whole numbers. b. Add, subtract, and multiply decimals. c. Solve real-world problems involving addition, subtraction, and multiplication of whole numbers and decimals.
M.EE.SP.IC.1-2	Determine the probability of an event occurring when the outcomes are equally likely to occur.
M.EE.SP.CP.1-5	Identify when events are independent or dependent.

Essential Elements by Conceptual Area: Understand and use geometric properties of two- and three-dimensional shapes.

M.EE.K.MD.1-3	Classify objects according to attributes (e.g., big/small, heavy/light).
M.EE.K.G.2-3	Match shapes of the same size and orientation including circles, squares, rectangles, and triangles.
M.EE.1.G.1	Identify the relative position of objects that are on, off, in, and out.
M.EE.1.G.2	Sort shapes of the same size and orientation including circles, squares, rectangles, and triangles.
M.EE.2.G.1	Identify common two-dimensional shapes including squares, circles, triangles, and rectangles of various sizes and in various orientations.
M.EE.3.G.1	Describe attributes of two-dimensional shapes.
M.EE.4.MD.5	Recognize angles in geometric shapes.
M.EE.4.MD.6	Compare angles as larger than, smaller than, or the same size as another angle.
M.EE.4.G.1	Recognize parallel lines and intersecting lines.
M.EE.4.G.2	Describe the defining attributes of two-dimensional shapes.
M.EE.5.MD.3	Identify common three-dimensional shapes.
M.EE.5.G.1-4	Sort two-dimensional figures and identify the attributes, including number of angles, sides, or corners that they have in common.
M.EE.7.G.1	Match two similar geometric shapes that are proportional in size and have the same orientation.
M.EE.7.G.2	Recognize geometric shapes with given conditions.
M.EE.7.G.3	Match a two-dimensional shape with a three-dimensional shape that shares an attribute.
M.EE.7.G.5	Recognize angles that are acute, obtuse, and right.
M.EE.8.G.1	Recognize translations, rotations, and reflections of shapes.
M.EE.8.G.2	Identify shapes that are congruent.
M.EE.8.G.4	Identify similar shapes with and without rotation.
M.EE.8.G.5	Compare any angle to a right angle, and describe the angle as greater than, less than, or congruent to the right angle.
M.EE.G.C..1	Know the attributes of geometric elements and two-dimensional shapes. a. Know the attributes of points, rays, lines, line segments, perpendicular lines, parallel lines, and angles. b. Know the attributes of two-dimensional shapes
M.EE.G.CO.4-5	Given a geometric figure and a rotation, reflection, or translation of that figure, identify the components of the two figures that are congruent.
M.EE.G.CO.6-8	Identify corresponding congruent and similar parts of shapes.
M.EE.G.GMD.4-6	Use properties of geometric shapes to describe real life objects.

Essential Elements by Conceptual Area: Solve problems involving area, perimeter, and volume.

M.EE.1.G.3	Put together two shapes to make a shape that relates to the whole (e.g., two semicircles to make a circle, two squares to make a rectangle).
M.EE.3.G.2	Recognize shapes that have been partitioned into equal areas.
M.EE.4.MD.3	Determine the area of a square or rectangle by counting units of measure (unit squares).
M.EE.4.G.3	Recognize that lines of symmetry partition shapes into equal areas.
M.EE.5.MD.4-5	Determine the volume of a rectangular prism by counting units of measure (unit cubes).
M.EE.6.G.1	Solve real-world and mathematical problems about area using unit squares.
M.EE.6.G.2	Solve real-world and mathematical problems about volume using unit cubes.
M.EE.7.G.4	Determine the perimeter of a rectangle.
M.EE.7.G.6	Determine the area of a rectangle using the formula for length x width, and confirm the result using tiling or partitioning into unit squares.
M.EE.8.G.9	Use the formulas for perimeter and area of rectangles, and volume of rectangular prisms to solve real-world and mathematical problems.
M.EE.G.GPE.7	Find perimeters and areas of squares and rectangles to solve real-world problems.
M.EE.G.GMD.1-2	Estimate the volumes of containers, the areas of figures, and the perimeters of figures. Check the estimates using formulas or models.

Essential Elements by Conceptual Area: Understand and use measurement principles and units of measure.

M.EE.1.MD.1-2	Compare lengths to identify which is longer/shorter or taller/shorter.
M.EE.1.MD.3	Understand concepts of time. a. Demonstrate an understanding of tomorrow, yesterday, and today. b. Demonstrate an understanding of morning, afternoon, day, and night. c. Identify activities that come before, next, and after. d. Demonstrate an understanding that telling time is the same every day.
M.EE.2.MD.1	Measure the length of objects using non-standard units.
M.EE.2.MD.3-4	Order objects by length using non-standard units.
M.EE.2.MD.5	Increase or decrease length by adding or subtracting unit(s).
M.EE.2.MD.6	Use a number line to add one more unit of length.
M.EE.2.MD.7	Identify, on a digital clock, the hour that matches a routine activity.
M.EE.2.MD.8	Recognize that money has value.
M.EE.3.MD.1	Tell time to the hour on a digital clock.
M.EE.3.MD.2	Identify the appropriate measurement tool to solve one-step word problems involving mass and volume.
M.EE.3.MD.4	Measure the length of objects using standard tools including rulers, yardsticks, and meter sticks.
M.EE.4.MD.1	Identify the smaller measurement unit that comprises a larger unit within a measurement system including inches/foot, centimeter/meter, minutes/hour.
M.EE.4.MD.2	Apply concepts of measurement. a. Tell time using a digital clock. Tell time to the nearest hour using an analog clock. b. Measure mass and volume using standard tools. c. Use standard measurement to compare lengths of objects. d. Identify coins including penny, nickel, dime, quarter, and their values.
M.EE.5.MD.1	Extend understanding of measurement concepts. a. Tell time using an analog or digital clock to the half or quarter hour. b. Use standard units to measure weight and length of objects. c. Indicate relative value of collections of coins.
M.EE.N.Q.1-3	Express quantities as a precise measurement based on the context.

Essential Elements by Conceptual Area: Represent and interpret data displays.

M.EE.1.MD.4	Organize objects into categories by sorting (e.g., color, size, shape). Count the number of objects in each category and identify the category with the most objects.
M.EE.2.MD.9-10	Create physical bar graphs from sorted objects (e.g., line up different colored toy cars).
M.EE.3.MD.3	Use picture or bar graphs to answer questions about the data.
M.EE.4.MD.4	Represent and interpret data on a picture or bar graph. given a model and a graph to complete.
M.EE.5.MD.2	Represent and interpret data on a picture graph, line plot, or bar graph.
M.EE.6.SP.1-2, 4	Display data on a graph or table that shows variability in the data.
M.EE.6.SP.5	Summarize data distributions shown in graphs or tables.
M.EE.7.SP.1-2	Answer a data related question, given a model of the data from a student experiment or collection.
M.EE.7.SP.3	Compare two sets of data found within a single data display such as a picture graph, line plot, or bar graph.
M.EE.7.SP.5-7	Describe the probability of events occurring as possible or impossible.
M.EE.8.SP.4	Construct a graph or table from given categorical data and interpret the data.
M.EE.SP.ID.1-2	Given data, construct a graph (including line, pie, bar, or picture) or table and interpret the data.
M.EE.SP.ID.3	Interpret trends on a graph or chart.
M.EE.SP.ID.4	Calculate the mean of a given data set.

Essential Elements by Conceptual Area: Use operations and models to solve problems.

M.EE.K.OA.1	Represent addition as "putting together" or subtraction as "taking from" in everyday activities (e.g., with objects, fingers, sounds, drawings, verbal explanations, or numbers).
M.EE.1.OA.1	Represent addition and subtraction e.g., by using objects, fingers, mental images, drawings, sounds (e.g., claps), or acting out situations.
M.EE.1.OA.2	Use "putting together" to solve problems with two sets.
M.EE.1.OA.5	Relate counting to addition and subtraction. a. Use manipulatives or visual representations to indicate the number that results when adding one more. b. Apply knowledge of "one less" to subtract one from a number.
M.EE.1.OA.7	Recognize two groups that have the same or equal quantity.
M.EE.2.OA.3	Equally distribute even numbers of objects between two groups.
M.EE.2.OA.C.4	Use addition to find the total number of objects arranged within equal groups up to a total of 10.
M.EE.3.OA.1-2	Use repeated addition to find the total number of objects and determine the sum.
M.EE.3.OA.7	Solve one-step real-world problems within 20, using addition or subtraction.
M.EE.4.OA.1-2	Demonstrate the connection between repeated addition and multiplication.
M.EE.4.OA.3	Solve one-step real-world problems using addition or subtraction within 100.
M.EE.4.OA.4	Use an understanding of multiplication to show at least one way to determine a product.
M.EE.6.EE.1-2	Evaluate if an equation is true.
M.EE.6.EE.3	Apply the properties of addition to identify equivalent numerical expressions.
M.EE.6.EE.5-7	Match an equation to a real-world problem in which variables are used to represent numbers.
M.EE.7.EE.1	Use the properties of operations as strategies to demonstrate that expressions are equivalent.
M.EE.7.EE.4	Use the concept of equality to solve one-step addition and subtraction equations with models.
M.EE.8.EE.7	Solve simple algebraic equations with one variable using addition and subtraction.
M. EE.A.SSE.1	Identify an algebraic expression involving one arithmetic operation to represent a real-world problem.
M.EE.A.SSE.3	Solve simple algebraic equations with one variable using multiplication and division.
M.EE.A.CED.1	Determine an equation involving at least one variable that can be used to solve a real-world problem.
M.EE.A.CED.2-4	Solve one-step inequalities.

Essential Elements by Conceptual Area: Understand patterns and functional thinking.

M.EE.3.OA.8	Identify arithmetic patterns.
M.EE.4.OA.5	Use repeating patterns to make predictions.
M.EE.5.OA.3	Identify and extend numerical patterns.
M.EE.7.EE.2	Identify an arithmetic sequence of whole numbers with a whole number common difference.
M.EE.8.EE.2	Identify a geometric sequence of whole numbers with a whole number common ratio.
M.EE.8.EE.5-6	Graph a simple ratio by connecting the origin to a point representing the ratio in the form of y/x. For example, when given a ratio in standard form (2:1), convert to 2/1, and plot the point (1,2).
M.EE.8.F.1-3	Given a table of data for a linear function containing at least 2 complete ordered pairs, identify a missing number that completes another ordered pair.
M.EE.8.F.4	Determine the values or rule of a function using a graph or a table.
M.EE.8.F.5	Describe how a graph represents a relationship between two quantities.
M.EE.A.SSE.4	Determine the successive term in a geometric sequence given the common ratio.
M.EE.A.REI.10-12	Interpret the meaning of a point on the graph of a line. For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.
M.EE.F.IF.1-3	Use the concept of function to solve problems.
M.EE.F.IF.4-6	Construct graphs that represent linear functions with different rates of change and interpret the graphs. For example, which rate is faster/slower or higher/lower.
M.EE.F.BF.1a-b	Select a graph from the first quadrant of the coordinate plane that represents a situation involving constant rate of change.
M.EE.F.BF.2	Determine an arithmetic sequence with whole numbers when provided a recursive rule.
M.EE.F.LE.1-3	Model a simple linear function such as y=mx to show that these functions increase by equal amounts over equal intervals.